Long swings in Japan’s current account and in the yen∗

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December 19, 2003

Abstract

The yen has experienced several big swings over recent decades. Overall, the currency has appreciated substantially. This paper argues that the fluctuations of the Japanese exchange rate were mainly the result of corresponding movements in the current account, which affected the demand for yen relative to other currencies. The paper investigates a vector error correction model for the nominal exchange rate and the current account, based on the idea that the exchange rate and its economic fundamental do not move too far apart over time. In addition, the model allows for a Markov-switching stochastic trend in the current account. Regime changes occur at uncertain dates whenever large changes in the value of the yen, or other economic developments, provoke a current account reversal. Bayesian estimation proceeds using a Gibbs-sampling procedure. The empirical results suggest that recurrent structural breaks in the yen’s fundamentals account for the large fluctuations of the Japanese exchange rate.

JEL classification: F31, F32, C32, C11, C15

Keywords: Japanese exchange rate; current account; exchange rate fundamental; Markov-switching; cointegration; Gibbs-sampling; purchasing power parity puzzle

∗I thank Danny Quah for very helpful advice. I further thank Charles Goodhart, Peter Kenen, Nobuhiro Kiyotaki, Richard K. Lyons, Wojciech S. Maliszewski, Alexander Michealides, Ellen Meade, Bob Nobay and Frank Smets for their comments. I am grateful for suggestions from seminar participants at the Economic Research Department of the ECB, the International Financial Stability Programme at the CEP (LSE), the 17th Annual Congress of the EEA in Venice and the Conference on Exchange Rates of the Applied Econometrics Association (AEA) in March 2003 in Marseille.

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1 Introduction

Along with its rise in the post-war period to one of the world’s largest economies, Japan has experienced a sustained appreciation of its currency, the yen. Less often noticed—but just as remarkable—is the fact that the yen’s value has fluctuated widely over the years, both in nominal and real terms, since it started to float in the early 1970s.

**Exchange rate fluctuations.** Some numbers help to illustrate the magnitudes involved. Between 1976Q4 and 1978Q3 for instance, that is in less than two years, Japan’s (quarterly) nominal effective exchange rate appreciated by 45%. It subsequently depreciated by 24% in the period through 1980Q1 but in the following year the loss in value was more than offset when the yen appreciated again, this time by 25%. What followed was a short period of relative exchange rate tranquility, despite a strong and rising current account surplus. Compared to other industrial countries, Japan liberalized its capital markets at a rather late stage. In the first half of the 1980s, the opening of its financial account induced strong capital outflows, which helped to contain the yen’s strength for a while. Once the initial wave of capital outflows was over, however, the nominal exchange rate shot up by 61% between 1985Q3 and 1988Q4 (39% in the year from 1985Q3 through 1986Q3 alone). After reaching a peak, the currency first weakened and then recovered, to return to almost the 1988 level in 1992. Yet after 1992Q3, the yen’s trade-weighted value rose once again by 52% in the period through 1995Q2. It then depreciated by 35% through 1998Q3, only to appreciate again by 40% in the following two years.

The sizeable and prolonged swings of Japan’s nominal exchange rate translated into very similar movements of the real exchange rate throughout the floating period. The yen appreciated less in real terms than in nominal terms, however. From 1980Q1 to 2000Q1, Japan’s annual inflation rate has stayed 1.7% below the weighted inflation rates of its trading partners, with little variation. By contrast, Japan’s nominal effective exchange rate rose 4.5% per year on average, implying a substantial real appreciation of the yen over the years. Clearly, purchasing power parity has been the exception rather than the rule in Japan.

What explains these massive exchange rate movements? This paper takes the view that conventional theories of exchange rate determination cannot explain the yen’s performance in a satisfactory way. First, the actual fluctuations of the Japanese currency seem simply too large for many models. For instance, it seems difficult to think that interest rate parity, or monetary models of the exchange rate, could explain the double-digit percentage movements of the yen that often occurred within a matter of months. On the other hand, there are other, productivity-based theories that can potentially explain the long-term appreciation of the yen; yet they
have practically nothing to say about the episodes during which the yen depreciated (Engel, 1999). The Balassa-Samuelson effect is one important example, but the criticism extends to theories that link net foreign asset accumulation to real exchange rate levels.

**Trade and capital flows.** This paper tells a different story. It argues that it was in the first place the Japanese current account that caused the large fluctuations of the Japanese currency over the past three decades as well as its sustained appreciation. In this interpretation, Japan’s large surpluses in the 1980s and 1990s influenced the demand for yen relative to other currencies and helped to strengthen the currency. The variability of the current account thus implied large changes in the currency’s value. The interpretation seems to be confirmed by Brooks, Edison, Kumar and Słoś (2001) who find that the yen exchange rate has remained closely tied to the current account over recent years; portfolio flows appear to have been less relevant for the yen-dollar exchange rate than, say, for the euro-dollar exchange rate. The argument is moreover closely related to the traditional flow market model of the foreign exchange market (see, for example, Mussa, 1979)—which is also sometimes being referred to as the balance of payments flow approach to the exchange rate (Rosenberg, 1996)—and appears consistent with findings in the recent literature on exchange rate microstructure (Evans and Lyons, 2002).

Yet capital flows, too, mattered in Japan. In particular, the large purchases of foreign debt securities by Japan as a result of its large export surpluses tended to ease the pressure on the yen and to allow for a more gradual and delayed adjustment of the exchange rate to the movements of the current account.

