Welfare Implications of Endogenous Credit Limits with Bankruptcy

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Abstract

This paper studies the aggregate welfare consequences of changes in the penalties for credit default and in social insurance policies when borrowing limits may respond to these changes. It uses a dynamic general equilibrium model of an exchange economy with incomplete markets. The novel feature is that the borrowing limit is determined in the equilibrium so as to reflect that financial institutions allow for a certain aggregate degree of default and that defaulting agents are excluded from the economy for a fixed period of time. The effect on the stationary equilibrium of an exogenous reduction in the exclusion period is explored numerically. For comparison purposes, the same experiment is carried out under the assumption made in related studies that the borrowing limit is fixed. The welfare gains with endogenous borrowing constraints are appreciably smaller than with exogenous borrowing constraints. The same result holds for an exogenous change in social policy that reduces individual income variability.

Key words: borrowing constraints, incomplete markets, default
JEL: D52, D90, G10, G33

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1 Introduction

A large fraction of US families are liquidity constrained, and estimates point to a substantial effect of credit limits in containing household levels of debt. Thus borrowing constraints seem to be a pervasive feature of financial markets. Recent research shows their existence may be important for a variety of issues in macroeconomics and finance when financial markets are incomplete. But what determines changes in the level of these borrowing constraints? Are these changes important to understand the consequences of policies or other events on the economy?

The aim of this paper is to take a step towards assessing the significance for the economy of the endogenous determination of borrowing constraints. Two objectives are pursued to this end. First, it seeks to setup a theoretical model where the borrowing constraint is determined endogenously. Second, it intends to study within this model the role of the response of the borrowing constraint for the positive and welfare consequences of changes in economic factors and institutions. Attention will be drawn to the institutions or rules that deal with financial default since there is compelling evidence that these factors influence the availability of credit. Social insurance policies – such as unemployment compensation – will also be considered since they presumably have consequences for the choices about borrowing and the repayment of debts.

The paper studies the equilibrium determination of the borrowing constraint in an economy where financial intermediaries target the default rate and where defaulting agents are excluded from the economy for a fixed period of time. The analysis is based on a version of Huggett (1993)’s dynamic competitive general equilibrium model of incomplete markets with idiosyncratic risk. In that model, a bond is the only asset agents can trade in order to partially insure consumption subject to an exogenous borrowing limit. Most importantly, there is full commitment of these individuals in meeting their debts. The present research departs from this assumption on agents’ commitment.

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1 Hall and Mishkin (1982) and Jappelli (1990) measure the incidence of liquidity constraints. Gross and Souleles (2002) and Cox and Jappelli (1993) estimate the effect or removing credit limits on levels of debt.
3 Gropp, Scholz, and White (1996) find that state personal bankruptcy exemptions have a significant, positive effect on the probability that households will be turned down for credit or discouraged from borrowing. Berkowitz and White (2002) show that supply of credit falls when non-corporate firms are located in states with higher bankruptcy exemptions.
Under these circumstances, borrowing limits arise endogenously to reflect the penalties inflicted upon defaulting and the rate of default targeted by financial intermediaries. Concerning the former, the punishment to a borrower for repudiating her debts consists of the exclusion from future trades for a certain period of time. On the latter, financial institutions set a certain target for the fraction of current loans that will fail to be honored, i.e., the default rate. The default rate and the length of the period of forced autarky are seen as arising from social norms or institutions that are exogenous parameters to the analysis. Clearly the analysis must introduce the existence of default as an individual choice and its implications in general equilibrium. The equilibrium borrowing constraint is determined so that certain participation conditions consistent with the default rate are satisfied.

The model is calibrated to grossly match U.S. observations, including default rates and interest rates. The exploration of alternative parametric settings uncovers important properties of the model relating to existence, uniqueness, and convergence to a stationary distribution. The effect on the stationary equilibrium of exogenous changes in two parameters of the model are investigated numerically. The first is a reduction in the exclusion period inflicted on a defaulting individual. When this prescribed punishment for default is eased, banks tighten up the borrowing limit, and welfare decreases, the fall in interest rates notwithstanding. The same experiment is repeated but holding constant the borrowing constraint and adding some (conservative) restrictions on the conditions required to declare bankruptcy. In this case, there is a welfare gain as individuals will be more inclined to smooth consumption in bad states even if this is at the expense of a higher exposure to default situations. Quantitatively, with an exogenous borrowing constraint the welfare gain is notoriously larger than when the borrowing limit is endogenous. The second exogenous change is a mean-preserving reduction in the variability of individual income realizations. It leads to a tighter borrowing limit which washes out much of the risk-sharing welfare benefits impact of such a policy. Just as before, it is also true that if the borrowing constraint were instead to be held exogenously fixed then the default level would rise but welfare would show a much larger improvement.

There are papers with endogenous borrowing constraints in incomplete-markets economies with individual risk. Zhang (1997) also derives the constraint based on the threat of exclusion from trade if default takes place but, unlike the present paper, it is set to the level that precisely prevents any default in equilibrium and the exclusion is permanent. Therefore that model, consisting of two types of agents only, cannot address the questions dealt with here. It focuses on effects of preference parameters on the risk-free interest rate. In

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4 U.S. law allows credit bureaus to report past bankruptcies up to ten years old. Musto (1999) finds this 'bankruptcy flag' has a big effect on credit access.
Chatterjee et al. (2002) credit limits are also determined endogenously. That paper studies contracts whose price depends on the loan size and the borrowers’ observable type. Default in equilibrium arises largely because agents experience idiosyncratic shocks to preferences, although productivity shocks also occur. The present paper makes instead the simplifying assumption that there is a single type of contract offered to all borrowers irrespective of their individual types or contract size, and default arises because the borrowing constraint is loose enough to permit borrowing to a point where agents may be unwilling to repay their high debts depending on the realization of an individual productivity shock. On the other hand, their model of a small-open economy sets exogenously the risk-free interest rate. The present paper determines the interest rate in equilibrium and can therefore assess general equilibrium effects which prove to be important to understand the significance of endogenous borrowing constraint. Their paper analyzes the consequences of proposed changes in the US bankruptcy law that can be related to the experiments in the present paper.