The paper seeks to illustrate these economic relationships using a nonlinear multivariate time series framework. The model consists of the current account and the nominal effective exchange rate. While being individually nonstationary, the two variables turn out to be cointegrated. A vector error correction model is therefore regarded as an appropriate framework for modelling their joint dynamics. However, the model is modified in an important respect in that the intercept of the current account equation is allowed to switch between two states, or regimes, according to an unobservable Markov process. This is to take account of the recurrent structural breaks in the current account variable, which is upward-trending in certain periods and downward-trending in others.

The empirical specification appears to be a natural choice upon visual inspection of the Japanese data. Long swings in Japan’s current account appear to have induced similar long swings in the country’s exchange rate, at least over the last three decades. But the setup also emphasizes the theoretical idea that exchange rate fundamentals change their dynamic pattern occasionally and that exchange rate fluctuations reflect these structural breaks.
Outline. The paper is organized as follows. Section 2 explains why current account fluctuations help to explain the performance of the yen. Section 3 sets out the empirical model. Section 4 describes how Gibbs-sampling can be applied for Bayesian inference on the model. Section 5 discusses the empirical results. Section 6 provides conclusions.

2 Shifts in the demand for yen

2.1 Time series evidence

Figure 1: Japanese current account and exchange rate (1980s and 1990s). Japanese current account (left scale, in trillions of yen) and nominal effective exchange rate (right scale, in logarithms), period from 1977Q1 to 2001Q1. Source: International Financial Statistics (IMF).

This paper holds that the strong desire of foreigners to purchase goods and services from Japan is key for understanding the yen’s performance in the recent past. Let us begin by looking at some time series evidence. Figure 1 plots Japan’s current account and nominal effective exchange rate for the period from 1977Q1 to 2001Q4. As in the rest of this paper, the nominal exchange rate is defined as the foreign-currency price of the domestic currency, that is, a rise in the nominal exchange rate implies an appreciation of the domestic currency. One can observe that the current account went through four big swings. The nominal exchange rate followed these movements quite closely. It similarly experienced large, protracted swings which seem related to those of the current account. The statistical part of this paper will show that both variables are indeed cointegrated over this period.

The relationship is less clear only after 1981, when the yen suddenly weakened
for several quarters. At that time, the current account was rising. As mentioned in the introduction, large capital outflows occurred at that time due to the liberalization of Japan’s capital account. (Note that the US dollar experienced a sharp appreciation during the pre-1985 period even after US interest rates had fallen from their record levels of the early 1980s.)

The exchange rate movements seem to follow the current account movements with a substantial lag. Sometimes, the turning points of both time series coincide approximately. At other times, however, the corresponding peaks and troughs lie more than two years apart. Another way to look at it is to note that the yen appreciated most at times when the current account was in strong surplus. 1978, 1986, 1992 and 1998—and 1971, as we are about to see—were the years in which the current account reached its peak; and these were also the dates at which the yen’s value increased most dramatically.

Quarterly data is available only from 1977. Before that year, data exists for both variables only at a biannual frequency. Consider Figure 2, which again plots the same variables as Figure 1 this time for the period from 1970 until the end of 1979, allowing for a little time overlap in both plots. Figure 2 shows another swing of the current account in the first half of the 1970s, with a corresponding up-and-down movement of the exchange rate. Again, one can observe a short lag between both variables. As in Figure 1, the yen appreciates very strongly at a time when the current account reaches a temporary peak, namely in the years 1971 and 1972.
2.2 High demand for yen

2.2.1 The current account

Trade surpluses. What has caused the sustained appreciation of the yen and its apparent long swings? What kind of role does the current account play? The answer given in this paper is that the demand for yen has been fluctuating over the years, along with the varying performance of Japanese exports. Over time, the current account has provided a good indicator of the changing desire of economic agents to obtain and spend Japanese currency.

An illustration at the level of actual current account transactions may be helpful. Suppose for example that a Japanese firm exports cars to another firm in the United States. This is recorded as a credit in the Japanese current account, or more specifically in its trade balance. If payment is made in yen, then the US importer, or her bank, has to obtain yen in order to pay for the goods. If the price of the cars is denominated in US dollars, then it is likely that the Japanese exporter, or her bank, will convert the dollars she receives into yen. In any event, the demand for yen is likely to rise relative to that for US dollars.

A surplus item on the Japanese services balance can similarly lead to an increase in the demand for yen, for instance if foreigners have to pay interest on debt that they owe to Japanese banks. If a Japanese worker working abroad sends money to his family at home, this would be recorded in the transfer balance and lead to a credit in the current account. It likewise implies a conversion of foreign currency into yen.

The argument that the current account matters for the exchange rate reminds of the recent attempts in the microstructure literature on exchange rates to link exchange rate movements to order flows in the foreign exchange market (Evans and Lyons, 2002; Evans and Lyons, 2003). It is also closely related to the balance of payments (BOP) flow approach to exchange rate determination in macroeconomics (see for example chapter 12 in Kenen, 1996). For a long time, the conventional way of analyzing exchange rate behaviour was to monitor the flow supplies of, and demands for, foreign currencies in the foreign exchange markets (Rosenberg, 1996). However, the BOP flow approach lost much of its appeal to academic economists in the 1970s when more stock-oriented exchange rate theories came into vogue. Rosenberg (1996, page 69) has pointed out, however, that despite the sceptical view of academics, “most market participants today probably still rely on some variant of the BOP flow approach in their analysis of exchange rate movements and in their formulation of international investment strategy”.