The welfare impact of changes in personal bankruptcy law is also the subject of Athreya (2002) and Li and Sarte (2002). These models share many fundamental characteristics with the one used here, except for their assumption of exogenous borrowing constraints. Therefore they cannot account for the effect implied by the response of borrowing constraints which is the subject of the present paper. These papers run experiments which are analogous to the experiment with exogenous borrowing constraint also considered in the present paper for comparison purposes. Our results indicate that those works might be overestimating the gains from bankruptcy law reform. On the other hand, the role of social insurance policies in the presence of default risk considered here do not appear to have been studied in other papers.

All the cited papers take market incompleteness as given. Yet there is an important literature that studies the endogenous determination of trade restrictions in otherwise complete-market models. These include Kehoe and Levine (2001), Alvarez and Jermann (2000), and Krueger (2000). Other papers study long-term optimal contracts. In Cooley at al. (2002) the credit limits prescribed by recursive contracts have implications for firms’ investment. Quadrini (2002) studies optimal contracts between a firm and a investor which involve default.

Section 2 lays down the model and the equilibrium concept, and discusses some of its novel aspects. Section 3 describes the choice of numerical benchmark and characterizes the properties of the associated equilibrium. Section 4 contains the numerical experiments. Section 5 concludes.
2 The model and equilibrium

This paper studies an exchange economy with incomplete markets where borrowing constraints emerge to reflect the rules that deal with default in credit markets. The section sets out the model first, and then defines the equilibrium.

2.1 The model

There is a continuum of agents of total mass equal to one. Preferences are defined over stochastic processes for consumption, \( c_t \), and represented by the utility function

\[
E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \text{where } \beta \in (0, 1), \ \sigma > 1.
\]

Each period each agent receives an endowment \( y \) of a perishable good. The set of possible endowments is \( Y = \{y_1, y_2\} \) with \( y_1 > y_2 \). The individual endowment follows a Markov process with stationary transition probabilities \( \pi(y'|y) \) for \( y', y \in Y \).

An agent may or may not be permitted to borrow or lend. Let \( z_t \) denote the number of periods before the agent is given access to markets as of time \( t \). It takes values in the set \( Z = \{0, 1, 2, ..., T\} \). When \( z_t = 0 \) the agent can trade a one-period bond or credit balance. Let \( b_{t+1} \) denote the individual amount of bonds held between period \( t \) and the next, and \( q_t \) the price of one such bond. There is a credit limit or borrowing constraint \( b \) so that \( b_{t+1} \) belongs in \([b, \bar{b}]\) where \( \bar{b} \) is some non-binding upper bound. An agent with \( z_t = 0 \) may decide not to repay her negative credit balances at any time by choosing the default variable \( d_t = 1 \), or otherwise stay in trade by choosing \( d_t = 0 \). In the former case, the agent will be excluded from trade for \( T \) periods as \( z_{t+s} = T \) and \( z_{t+s} > 0 \) for \( s = 1, ..., T \). Agents with \( z_t > 0 \) are excluded from trade and cannot hold any credit balances. This exclusion penalty \( T \) is assumed fixed by the law.

Trade in bonds or credit balances takes place through competitive banks or financial intermediaries. Banks operate as a clearing house which takes deposits from agents holding positive balances and lends to agents on short positions. It is assumed that banks do not screen individual borrowers’ types even though some types may be more likely to default than others. The conditions of credit, including the interest charged \( 1/q - 1 \) and the borrowing limit \( b \), are thus the same for all agents. This financial setting resembles today’s securitized mortgage markets or securitized credit card markets where buyers of
the asset receive a pro-rata share of all different sellers’ deliveries. The price $q$ of bonds is determined competitively so that the supply and demand for credit are equalized. The level of the credit limit $b$ is determined by financial intermediaries so that the proportion of repudiated debts over outstanding liabilities, or the rate of default, coincides with some exogenously set target $\lambda$. This is a simplifying rule of behavior which will nonetheless be useful to highlight the differences with the opposite view of an exogenous borrowing constraint and an endogenous default rate which is taken in other papers such as Athreya (2002). A minimal rationality condition will be imposed on the behavior of banks though, which is that any loan made must have some positive probability of being honoured. This calls for a credit limit which is tight enough to rule out loans of a size which fail regardless of the borrower’s situation when the repayment is due.

Free-entry in financial intermediation means that banks balance their books. Because of the presence of default there is a fraction of positive credit balances that will not be honored and banks have to charge this loss to depositors. Therefore, agents with positive balances know the fraction $\lambda$ of their assets will be irrecoverable. Individual agents take the default rate, the price of assets, and borrowing limit as given.

### 2.2 Equilibrium

One will study equilibrium situations where the price of bonds and the credit limit are constant over time. The individual state space is $S \equiv B \times Y \times Z$ with elements $s = (b, y, z) \in S$ and $\mathcal{A}_S$ its Borel $\sigma$-algebra. The aggregate state then consists of a probability measure $\Phi$ over $S$ that describe the distribution of agent types. In a stationary equilibrium this distribution must be constant. A stationary equilibrium can be formulated recursively. Given $\lambda$, $T$, and the rest of parameters, an equilibrium is a probability measure $\Phi$ on the measurable space $(S, \mathcal{A}_S)$, a price of bonds $q$, a credit limit $b$, a value function $v(\ldots)$, and decision rules for bonds $b'(\ldots)$ and defaulting $d(\ldots)$ such that:

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5. This *anonymity* is characteristic of the the research on bankruptcy in general equilibrium in Dubey, Geanakoplos and Shubik (2000). It is also used in other papers like Li and Sarte (2002), and Athreya (2002). In Chatterjee et al. (2002) contracts are instead type and size-contingent.