1As a matter of fact, Japan’s trade with the United States is often dollar-denominated whereas trade with its Asian trading partners usually takes place in yen.
Japan’s lending abroad. A puzzling feature of the time series plotted in Figures 1 and 2 is that the nominal exchange rate responds to the current account movements with an often substantial lag. What are the exchange rate lags due to? According to the view described above, current account movements imply changing flows of foreign exchange. However, the cash flow may not always occur immediately. First, current account transactions may lead to deferred payments, for example when goods are shipped abroad and the foreign importer does not have to pay for them immediately, due to a trade credit for instance. Second, and more importantly, a country-wide current account surplus may induce banks and other economic agents to lend more extensively abroad. The additional supply of yen in the foreign exchange markets helps to contain part of the exchange rate appreciation. However, to the extent that the exchange rate does not appreciate right away, it may do so at a later stage when loans received by foreigners from Japanese agents have to be repaid—say, after a few months or even years. The consequence is that a current account imbalance can have a prolonged impact on the demand and supply conditions in the foreign exchange markets. Empirical modelling in Section 3 will take into account the possibility that exchange rates respond only gradually to current account movements.


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Evans and Lyons (2003) have recently analyzed the price impact of end-user order flows on exchange rates. Interestingly, they find that currency trades originating from non-financial corporations have more persistent price effects than, say, trades originating from leveraged traders (such as hedge funds), and that their explanatory power increases with the horizon. These findings appear consistent with the arguments presented here.
Consider Figure 3, which plots the Japanese current account together with the debt securities balance. Debt securities are part of the portfolio investment balance and they are arguably the most important item in Japan’s financial account. From Figure 3, it is clear that Japan’s securities lending abroad has been mirroring the evolution of its current account surplus for a long time. Notice that the lag between the current account and exchange rate series in Figure 1 started to become much more sizeable after Japan began to liberalize its capital account. This evidence favours the interpretation put forward here, namely that capital flows can temporarily buffer the exchange rate pressure stemming from current account movements. As a result, the exchange rate adjusts only gradually whenever the current account reverses its trend.

**Reserves.** Japan has been accumulating vast reserves of foreign exchange over recent years. Purchases of foreign exchange appear to have been particularly heavy during those years in which the nominal exchange rate appreciated most strongly. Although these purchases could never fully offset the appreciation of the yen, they did have a—statistically significant (see Section 3)—moderating impact. Altogether, it appears that reserves played more of an endogenous role, reacting whenever economic fundamentals put too strong upward pressure on the exchange rate. This view is also supported by Girton and Roper (1977) who suggest that both exchange rate adjustments and reserve changes serve as indicators of exchange market pressure. In the multivariate model of Section 3, I choose not to include foreign exchange reserves as an additional variable in order to preserve a parsimonious setup.

### 2.2.2 Case study of Japan

What is the merit of studying the exchange rate performance of Japan which is but a single economy? Wouldn’t we need to observe similar time series patterns as in Japan elsewhere as well in order to make the above a convincing theory? The first thing to note here is that Japan’s export boom in the 1980s and 1990s has been truly exceptional. As Figure 4 demonstrates, Japan’s current account surpluses were far greater than the surpluses of any other country; changes in the balance of payments were also much more sizeable than those in other countries. As for the capital account, other advanced economies, such as the United States or the European economies, have traditionally been far more integrated into world financial markets than Japan with its well-documented home bias which can explain why it is the current account, rather than the capital account, that mattered most for the Japanese exchange rate (Brooks et al., 2001).

Upon closer inspection, one can also find many episodes in other countries
where the current account influenced the exchange rate in much the same way as in Japan. Currency crises are the case in point. In many of the recent exchange rate crises, the sequence of events, other factors notwithstanding, has been quite similar. The countries affected were typically running large current account deficits prior to their crises. At the same time, the countries were usually recipients of large capital inflows which helped to keep the exchange rate from depreciating. But as soon as the capital inflows dried up or reversed in sign, the national currencies got into trouble. Eichengreen (2003, chapter 8) and Bussiere and Mulder (1999) for instance have shown that a small set of variables—including the current account as a percentage of GDP, export growth, international reserves and short-term foreign debt relative to reserves—do a very good job in predicting the EMS crisis in 1992–1993, the Mexican crisis in 1994–1995 as well as the Asian crisis in 1997.

Japan allows us to study the impact of trade and capital flows on exchange rate behaviour under more stable conditions. Since Japan has been running large export surpluses for a long time, the country only needed to decide on how to fend off the upward pressure on its exchange rate and how to invest its export revenues. Conditions were similar for Germany in the 1980s, before German unification, when it also ran substantial current account surpluses (see Figure 4). During these years, the German exchange rate responded to current account movements in much the same way as the yen did in Japan (Figure 5). For deficit countries, the situation is quite different since these countries are confronted with issues such as current...

Figure 4: Large current account surpluses. Current account balances of countries with large current account surpluses (in billions of US dollars). Countries are selected and ordered according to the highest current account balance they have achieved in any single quarter in the period from 1977Q1 to 2001Q3. Source: International Financial Statistics (IMF).
account sustainability and finiteness of reserves that need to be resolved in one way or the other, for instance by attracting capital flows from abroad. This might explain why in the United States, the country with the largest current account deficits in the world and with very large and open financial markets, exchange rate movements have been much more strongly tied to capital flow determinants such as international return differentials (Brooks et al., 2001; Eichengreen, 1996).

![Figure 5: German current account and nominal effective exchange rate in the 1980s. German current account (left scale, in German mark) and nominal effective exchange rate (right scale, in logarithms), period from 1977Q1 to 1990Q4. Source: International Financial Statistics (IMF).](image)

### 2.3 Alternative explanations

As has already been pointed out in the introduction, one should expect many standard exchange rate theories to have difficulty to explain the massive upswings and downswings of the Japanese currency over the years. For example, interest and return differentials of Japan vis-à-vis other industrial countries appear stationary in the data—in contrast to the exchange rate which is found to be nonstationary—and they did not exhibit large swings over time as did the exchange rate. In addition, whereas interest and return differentials seem to have played an important role for the US dollar’s performance, possibly through their influence on capital flows, it is much harder to establish a similarly significant link for the yen. All of this suggests that return differentials may only have had a temporary and rather weak impact on the Japanese exchange rate.

Rather than discussing the merits and drawbacks of the many other exchange rate determinants proposed by the literature, I will focus here on only on one theory.
that is particularly relevant for Japan in the opinion of many economists.

**Balassa-Samuelson effect.** For many economists, Japan is the showcase for the Balassa-Samuelson effect that predicts an appreciation of the real exchange rate in countries with high productivity growth (Balassa, 1964; Samuelson, 1964). This theory has the potential to explain large long-run movements of the real exchange rate. It is indeed the case that Japan experienced both strong productivity growth and a sustained increase in the real value of its currency over the past fifty years. However, other implications of the theory are less well matched by the data. According to the theory, the real exchange rate movements are brought about by movements in the ratio of nontraded versus traded goods prices. However, Engel (1999) recently found that relative prices of nontraded goods account for almost none of the movements in real exchange rates in major industrial countries, irrespective of the time horizon (the method he used was based on a decomposition of the mean squared error of real exchange rate changes). As Engel points out, relative prices of nontraded goods in Japan increased by about 40 percent since the 1970s but the appreciation of the real exchange rate was 90 percent. However, since the relative price of nontradables in the United States closely mirrored the relative price of nontradables in Japan, the Balassa-Samuelson effect was effectively neutralized. As a final argument, there have not been any large downswings in the relative price of nontradable goods in Japan but, as we have seen, there have been several instances of a dramatic depreciation of the yen.

### 3 Empirical modelling

This section presents an empirical model based on the close relationship between the current account and nominal exchange rate in Japan. The model allows for large swings in the Japanese current account. At the same time, it takes into account the repercussions of these swings on the performance of the yen.

#### 3.1 Cointegration of exchange rate and economic fundamental

The statistical analysis of Japan’s nominal effective exchange rate as well as its current account reveal that both variables are non-stationary. Table 1 reports the results of the augmented Dickey-Fuller tests for both variables. The null hypothesis of a unit root cannot be rejected for either variable.

While individually I(1), I find the Japanese current account and nominal effective exchange rate to be cointegrated, that is, linear combinations of both variables exist that are I(0). To test for cointegration, I apply Johansen’s (1988) methodol-
<table>
<thead>
<tr>
<th>Constant</th>
<th>Trend</th>
<th>Lags</th>
<th>Coefficient</th>
<th>Std. error</th>
<th>t statistic</th>
<th>5% crit.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal effective exchange rate (1958Q3-2001Q2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>5</td>
<td>1.0014</td>
<td>0.0355</td>
<td>1.888</td>
<td>-1.942</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>5</td>
<td>0.9991</td>
<td>0.0355</td>
<td>-0.134</td>
<td>-2.878</td>
</tr>
<tr>
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<td>Yes</td>
<td>3</td>
<td>0.9395</td>
<td>0.0351</td>
<td>-3.056</td>
<td>-3.437</td>
</tr>
</tbody>
</table>

| Current account (1978Q3-2001Q1) |
| No       | No    | 5    | 0.9849      | 0.4856     | -0.728     | -1.944   |
| Yes      | No    | 4    | 0.9115      | 0.4802     | -2.283     | -2.895   |
| Yes      | Yes   | 4    | 0.8120      | 0.4681     | -3.298     | -3.461   |

Table 1: Unit root tests. Augmented Dickey-Fuller tests. Lag length selection based on Akaike information criterion.

\[
\begin{array}{cccc}
H_0 & H_A & \text{Trace test} & \text{p-value} \\
\hline
r = 0 & r > 0 & 20.860 & [0.006] ** \\
r \leq 1 & r > 1 & 1.6035 & [0.205] \\
\end{array}
\]

Table 2: Testing for cointegration. Testing for the number of distinct cointegrating vectors, using 5 lags. Double asterisks (**) mark significance at the 99% level.

Given that both variables are cointegrated, their joint dynamic behaviour can conveniently be represented by a vector error correction model. Estimation can proceed along traditional lines using maximum likelihood methods.

3.2 A Markov-switching vector error-correction model

Motivated by the inspection of the data in Section 2, I seek to analyze a modified vector error correction model. I make two alterations to the conventional setup. First, the intercept of the current account equation is allowed to switch between two unknown values according to a two-state Markov process with constant, unknown transition probabilities. While the current account is non-stationary, the visual inspection of its time series in Figures 1 and 2 suggests that it is subject to two kinds of drifts, an upward drift in some periods and a downward drift in other periods.
Second, seasonal dummies are present only in the current account equation, not in the exchange rate equation. This seems justified since the exchange rate, unlike the current account, does not appear to exhibit any seasonality. From an economic point of view, it is also hard to conceive why the exchange rate, by itself, should be fluctuating seasonally.