6. Existing literature, such as Alvarez and Jermann (2000) and Kehoe and Levine (2001), has considered the particular case where contracts are fully enforced, i.e. $\lambda = 0$. The possibility of positive default contemplated in the present paper, besides being characteristic of actual data, is no more arbitrary than that of exactly zero default. A more detailed model of the banking sector consistent with positive default is not the object of this paper.
• Given \( q \) and \( b \), the functions \( b'(\cdot,\cdot,\cdot), \ d(\cdot,\cdot,\cdot) \) and \( v(\cdot,\cdot,\cdot) \) solve the problem

\[
v(b, y, z) = \max_{b',d} u(c) + \beta \sum_{y' \in Y} \pi(y'|y) v(b', y', z')
\]

s.t. \( b' \in [b, \bar{b}], \ d \in \{0, 1\} \)

\[
c + qb' = y + (1 - d)(b - \max\{0, \lambda b\})
\]

\[
b' = 0 \text{ if } z > 0, \text{ or } z = 0 \text{ and } d = 1
\]

\[
z' = \begin{cases} 
  z \text{ if } z = 0 \text{ and } d = 0 \\
  T \text{ if } z = 0 \text{ and } d = 1 \\
  z - 1 \text{ if } z > 0
\end{cases}
\]

• Market clearing:

\[
\int_S b'(b, y, z) d\Phi = 0.
\]

• Stationary distribution:

\[
\Phi(A) = \int_S Q(s, A) d\Phi \text{ for } A \in A_S,
\]

with \( Q : S \times A_S \to [0, 1] \) being the transition function derived from the decision rules \( b' \) and \( d \), and the transition probabilities \( \pi(y'|y) \).

• Default rate is consistent with banks’ target:

\[
\int_S bd(s) d\Phi = \lambda \int_S \min\{0, b\} d\Phi.
\]

• Ruling out sure default: there exists some \( y \in Y \) such that \( d(b, y, 0) = 0 \) for all \( b \in [b, \bar{b}] \).

The fourth condition simply states that the equilibrium must produce a default rate which is consistent with the banks’ targets. The last condition goes to ensure that banks only make loans that have a certain positive probability of repayment.

In this definition the state space has three variables, including \( z \). However the high dimensionality is only apparent since for any \( z \neq 0 \) the household is in autarky and her decision problem is trivial. An equilibrium can in effect, be written in a more manageable form by regarding it as a situation where a certain participation constraint holds with equality for a certain type of agents in \( S \). Let \( S_0 \subset S \) denote the set of types that can trade (\( z = 0 \)) and who do not default (\( d(s) = 0 \)). With this notation, the equilibrium can be characterized as follows. For given \( T \) and \( \lambda \), an equilibrium consists of \( \bar{b}, q \), and \( S_0 \), and \( \Phi \), \( v(\cdot,\cdot,0) \), and \( b'(\cdot,\cdot,0) \) such that:
Given $b$, $S_0$, and $q$, for each type $(b, y, 0) \in S_0$,

$$v(b, y, 0) = \max_{b' \in [b, \bar{b}]} u(c) + \beta \left[ \sum_{y' : s' \in S_0} \pi(y'|y)v(b', y', 0) + \sum_{y' : s' \notin S_0} \pi(y'|y)v^{AU}(y') \right]$$

s.t. $c + qb' = y + b - \max\{0, \lambda b\}$.

where

$$v^{AU}(y) \equiv E \left[ \sum_{t=0}^{T-1} \beta^t u(y_t) + \beta^T v(0, y_T, 0) \mid y_0 = y \right].$$

(ii) Market clearing:

$$\int_{S_0} b'(b, y, z)d\Phi = 0.$$

(iii) Stationary distribution:

$$\Phi(A) = \int_S Q(s, A)d\Phi \text{ for } A \in A_S.$$

(iv) Participation constraint: For and only for $s \in S_0$

$$v(b, y, 0) \geq v^{AU}(y).$$

(v) Default rate:

$$\int_{S - S_0} bd\Phi = \lambda \int_S \min\{0, b\} d\Phi.$$

(vi) Ruling out sure default: there exists some $y \in Y$ such that $(b, y, 0) \in S_0$ for all $b \in [\underline{b}, \bar{b}]$.

The advantage of this definition over the original one is that the consumer’s problem becomes simpler as it has to be solved only for agents that do not currently default. The defaulting choice has been replaced by the determination of such a set of agents $S_0$ through the participation constraint. Note that the value of exclusion is well defined since $v(0, y, 0) \geq v^{AU}(y)$ holds.

2.3 Characterization

The notable features of this equilibrium are the determination of the non-default set $S_0$, point (iv), and the borrowing constraint $\bar{b}$, points (v) and (vi). First, one can gain some insight as to how the set $S_0$ is shaped. To bring this more clearly, condition value and policy functions by the underlying $\underline{b}$ and $S_0$. Following this convention, the participation condition $v(b, y, 0 \mid \underline{b}, S_0) - v^{AU}(y \mid b, S_0)$ is increasing in $b$. For an agent with income $y \in \{y_1, y_2\}$, one can define $b(y)$ as the value of $b \in [\underline{b}, \bar{b}]$ where this expression becomes non-negative.
Clearly, $b(y) < 0$ since $v(0, y, 0 \mid \underline{b}, S_0) - v^{AU}(y \mid \underline{b}, S_0) > 0$. Given the pair $(b(y_1), b(y_2))$, the no-default set $S_0$ is given by points to the right of this for every $y$:

$$S_0 = S_0(b(y_1), b(y_2)) \equiv \{ s \in S : z = 0, b \geq b(y), \forall y \in Y \}.$$ 

Defaulting occurs among agents with high debts. Clearly $S_0$ influences $b(y)$’s which in turn determine the set $S_0$. It will prove useful to define the participation constraint for an agent with income $y_i$ for $i \in \{1, 2\}$ as

$$\text{PART}_i(b(y_i)) \equiv v(b(y_i), y, 0 \mid \underline{b}, S_0(b(y_1), b(y_2))) - v^{AU}(y_i \mid \underline{b}, S_0(b(y_1), b(y_2))).$$

With this notation, it is straightforward to establish that in equilibrium $b(y_i)$ must be such that $\text{PART}_i(b(y_i))$ becomes non-negative, for $i = 1$ and 2 respectively. Thus in order to find the equilibrium default set one needs to understand how the $b(y)$’s influence the participation constraints. For a fixed $y_i$, there are two effects of changes in $b(y_i)$ on the participation condition $\text{PART}_i(b(y_i))$. First, there is the direct impact on the first argument of the value function for participation $v(\ldots, 0 \mid \ldots)$. The sign of this effect must be positive. Second, there is an indirect effect through the changes in the no-default set, $S_0$, which is an argument of both the value of participation and the value of defaulting $v^{AU}(\ldots \mid \ldots)$. The sign of this effect may well depend on the specific circumstances. On the other hand, changes in the default threshold for other income levels $b(y_j)$ for $j \neq i$, will also affect the participation condition for income $y_i$ via $S_0$ in ways which are hard to establish at this level of generality. One important question is the uniqueness of the pair $(b(y_1), b(y_2))$. These issues will be addressed below within specific numerical settings.

Second, turning to the determination of the borrowing constraint, a given $\underline{b}$ implies a certain equilibrium default rate. In equilibrium the value $\underline{b}$ is determined so that such a default rate equals the value $\lambda$ targeted by the banks according to point (v) in the definition. Observe that a change in $\underline{b}$ will in general bring about a variety of direct as well as general-equilibrium indirect effects on the default rate that are difficult to sort at this stage. The direct effect will tend to lead to more default the looser is the borrowing constraint, i.e., the smaller $\underline{b}$. The indirect effects will work through the adjustment of the interest rate, and shifts in the distribution of population across levels of wealth and bankruptcy status.

Without further conditions, the equilibrium could be a situation where some loans are made which bear a probability 1 of default. Point (vi) in the definition requires that $\underline{b}$ be tight enough that this type of situations are ruled out. Given the discussion on the determination of $S_0$ above, this means that in equilibrium it must be true that $b(y_i) = \underline{b}$ for at least one $i \in \{1, 2\}$, that is there is an income type which will never default or all default will be done by agents in a certain income category. Notice that since this condition will generally limit
the extent of default, the existence of an equilibrium \( \hat{b} \) might fail if the target default rate \( \lambda \) is relatively high.\(^7\)

A general result on the existence and uniqueness of a stationary equilibrium distribution is not provided in this paper. Note that the results in Huggett (1993) do not guarantee existence of and convergence to such a solution in this case because of the ongoing flows of exit from and entry to the market by agents. One will thus rely largely on computational results. In the numerical explorations carried out in this research, convergence to the stationary distribution is not generally a problematic issue. However, convergence has been found to fail sometimes for given off-equilibrium interest rates and borrowing limits where the equilibrium condition (vi) does not hold, i.e., when some default happens in the two income groups.

The iterative procedure for computing an equilibrium is divided in two main steps: (1) Guess a value for \( \hat{b} \), and compute the equilibrium, i.e. \( S_0, q, v(\ldots), b'(\ldots,0), \) and \( \Phi \); (2) Verify the equilibrium is consistent with the default rate \( \lambda \), or update \( \hat{b} \) and start back in step 1. Step 1 is similar to solving Huggett (1993)'s model except for the fact that some types of agents may default so the set \( S_0 \) must be determined (in Huggett (1993) \( S_0 = S \)). This requires an extra round of iterations. The specific steps are as follows: (a) Fix a \( q \); (b) Initialize \( S_0 = S \); (c) Solve \( v(\ldots,0) \) and \( b'(\ldots,0) \); (d) Check that \( v(b,y,0|b) - v^{AU}(y|b) \geq 0 \) iff \( (b,y,0) \in S_0 \). Update \( S_0 \) and go to c; (e) Compute \( \Phi \); (f) Check market clearing. Update \( q \) and back to point b. The equilibria in this paper are situations where the procedure converges to a stationary distribution \( \Phi \). The appendix contains details on the the implementation of the computations.

3 The benchmark model

The equilibrium implications of this model will be analyzed numerically. Benchmark values for the parameters have to be selected. This section presents this choice and describes the properties of the corresponding equilibrium.

The parameters of the model related to the process of individual income are \( (y_1,y_2), \pi(y_1 | y_1), \) and \( \pi(y_2 | y_2) \). They will take on the values based on Kydland (1984) and used in Huggett(1993) to parameterize an economy where one period corresponds to two months and individual risk is associated with changes in unemployment status in the U.S. The upper bound on assets \( b \) is set

\(^7\) An earlier version of the paper studied equilibria without the restriction imposed by point (vi). The possibility of borrowing to never repay made the default rate too sensitive to changes in parameters, at times leading to exceedingly high default rates and causing severe existence problems.
so that it never binds. The target default rate \( \lambda \), and the exclusion period \( T \) will be selected so that they look reasonable attending to evidence on default rates and the penalties for defaulting. An annual default rate around 5-6 percent is close to the average behavior for corporate bonds and personal unsecured loans, largely credit card lines.\(^8\) There is little firm evidence about the period of exclusion after default. Following Athreya (2002), it will be chosen to lie in the region of 4 years.

The two remaining parameters are the elasticity of substitution \( \sigma \), and the discount rate \( \beta \). They will be set so that two requirements are met. First, the equilibrium must be consistent with evidence on interest rates. In the model the default risk causes a spread \( \lambda \) between the borrowing rate and the lending rate. Hereafter the bimonthly borrowing rate \( 1/q - 1 \) will be denoted by \( r \). In annual terms, it should lie between the 5 percent S&P Index average return and a 15 per cent on credit cards. Second, an equilibrium value of \( b \) must exist which leads to a default rate which matches \( \lambda \) and where agents of a certain income level \( y \) never default \( b(y) = b \) (i.e. points (v) and (vi) in the definition).

### Table 1. Benchmark Model

| Parameters:  | \( \pi(y_1|y_1) \) | \( \pi(y_2|y_2) \) | \( y_1 \) | \( y_2 \) | \( \beta \) | \( \sigma \) | \( T \) | \( \lambda \) | \( \bar{b} \) |
|--------------|-----------------|-----------------|---------|---------|---------|---------|------|--------|------|
| \( \pi(y_1|y_1) \) | 0.925 | 0.50 | 1.00 | 0.10 | 0.990 | 1.08 | 25 | 0.008 | 4.00 |

<table>
<thead>
<tr>
<th>Equilibrium:</th>
<th>( r )</th>
<th>( \bar{b} )</th>
<th>( b(y_1) )</th>
<th>( b(y_2) )</th>
<th>( j_1 )</th>
<th>( j_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>0.0181</td>
<td>-4.10</td>
<td>-4.10</td>
<td>-4.022</td>
<td>1</td>
<td>14</td>
</tr>
</tbody>
</table>