### 3.2.1 The model

The model I analyze takes the following form:

\[
\begin{bmatrix}
\Delta s_t \\
\Delta z_t
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
\psi_{z,1} & \psi_{z,2} & \psi_{z,3}
\end{bmatrix}
\begin{bmatrix}
d_{1,t} \\
d_{2,t} \\
d_{3,t}
\end{bmatrix}
+ \begin{bmatrix}
\pi_{0,1} \\
\pi_{0,2}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\nu_z
\end{bmatrix} R_t + \sum_{i=1}^{h-1} \begin{bmatrix}
\pi_{i,11} & \pi_{i,12} \\
\pi_{i,21} & \pi_{i,22}
\end{bmatrix}
\begin{bmatrix}
\Delta s_{t-i} \\
\Delta z_{t-i}
\end{bmatrix}
+ \begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix}
\begin{bmatrix}
\beta_1 & \beta_2
\end{bmatrix}
\begin{bmatrix}
s_{t-1} \\
z_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{s,t} \\
\varepsilon_{z,t}
\end{bmatrix}
\]

(1)

where

\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix}
\begin{bmatrix}
\beta_1 & \beta_2
\end{bmatrix}
= \alpha \beta' =
\begin{bmatrix}
\pi_{11} & \pi_{12} \\
\pi_{21} & \pi_{22}
\end{bmatrix}
\]

\[\nu_z > 0\]

\[\text{Prob}(R_t = 1|R_{t-1} = 1) = p, \quad \text{Prob}(R_t = 0|R_{t-1} = 0) = q\]

In this representation, \(s_t\) is the nominal effective exchange rate of Japan and \(z_t\) is the Japanese current account (in the domestic currency). The vector \([d_{1,t}, d_{2,t}, d_{3,t}]'\) contains seasonal dummies.

In period \(t\), the system is in one of two regimes, 0 or 1, according to the random variable \(R_t\). The variable \(R_t\) follows a Markov process with transition probabilities \(p\) and \(q\). The regime affects the intercept of the current account equation, which switches between a lower level, \(\pi_{0,2}\), and a higher level, \(\pi_{0,2} + \nu_z\). The current account is drifting downward whenever the system is in regime 0; it is drifting upward whenever the system is in regime 1.

The transition probabilities are assumed constant here in order to avoid a too complex model, as is done in most applications of Markov-switching models. However, as ? demonstrates using a Markov-switching framework with time-varying probabilities, the frequent trend reversals of the Japanese and German current accounts during recent decades—when both countries experienced large
export booms—were triggered by the often considerable movements of the real exchange rate in these two countries.

By collecting both variables in a vector \( y_t = [s_t, z_t]' \), the model can be written as:

\[
\Delta y_t = \Psi d_t + \pi_0 + \nu R_t + \sum_{i=1}^{h-1} \Pi_{t-i} \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t
\]

\[
= \Psi d_t + \pi_0 + \nu R_t + \sum_{i=1}^{h-1} \Pi_{t-i} \Delta y_{t-i} + \alpha \beta'y_{t-1} + \varepsilon_t \tag{2}
\]

\[
= \Psi d_t + \pi_0 + \nu R_t + \sum_{i=1}^{h-1} \Pi_{t-i} \Delta y_{t-i} + \alpha \eta_{t-1} + \varepsilon_t
\]

\( \varepsilon_t \sim \text{i.i.d. } N(0, \Sigma) \)

As equation (2) indicates, the matrix of long-run responses, \( \Pi \), is the product of the feedback vector, \( \alpha = [\alpha_1, \alpha_2]' \), and the cointegrating vector, \( \beta' = [\beta_1, \beta_2] \). The vector \( \alpha \) is called feedback vector since it measures the system’s response to the error from the long-run equilibrium relation. This error is given by \( \eta_{t-1} = \beta'y_{t-1} \).

The model may be written still more compactly as:

\[
Y = D\Psi' + R\nu' + X\Gamma + Z\beta\alpha' + E
\]

\[
= D\Psi' + R\nu' + WB + E \tag{3}
\]

where

\[
Y = \begin{bmatrix} \Delta y'_1 \\ \vdots \\ \Delta y'_T \end{bmatrix}, \quad D = \begin{bmatrix} d'_1 \\ \vdots \\ d'_T \end{bmatrix}, \quad R = \begin{bmatrix} r_1 \\ \vdots \\ r_T \end{bmatrix}, \quad X = \begin{bmatrix} 1 & \Delta y'_0 & \Delta y'_{-1} & \cdots & \Delta y'_{-h} \\ 1 & \Delta y'_1 & \Delta y'_0 & \cdots & \Delta y'_{-h-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \Delta y'_{T-1} & \Delta y'_{T-2} & \cdots & \Delta y'_{T-h+1} \end{bmatrix},
\]

\[
Z = \begin{bmatrix} y'_0 \\ \vdots \\ y'_{T-1} \end{bmatrix}, \quad E = \begin{bmatrix} e'_1 \\ \vdots \\ e'_T \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \pi'_1 \\ \pi'_2 \\ \vdots \\ \pi'_{T-1} \end{bmatrix}, \quad W = [X Z \beta] \quad B = \begin{bmatrix} r' \\ \alpha' \end{bmatrix}.
\]

4 Bayesian inference with Gibbs-sampling

The model introduced in the previous section is nonlinear and difficult to estimate by classical statistical methods. This paper demonstrates how Bayesian inference, and in particular the simulation tool of Gibbs-sampling, can be used to estimate the model. The key of the approach taken here is that the unobserved regimes can be treated as additional unknown parameters (see Albert and Chib, 1993). They can
then be analyzed along with the model’s unknown parameters via the simulation method of Gibbs sampling.