Extensive experimentation shows that these two requirements can be met only if the parameters in question belong in a narrow region of values as follows: \( (\sigma, \beta) \in [1.08, 1.15] \times [0.9895, 0.992] \). It turns out that \( \beta \) appears to have a major negative effect on the default rate, and that \( \sigma \) has a negative impact on the market-clearing interest rate. In general, however, for a given \( \sigma \), a value of \( \beta \) that is consistent with the conditions on equilibrium default may fail to exist. In particular, if \( \sigma \) is large, a calibration must necessarily involve a relatively large \( \beta \) to meet the target \( \lambda \), but then it may be too large to keep default at zero for one income type. For relatively small values of \( \sigma \), consistent \( \beta \)'s can be found over a broader range of values. However, the small \( \sigma \) may imply an interest rate which is too large. Among the various choices for \( \sigma \) and \( \beta \) that

\(^8\) Moody’s KMV historical default report documents that over the post-1970 period, default rates and average default losses have reached averages of 6.47 percent with lower rating categories and a 3.33 percent for the single B category (see http://riskcalc.moodysrms.com/us/research/defrate.asp). The Federal Reserve Board releases charge-off and delinquency annual rates on unsecured credit card loans around 5 per cent since 1996 (see http://www.federalreserve.gov/releases) .
look reasonable, the one made here implies an annual interest of 10.0 percent (i.e., \( r = 0.0181 \)) which lies in the middle range of observed rates. The choice of benchmark is displayed in the top section of Table 1 below.\(^9\)

The key endogenous variables are the borrowing constraint \( b \), the interest rate \( r \), the default levels of assets \( b(y_1) \) and \( b(y_2) \) (or \( S_0 \)), along with the wealth distribution \( \Phi \), and the individual asset policy function \( b' \). Attention will be drawn first to the equilibrium determination of the default decisions and the borrowing constraint displayed in the bottom part of Table 1. It also includes the interest rate \( r \), and the symbols \( j_1 \) and \( j_2 \) which denote the indexes in the grid corresponding to the default levels of assets.

A feature of the equilibrium is that a high realization of income, \( y_1 \), will never lead to default, and all default is incurred by agents who hit the low-income state, \( y_2 \), and are deep enough into debt, \( b < b(y_2) \). To have a visual

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\(^9\) Calibrations of the individual income process based on Heaton and Lucas (1996) are also commonly used. Compared to the present setting, such a calibration would imply a narrower spread and more symmetric persistence: \( y_1 = 1.1, y_2 = 0.66, \pi(y_1|y_1) = \pi(y_2|y_2) = 0.945 \). However, in the present model a calibration for the rest of parameters that meets all the targets does not appear to exist for this income process. One can conjecture the trouble is that the rich and the poor (income wise) are too similar and so must be their default behavior. This makes it hard to find configurations matching the target default \( \lambda \) with default by only one income type.
impression of who defaults, Figure 1 depicts the cumulative distribution over asset levels for low-income individuals in the equilibrium. Default happens in a very narrow region in the low tail of the distribution which is yet sufficient to produce an empirically plausible default rate of about 5 per cent per year.

In terms of the notation set out earlier, \( b(y_1) = b \) since \( PART_1(b) > 0 \), and \( b(y_2) > b \) where \( PART_2(b(y_2)) = 0 \) since \( PART_2(b) < 0 \). Given the equilibrium \( b \) and \( r \), the solid curves in Figure 2 depict the properties of the participation condition \( PART_i(b(y_i)) \) for both income levels \( i = 1, 2 \). Points labeled 'equil' are associated with the equilibrium decisions on \( b(y_1) \) and \( b(y_2) \) respectively, which are consistent with the characterization developed in Section 2.2.

![Equilibrium S0](image)

This graphic also shows the typical downward discontinuity of the curve corresponding to the high-income state at \( b(y_1) = b(y_2) \) and the existence of a zero at \( b(y_1) > b(y_2) \) indicated as 'non-equil'. For the given \( b \) and \( r \) of the benchmark equilibrium, this may give way to the existence of another set \( S_0 \) characterized by the property that, unlike in the benchmark equilibrium, high-income individuals are more inclined to default in the sense that \( b(y_1) > b(y_2) \). Specifically, this alternative default set exists and is characterized by the two threshold values \( (b(y_1), b(y_2)) = (-3.82, -4.01) \). Nonetheless, this type of situation where \( b(y_1) > b(y_2) \) never arises when \( b \) and \( r \) adjust to re-establish an equilibrium under this type of default rules. Supporting arguments are provided in the next subsection.

Because the multiplicity of \( S_0 \) for given \( b \) and \( r \) is driven by the discontinuity in the participation schedule for high income, it is worth studying the reason for this downward jump. It turns out this happens when \( b(y_1) \) reaches \( b(y_2) \) from below. It helps describing the shift in individual decisions when \( b(y_1) \) changes exogenously near \( b(y_2) \). To do so, consider an individual who in a certain

---

10 Since this set of choices involves a \( b(y_1) \) larger than in the benchmark, this also leads to a downward shift of the positive section of \( PART_2 \).
period \( t \) has high income, \( y_1 \), and a bond position which coincides with the default level mark, \( b_t = b(y_1) \). Start with a situation where \( b(y_1) < b(y_2) \). At \( t \) she chooses a value of bond holdings for next period \( t + 1 \), \( b_{t+1} = b'(b, y_1, 0) \), such that she will continue participating in \( t + 1 \), i.e., \( b(y_1) < b(y_2) < b_{t+1} \). If in period \( t + 1 \) income is low, she will choose a level of bonds for period \( t + 2 \), \( b_{t+2} = b'(b_{t+1}, y_2, 0) \), which will ensure participation in period \( t + 2 \) if income is high in that period but will lead to default if income turns out to be low, i.e., \( b(y_1) \leq b_{t+2} < b(y_2) \). This is the behavior which is optimal when \( b(y_1) \) falls slightly short of \( b(y_2) \). What if \( b(y_1) \) rises and the situation is reversed? Now with \( b(y_1) > b(y_2) \) the previous optimal contingent plan for \( b_{t+2} \) is clearly no longer feasible. If the agent wants to choose a \( b_{t+2} \) which will lead to default with a bad realization of income she will have to give up participation if income turns out to be high as well, or else she will have to set \( b_{t+2} \) so default will never occur. This narrows severely the agent’s set of choices and creates the drop in the value function depicted in Figure 2. More specifically, the optimal reaction will be for the agent to let go the option to participate in \( t + 2 \) by borrowing to the limit in \( t + 1 \), \( b_{t+1} = \underline{b} \). But then, it also becomes optimal to do so right in the initial period \( t \) by choosing \( b_{t+1} = \underline{b} \).

### 3.2 Borrowing constraint

The values in Table 1 correspond to an equilibrium in the sense that a larger \( \underline{b} \) (i.e., tighter constraint) reduces the default rate below the target \( \lambda \), eventually leading to zero default when \( b(y_1) = b(y_2) = \underline{b} \). Conversely, a lower \( \overline{b} \) (i.e., looser constraint) will lead to a higher default rate, involving from a certain point not only low-income but also high-income individuals. The equilibrium would not exist if, in order to raise equilibrium default rate up to the value of \( \lambda \), one such a loose constraint were needed. By construction, in the calibrated model this is not an issue.

The discussion in the preceding subsection suggests that more than one equilibrium could exist, perhaps involving default taking place in the high-income state. This notwithstanding, on the basis of extensive experimentation, it should not be a cause of concern in practice. To examine this, Figure 3 represents the equilibrium default rate as a function of \( \underline{b} \). An equilibrium corresponds to a value where the default rate implied by the model coincides with the fixed value \( \lambda \). For a given level of the credit limit there may be two default rates implied, each associated with one of the two different types of default rule. For more extreme values of the borrowing limit, only one type of default rate.

\[ \text{Since the default threshold } b(y) \text{ is computed on the coarse grid, numerical calculations may in some cases show apparent discontinuities in the equilibrium default rate.} \]
rule will exist. The solid line draws the default rate under a default rule with \( b(y_1) < b(y_2) \), and the dashed line under a rule with \( b(y_1) \geq b(y_2) \). Although the two lines may exist for an intermediate range of values, the only equilibrium consistent with the target default rate \( \lambda \) is of the former type which corresponds to the benchmark equilibrium at hand. In addition, the other type of rule \( b(y_1) \geq b(y_2) \) invariably fails to satisfy the equilibrium condition required in this case that \( b(y_2) = b \).\(^{12}\)

\[
\begin{align*}
\text{Default rate} & \\
\lambda & \\
b & \text{multiple } S_0
\end{align*}
\]

Figure 3. Equilibrium \( b \).

4 Numerical experiments

This section studies changes in the exogenous parameter accounting for the time penalty on default, \( T \), and in the relative value of individual income realizations, \( y_1 \) and \( y_2 \).

4.1 Bankruptcy penalty

The exercise with \( T \) is interesting as it can be related to provisions set out in the bankruptcy law and the practices followed by financial institutions.

The first type of experiment traces the response of the endogenous variables to such a change within the equilibrium concept considered so far. The endogenous variables then include the borrowing constraint and the default values of debt \( b(y) \), but the default rate \( \lambda \) is exogenous. In the second type of experiment, the borrowing constraint is instead held fixed at its benchmark levels and the default values of debt \( b(y) \) are constrained by bankruptcy law not

\(^{12}\) In cases where there is default in the two income groups the computation may fail to converge to a stationary equilibrium distribution.
to exceed the values determined in the benchmark equilibrium. Under these conditions, the default rate $\lambda$ becomes an endogenous variable. This latter experiment resembles in its assumptions the analysis in Athreya (2002) and Li and Sarte (2002). The purpose of this type of experiment is to gain an appreciation of the importance of the response of borrowing constraints and default by comparison with the first type of experiments.

Welfare $W$ is measured as the expected value function, $v(s)$, over assets $b$, income $y$, and all credit status $z$ according to the following:

$$W = \int v(b, y, z) d\Phi.$$ 

This is a measure of ex-ante welfare. In Table 2 below, the last column 'welfare' represents the percentage change in $W$ in equivalent consumption units relative to the corresponding benchmark, which is calculated as $(W/W^B)^{1/(1-\sigma)} - 1$, where $W^B$ is welfare in the benchmark equilibrium.

### Table 2. Effects of $T$

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\lambda$</th>
<th>$b$</th>
<th>$r%$</th>
<th>$b(y_1)$</th>
<th>$b(y_2)$</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark equilibrium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.008</td>
<td>-4.100</td>
<td>1.805</td>
<td>-4.100</td>
<td>-4.022</td>
<td>-</td>
</tr>
<tr>
<td><strong>Endogenous $b$ and $b(y)$’s, exogenous $\lambda$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.008</td>
<td>-2.820</td>
<td>1.515</td>
<td>-2.820</td>
<td>-2.807</td>
<td>-0.0382</td>
</tr>
<tr>
<td>17</td>
<td>0.008</td>
<td>-2.397</td>
<td>1.20</td>
<td>-2.397</td>
<td>-2.394</td>
<td>-0.0467</td>
</tr>
<tr>
<td><strong>Exogenous $b$, restricted $b(y)$’s, endogenous $\lambda$</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.016</td>
<td>-4.100</td>
<td>2.23</td>
<td>-4.100</td>
<td>-4.022</td>
<td>+0.1606</td>
</tr>
<tr>
<td>17</td>
<td>0.025</td>
<td>-4.100</td>
<td>2.805</td>
<td>-4.100</td>
<td>-4.022</td>
<td>+0.2860</td>
</tr>
</tbody>
</table>

Right under the first row reproducing the benchmark equilibrium, the second block of Table 2 reproduces figures corresponding to the equilibrium for a shorter exclusion period $T$ when the borrowing constraint is endogenous. As the exclusion cost upon defaulting declines, individuals have a stronger reason to default and the response of the banks is to tighten credit conditions to prevent that from happening. More constraining credit opportunities also drive the market clearing interest rate downwards as there is less opportunity for indebtedness. The welfare consequences of these changes are a decline in equivalent consumption units between 4 and 5 per cent. In sum, easing the penalty for default leads to more restrictive credit conditions which prevail in reducing total welfare in the economy.
What if instead there were no adjustment of borrowing limits and the default rate were let to reflect the changing financial choices of individuals? To look at this type of situation, the borrowing limit is held fixed and the default rate $\lambda$ will adjust. We will restrict default decisions somehow so that an individual with income $y$ is not permitted to default if her level of assets exceeds the threshold value $b(y)$ determined in the benchmark equilibrium. This restriction could be regarded as the provision for debt discharge under the typical bankruptcy legislation. Since a reduction in the cost of default is being considered one should expect this condition to become a binding one and, therefore, the ensuing welfare effects will provide a conservative assessment of the welfare gains from a lower $T$. The actual results are displayed in the bottom section of Table 2. It strikes the eye the large welfare gain that follows, of 16 and 28 per cent of consumption equivalent units. The increase in the lending interest rate, $r$, and the decline in the borrowing or deposit rate, $r - \lambda$, from about 0.010 down to near 0.006 and 0.003 respectively, would tend to reduce welfare. Hence this sizable increase in welfare must be entirely attributed to the shifts in default behavior that lurk behind the rise in the default rate $\lambda$. In effect, reducing the cost of defaulting leads more agents to borrow closer to the default region thus benefiting from increased risk-sharing.\textsuperscript{13}