Applying the method of Gibbs sampling to time series has recently become increasingly popular in the literature. Examples can be found in Albert and Chib (1993) and Kim and Nelson (1998). Kim and Nelson (1999) survey the literature and provide many illustrations. However, I have not so far encountered a Bayesian treatment of a model such as the one presented in this paper that combines a vector error correction model with Markov-switching elements.

For further use, define $\tilde{y}_t \equiv [y_1, y_2, \ldots, y_t]$ and $\tilde{R}_t \equiv [R_1, R_2, \ldots, R_t]$ for $t = 1, 2, \ldots, T$. The sample size is $T$, so $\tilde{y}_T$ is the vector of all observations and $\tilde{R}_T$ is the vector of all regimes, or states. Let $\theta$ denote the vector of all parameters of the model.

### 4.1 Bayesian estimation

Bayesian inference about $\theta$ is based on the posterior distribution:

$$g(\theta | \tilde{y}_T) \propto L(\theta | \tilde{y}_T) g(\theta)$$

where $g(\theta)$ is the prior and $L(\theta | \tilde{y}_T)$ is the likelihood function. Direct inference would not be practical here due to the difficulties involved in computing the likelihood function. The alternative is to simulate the posterior density using the Markov chain Monte Carlo (MCMC) method referred to as the Gibbs sampler. For an introduction to MCMC methods and the Gibbs sampler, see for example Gelman, Carlin, Stern and Rubin (1995) and Kim and Nelson (1999). A brief review is also given in Appendix A.

### 4.2 Gibbs sampling

My objective is to find a complete set of conditional distributions of all the parameters, on which the Gibbs sampling scheme can be run. It turns out that this task is facilitated once I treat the regimes $\tilde{R}_T$ as additional unknown parameters and analyze them jointly with $\theta$, the vector of all other parameters. Given $\tilde{R}_T$, conditional inference on $\theta$ is basically equivalent to inference on a vector error correction model. Given $\theta$ on the other hand, procedures are available that help us to retrieve the conditional distribution of the regimes.

I thus obtain a tractable conditional structure as a basis of the Gibbs simulations. Gibbs sampling proceeds by iteratively drawing from the following sequence of conditional distributions:

- $[\tilde{R}_T | \tilde{y}_T, \Psi, \nu, B, \beta, \Sigma, p, q]$
\[ \cdot [p, q \mid \tilde{R}_T] \]
\[ \cdot [\Psi, \nu \mid \tilde{y}_T, \tilde{R}_T, B, \beta, \Sigma] \]
\[ \cdot [\Sigma \mid \tilde{y}_T, \tilde{R}_T, \Psi, \nu, B, \beta] \]
\[ \cdot [B \mid \tilde{y}_T, \tilde{R}_T, \Psi, \nu, \beta] \]
\[ \cdot [\beta \mid \tilde{y}_T] \]

The details of the Gibbs sampling procedure are discussed in detail in Appendix A.

5 Empirical results

5.1 Parameter estimates

The model is estimated with two different assumptions regarding the prior for the transition probabilities. The exact way of choosing the prior is discussed in Appendix A.2. The estimations were carried out using a lag length, \( h \), of 4.

5.1.1 Using a non-informative prior

I first use a flat prior for \( p \) and \( q \). The problem of using a non-informative prior is that during some periods of the sampling, the draws for \( p \) and \( q \) are unreasonably low. Even though most of the density mass of the posterior density for these parameters lies in the region between 0.8 and 0.9, there is still substantial mass in the lower tail of the density. This indicates that the transition probabilities occasionally pick up seasonal and short-term fluctuations. Figure plots the estimated probability of being in regime 1. However, the graph shows that even with a non-informative prior, the shifts of the model from one regime to another are clearly discernible.

5.1.2 Using an informative prior

Problems with noninformative priors similar to the one described here also arise in the literature that applies Bayesian Markov-switching models to business cycle data (see for instance Kim and Nelson, 1998). The usual approach in that literature is to specify informative priors for the parameters, in particular the transition probabilities, incorporating what is known or believed about the average duration of business cycle phases. I proceed in a similar way here by choosing priors for \( p \) and
Figure 6:  **Regime probabilities (non-informative prior).** Probability of regime with high current account intercept, using a non-informative prior for transition probabilities.

$q$ that give a low weight to transition probabilities that would imply very frequent current account reversals (details in Appendix A.2).³

Table 3 presents the estimation results for a subset of the parameters using these priors. Figure 7 plots the Gibbs simulations for several parameters, along with a correlogram indicating the degree of serial correlation of the drawings. Also drawn are the simulated posterior densities of the parameters which all appear well-shaped. Note that the estimate for the cointegrating vector, $\beta$, $[1.0, -0.25018]'$, is relatively close to the estimate from an ordinary, non-switching vector error correction model, which is $[1.0, -0.26805]'$. Finally, Figure 8 plots the estimated probability of being in regime 1. Periods when the current account, and thus the exchange rate, are strengthening (regime 1), are again clearly distinguished from the other periods when the current account is downward-drifting (regime 0).