In sum, the exercise reported here shows that the welfare gains of easing the cost of defaulting are considerably smaller if the borrowing constraint adjusts to keep the default rate under check than if this constraint remains instead constant and higher default follows. This result appears to be robust across alternative calibrations.

### 4.2 Social insurance

The exercise with income dispersion consists of a mean-preserving change in the spread between $y_1$ and $y_2$, which can be related to a change in social insurance policy deployed through taxes and transfers across individuals in the two income groups. In this crude formulation, every individual participates in this scheme irrespective of her bankruptcy status.

As in the previous subsection, two types of experiments are run. The first experiment has the borrowing limit and default levels as endogenous variables, while holding the default rate as exogenous. In the second experiment the borrowing limit is instead fixed and the rate of default becomes endogenous. Table 3 displays the response of the endogenous variables to mean-preserving reductions in the spread of income realizations. The welfare figures represent

\textsuperscript{13} The welfare calculations do include the agents that have defaulted and are excluded from markets. The contribution of this group to aggregate welfare is minimal and, in any event, tends to narrow the difference between the two experiments.
changes in equivalent consumption units calculated as in the previous subsection.

Table 3. Effects of social insurance

<table>
<thead>
<tr>
<th></th>
<th>y_1</th>
<th>y_2</th>
<th>λ</th>
<th>( \hat{b} )</th>
<th>r%</th>
<th>b(y_1)</th>
<th>b(y_2)</th>
<th>welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark equilibrium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>.10</td>
<td>0.008</td>
<td>-4.100</td>
<td>1.805</td>
<td>-4.100</td>
<td>-4.022</td>
<td>–</td>
</tr>
<tr>
<td><strong>Endogenous ( \hat{b} ) and b(y)'s, exogenous λ</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.99</td>
<td>.17</td>
<td>0.008</td>
<td>-2.84</td>
<td>1.780</td>
<td>-2.84</td>
<td>-2.834</td>
<td>-0.014</td>
</tr>
<tr>
<td></td>
<td>.985</td>
<td>.20</td>
<td>0.008</td>
<td>-2.64</td>
<td>1.940</td>
<td>-2.64</td>
<td>-2.634</td>
<td>+0.021</td>
</tr>
<tr>
<td><strong>Exogenous ( \hat{b} ), restricted b(y)'s, endogenous λ</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.99</td>
<td>.17</td>
<td>0.022</td>
<td>-4.100</td>
<td>2.774</td>
<td>-4.100</td>
<td>-4.022</td>
<td>+0.442</td>
</tr>
<tr>
<td></td>
<td>.985</td>
<td>.20</td>
<td>0.034</td>
<td>-4.100</td>
<td>3.540</td>
<td>-4.100</td>
<td>-4.022</td>
<td>+0.773</td>
</tr>
</tbody>
</table>

The figures on the first row of Table 3 correspond to the benchmark equilibrium. The second block reproduces figures corresponding to the equilibrium for narrower income spreads when the borrowing constraint is endogenous. It turns out that the reduction of individual risk variability makes individuals more inclined to default since the temporal exclusion from financial markets now becomes less costly in terms of risk sharing. The response of the banks is to tighten credit conditions to prevent that from happening. Two forces with opposite sign influence the interest rate: the more restrictive credit opportunities push the interest rate down, however more social insurance drives the market clearing interest rate up as the demand for (precautionary) saving declines. The net effect is relatively small adjustment of the interest rate. The welfare consequences of these changes are modest variation of -1.4 and +2.1 per cent in equivalent consumption units. In sum, lower individual income variability leads to more restrictive credit conditions which nearly cancel out the direct social insurance gains.

The same changes in income spreads will be considered in the environment with fixed borrowing limits and endogenous default rate in the way described in the previous subsection. The actual results are displayed in the bottom section of Table 3. Large welfare gains follow, of 44 and 77 per cent equivalent consumption units. The increase in the lending interest rate, \( r \), and the decline in the borrowing or deposit rate, \( r - \lambda \) would tend to reduce welfare. Hence this welfare change must be attributed to the direct social-insurance impact which is reinforced by a wider resort to debt repudiation as reflected in higher default rates \( \lambda \).
In sum, this exercise demonstrates that the welfare gains from social insurance program are considerably smaller if the borrowing constraint adjusts to keep the default rate under check than if this constraint remains instead constant and higher default follows.

5 Final remarks

This paper develops a model of an exchange economy with incomplete markets and idiosyncratic risk where the borrowing constraint is determined in equilibrium to meet a certain default rate on loans. One contribution of the paper is to formulate and solve such a model, and characterize some of its equilibrium properties. The same setup can be used to accommodate situations where the borrowing constraint is instead fixed and it is the rate of default that is determined in equilibrium. The model is used to study the welfare impact of, first, a reduced period of exclusion from the markets which penalizes bankruptcy or default, and, second, social insurance. The conclusion depends a great deal on whether the borrowing constraint is endogenous. If the borrowing constraint is exogenous one may be grossly overstating the welfare gains from such a change. This result suggests that the assessment of certain policies may critically depend on the endogeneity of the credit limits.

This result is based on a very stylized model of the economy and financial institutions though. A model of a production economy with capital and which could account for the finer details of actual bankruptcy laws would be a necessary extension to establish more firmly the practical implications of the conclusions reached here.