### 5.2 Exchange rate simulations with and without current account shifts

I have carried out some simulations of the estimated model in order to assess the significance of its regime-switching component for the variability of the current account and the exchange rate. I simulate the model twice (with an identical sequence of random shocks hitting the variables in both cases). The simulation period is sixty years, not taking into account an initial warming-up period. In the first simulation, I generate artificial time series for the current account and exchange rate from the estimated model. In the second simulation, I set the regime-switching intercept of the current account to be zero, thereby removing the regime-switching component of the model. The estimated probability of being in regime 1. Periods when the current account, and thus the exchange rate, are strengthening (regime 1), are again clearly distinguished from the other periods when the current account is downward-drifting (regime 0).

³Alternatively, one could smooth the current account to remove its short-term fluctuations and then estimate the model using a noninformative prior.
Table 3: **Parameter estimates.** Posterior estimates of selected parameters. Mean, median and 95% interval of the Gibbs simulations for each parameter, after discarding initial simulations (see Section A.6).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Median</th>
<th>95% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_2$</td>
<td>-0.25018</td>
<td>-0.24473</td>
<td>-0.36351, -0.15685</td>
</tr>
<tr>
<td>$\nu_z$</td>
<td>0.84177</td>
<td>0.84045</td>
<td>0.62283, 1.0627</td>
</tr>
<tr>
<td>$\psi_{z,3}$</td>
<td>0.16942</td>
<td>0.17142</td>
<td>-0.10454, 0.46098</td>
</tr>
<tr>
<td>$\pi_{0,1}$</td>
<td>0.28296</td>
<td>0.28227</td>
<td>0.11201, 0.45258</td>
</tr>
<tr>
<td>$\pi_{0,2}$</td>
<td>-1.9026</td>
<td>-1.8453</td>
<td>-3.8404, -0.22604</td>
</tr>
<tr>
<td>$\pi_{1,11}$</td>
<td>0.33487</td>
<td>0.33151</td>
<td>0.15985, 0.51812</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-0.075799</td>
<td>-0.075882</td>
<td>-0.12087, -0.030032</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.35584</td>
<td>0.33914</td>
<td>-0.088329, 0.85646</td>
</tr>
<tr>
<td>$\sigma_s^2$</td>
<td>0.0022798</td>
<td>0.0021254</td>
<td>0.0016068, 0.0033567</td>
</tr>
<tr>
<td>$\sigma_{sz}$</td>
<td>-0.0014129</td>
<td>-0.0013198</td>
<td>-0.0058805, 0.0027598</td>
</tr>
<tr>
<td>$q$</td>
<td>0.90953</td>
<td>0.91081</td>
<td>0.87419, 0.94077</td>
</tr>
<tr>
<td>$p$</td>
<td>0.90649</td>
<td>0.90756</td>
<td>0.87102, 0.93783</td>
</tr>
</tbody>
</table>

The current account to its time-invariant average, which I can evaluate on the basis of the estimated values for $\pi_{0,2}$, $\nu_z$, $p$ and $q$. This exercise is meant to give us an idea of what would happen to the exchange rate once all structural breaks of its fundamental are eliminated. The results, which are shown in Figure 9, indicate that without the long swings in the current account, exchange rate fluctuations over the longer term could be much smoother.

5.3 Capital flows

The emphasis so far has been entirely on the current account. Even if it is taken for granted that the current account influences relative currency demands, what about capital movements and their impact on the flow of foreign exchange?

I have analyzed this question by testing for a cointegration between the exchange rate and various parts of the balance of payments, the current account, the equity securities balance and changes in reserves. Note that including all components of the balance of payments in a multivariate setup is not possible as this would imply perfect multicollinearity. Therefore a selection of variables has to be made. In particular, I choose to keep the Japanese debt securities balance out of the analysis because of its strong correlation with the current account.

The results, which are summarized in Appendix B, show two things. First, there exists a strong cointegrating relationship between the variables. In the esti-
Figure 7: **Gibbs simulations (informative prior).** Posterior distributions of selected parameters, using an informative prior for transition probabilities.

estimated cointegrating vector, all variables have the expected sign. A current account surplus is associated with an appreciating currency while, say, equity outflows and purchases of reserve assets put downward pressure on the exchange rate. Notice that the parameter estimate for the current account variable is likely to understate the current account’s true impact on the exchange rate as it also reflects the moderating effect of Japan’s investments in debt securities and other investments abroad.

In the empirical model of this paper, I focus on the current account as the main fundamental of the Japanese exchange rate. This is a way to preserve a parsimonious setup. The time series evidence in Figures 1 and 2 has made it quite clear that, notwithstanding other influences, the yen’s valuation has been closely related to the Japanese trade performance for at least three decades. However, Appendix B demonstrates that the main empirical results go through even if other exchange rate fundamentals, such as the equity securities balance, are included in the model.

6 Conclusions

I have estimated a Markov-switching vector error correction model of the Japanese nominal effective exchange rate and current account. Two general observations are underlying the model. First, Japan is a country where the exchange rate is predominantly influenced by a single economic fundamental, the current account. Second, the movement of the Japanese current account alters its direction from time to time. Taken together, these two findings imply that the current account exhibits large swings over the years, which translate into similarly large swings of the exchange rate.
In the model, the current account contains a stochastic trend that switches between two regimes, or states, according to an unobservable Markov process. The exchange rate itself is modelled as independent of the regimes. However, since it is cointegrated with the current account, it follows the movements of the current account and adjusts to current account reversals whenever they occur. A simulation exercise demonstrates that the effect of the shifts between the regimes is rather large. The fluctuations of the exchange rate and its fundamental are large when regime switching is allowed for. They become much smaller once the model is simulated without the regime changes.