On the other hand, the results indicate that holding either default rates or borrowing constraints exogenous may prove misleading. Developing and using a quantitative theory where both variables are jointly determined – possibly along the lines of Chatterjee et al. (2002) – should therefore be one focus of further research.

A Details on computation

The transition function $Q$ can be constructed as

$$Q(s, A) = \lambda(\{u : g(s, u) \in A\}) \text{ for } A \in \mathcal{A}_s$$

where $g : S \times U \rightarrow S$ and $u$ is a random variable defined on a probability measure space $(U, \mathcal{U}, \lambda)$ with $g(s, u) = (g_1(s, u), g_2(s, u), g_3(s, u))$ where:
\( g_1(s, u) = b'(s) \)
\( g_2(s, u) = y_1 \) if \( s_2 = y_1, u \in (0, \pi(y_1|y_1)) \) or \( s_2 = y_2, u \in (0, \pi(y_1|y_2)) \) 
\( = y_2 \) if \( s_2 = y_1, u \in (\pi(y_1|y_1), 1) \) or \( s_2 = y_2, u \in (\pi(y_1|y_2), 1) \)
\( g_3(s, u) = 0 \) if \( s_3 = 0 \) and \( d(s) = 0 \)
\( = T \) if \( s_3 = 0 \) and \( d(s) = 1 \)
\( = s_3 - 1 \) if \( s_3 > 0 \)

**Discretization:** The state space \( S \) is discretized to calculate \( \Phi(s) \) as derived from the optimal decision rules \( b'(s) \) and \( d(s) \) (or \( S_0 \)) and the transition probabilities \( \pi(y'|y) \). I let \( b \in \{b_1, ..., b_j, ..., b_{j_{\text{max}}} \} \) and define

\[ S_0 = \{(b_{j(y)}, y, 0), ..., (b_{j_{\text{max}}}, y, 0)| y \in \{y_1, y_2\}\}, \]

where \( b_{j(y)} \) is the smaller value on the grid that is no smaller than the default threshold \( b(y) \) defined above. The case with zero default \( S_0 = S \) is equivalent to \( j(y_1) = j(y_2) = 1 \). The equilibrium \( b \) will be the one point on the discrete grid that implies a default rate closest to \( \lambda \). In general, this will not hit \( \lambda \) with arbitrary accuracy. The grid for bonds has 101 unevenly spaced elements with an upper bound \( \bar{b} = 4 \) (there are 303 points in the finer grid used to approximate the stationary distribution). The points on the grid \( b_k \) for \( k = 1, ..., N \) are determined as \( b_1 = \bar{b} \), and for \( k > 1 \),

\[ b_k = \bar{b} + (\bar{b} - b) \frac{k^{2.35}}{N^{2.35}}. \]

**Distribution:** On this discreet setup, the law of motion for the distribution is given by

\[ \Phi'(s') = \sum_s Q(s, s') \Phi(s), \]

where \( Q(., .) \) is the transition function. The natural way to compute the stationary distribution is to iterate this equation until convergence. It is useful to distinguish different cases:

- If \( z' = 0 \):

\[ \Phi'(b', y', 0) = \begin{cases} 
\sum_y \pi(y'|y) \Phi(b^{-1}(b', y), y, 0) & b' < 0 \\
\sum_y \pi(y'|y) \Phi(b^{-1}(b', y), y, 0) + \sum_y \pi(y'|y) \Phi(0, y, 1) & b' \geq 0 
\end{cases} \]

The \( b^{-1}(b', y) \) is the value bonds such that, given income \( y \), leads to a choice \( b' \). If \( b' > b'(b_{j_{\text{max}}}, y, 0) \) then \( \Phi(b^{-1}(b', y), y, 0) = \Phi(b_{\text{max}}, y, 0) \); on the other hand, if \( b' < b'(b_{j(y)}, y, 0) \) then \( \Phi(b^{-1}(b', y), y, 0) = 0 \).
• If \( z' \in \{ T - 1, T - 2, \ldots, 2, 1 \} \):

\[
\Phi'(b', y', z') = \begin{cases} 
0 & b' < 0 \\
\sum_y \pi(y'|y)\Phi(0, y, z' + 1) & b' \geq 0
\end{cases}
\]

• If \( z' = T \):

\[
\Phi'(b', y', T) = \begin{cases} 
0 & b' < 0 \\
\sum_y \pi(y'|y)\Phi(b_{y(y)}, y, 0) & b' \geq 0
\end{cases}
\]

In a stationary distribution, the mass of agents that participate and that are excluded will be constant. In the computations this condition is imposed in each step by normalizing the mass of agents within the economy to unity. This device improves the convergence properties of the procedure.

**Default:** The optimal defaulting decisions is given by a pair \((b(y_1), b(y_2))\). To calculate this, one has to search over pairs \((b(y_1), b(y_2))\) (i.e., in the computation \((j(y_1), j(y_2))\)). One potential difficulty is that both values affect the participation constraint of the two income levels at the same time. The procedure adopted here consists of searching for a pair where the two participation values become non-negative. To speed things along, the region of search is first narrowed as follows. Take an initial \(b(y_1) = b\). Then check the two participation values at \(b(y_2) = b\). If both are positive this is the pair sought, otherwise evaluate the participation value in the low-income state \(PART_2, y_2\), at \(b(y_2) = 0\). If it is negative then fix a new larger \(b(y_1)\) and evaluate the participation values at \(b(y_2) = b\) and start again. Otherwise, search for the first \(b(y_2) \in [b, 0]\) such that participation in the low-income state becomes positive. Check it is also positive for the high-income state, otherwise fix a new larger \(b(y_1)\) and evaluate the participation values at \(b(y_2) = b\) and start again. Note these calculations are done on the coarse grid.

The possibility of multiple \(S_0\) is handled as follows. Use the above iterative procedure starting with \(b(y_1) = b\). If it leads to \(b(y_1) > b(y_2)\) the pair found characterizes the unique \(S_0\). If it leads to \(b(y_1) \leq b(y_2)\), this pair may or may not be the unique \(S_0\). To check existence of type-A default, use iterative procedure of the previous paragraph but imposing \(b(y_1) \geq b(y_2)\). Such a type of equilibrium does not exist if either it does not reach an end or it delivers \(b(y_1) = b(y_2)\).

**References**

litical Economy, 103, 6, 1158-75.