The paper seeks to explain the strong appreciation of the yen and its large variability, which have had important implications for Japan’s economic performance in recent years. But the motivation of this paper goes further. Its more general intention is to highlight the issues on which empirical research on exchange rates should focus in the future.

First, an often raised question in exchange rate economics is whether exchange rates are driven by economic fundamentals (MacDonald, 1999). In an influential paper, Meese and Rogoff (1983) demonstrated twenty years ago that a simple random walk can outperform many fundamentals-based exchange rate models in terms of out-of-sample forecasting performance. What the present study of Japan seeks to demonstrate is that fundamentals are strongly relevant for medium- and long-run exchange rate fluctuations.\footnote{This is, by the way, just the view of market participants: Cheung and Wong (2000) have recently carried out a survey of practitioners in the interbank foreign exchange markets in Hong Kong, Tokyo, and Singapore. They report that at the medium-run horizon, between 29% (Tokyo) and 35% (Hong...}
Figure 9: Simulating exchange rate dynamics. Simulation of current account (top) and nominal exchange rate (bottom) from the estimated model, over a period of 60 years. Simulations of the original model with switching regimes (panels on the left), and of a single-regime version of the model where the stochastic trend of the current account variable is set to its estimated average (panels on the right).

Second, there has been a lot of controversy over the validity of purchasing power parity (PPP). The PPP puzzle (Rogoff, 1996) concerns the question why deviations from PPP are so persistent. The average half-life of a shock to PPP is estimated to be around three to five years, which is difficult to explain with nominal rigidities alone. By studying carefully the experience of an individual country, namely Japan, this paper finds that long-lasting deviations of the real exchange rate from purchasing power parity may be the result of large and persistent movements of the underlying economic fundamentals. Rather than asking why the law of one price does not lead to a faster adjustment towards PPP, the paper suggests that research should focus on the economic forces that produce deviations from PPP in the first place.

Third, partly as a consequence of the poor empirical performance of PPP models, researchers have recently turned to non-linear exchange-rate modelling (Michael, Nobay and Peel, 1997; Obstfeld and Taylor, 1997; Taylor and Peel, 2000). Non-linear time series models of exchange rates are often motivated by the idea that transportation costs reduce arbitrage opportunities in international goods markets, implying that small deviations from PPP can be persistent. This paper shows that what is unsatisfactory with existing nonlinear time series models of exchange rates...
is that they are generally univariate. Univariate models have difficulties to explain why exchange rates deviate from PPP or why deviations, even large ones, are so persistent.

A decade ago, Engel and Hamilton (1990) discovered "long swings in the US dollar". This paper has demonstrated that the Japanese exchange rate exhibits long swings as well. In contrast to the study by Engel and Hamilton, the analysis in this paper takes on a multivariate perspective, and it is suggesting that the large swings in the yen are linked to similar persistent movements of its economic fundamental.

The paper also offers a technical innovation. The vector error correction model that has been estimated in this paper contains a Markov-switching intercept only in the equation of the exchange rate fundamental. A model specified in this way has not been analyzed before. As the paper demonstrates, estimation is feasible in a Bayesian context using a Gibbs-sampling procedure.

**Appendices**

**A Gibbs-sampling**

Suppose the vector of all parameters, \( \theta \), is partitioned into vector components, \( \theta_1, \theta_2, \ldots, \theta_k \). There is thus a complete set of conditional distributions, \( g(\theta_1 | \tilde{y}_T, \theta_2, \theta_3, \ldots, \theta_k) \), \( g(\theta_2 | \tilde{y}_T, \theta_1, \theta_3, \ldots, \theta_k) \), \ldots, \( g(\theta_k | \tilde{y}_T, \theta_1, \theta_2, \ldots, \theta_{k-1}) \). Given some arbitrary starting values for the parameters, \( \theta_1^0, \theta_2^0, \ldots, \theta_k^0 \), the Gibbs algorithm involves iterating through the following cycle (with the current iteration denoted by \( i \)):

**Step 1** Draw \( \theta_1^i \) from \( g(\theta_1 | \tilde{y}_T, \theta_2^{i-1}, \ldots, \theta_k^{i-1}) \)

**Step 2** Draw \( \theta_2^i \) from \( g(\theta_2 | \tilde{y}_T, \theta_1^i, \theta_3^{i-1}, \ldots, \theta_k^{i-1}) \)

\[ \ldots \]

**Step k** Draw \( \theta_k^i \) from \( g(\theta_k | \tilde{y}_T, \theta_1^i, \theta_2^i, \ldots, \theta_{k-1}^i) \)

Steps 1 through k can be iterated \( T \) times, so as to obtain a full set of simulated parameters for every iteration. Under regularity conditions, the distribution of \( \theta^i \) converges to the distribution of \( \theta \) as \( T \) goes to infinity (see references quoted in Albert and Chib, 1993; Kim and Nelson, 1999). This suggests setting \( T = N + M \), so that when \( N \) initial simulations are discarded, the remaining \( M \) drawings of all parameters can be used as an approximate simulated sample from \( g(\theta_1, \theta_2, \ldots, \theta_k) \).
A.1 Generating the regimes, $\tilde{R}_T$

To generate the regimes, $\tilde{R}_T$, I employ multi-move Gibbs-sampling (see Kim and Nelson, 1999). Multi-move Gibbs-sampling refers to the simulation of all the regimes as a block from the joint conditional distribution:

$$g(\tilde{R}_T|\theta_{-\tilde{R}_T}, \tilde{y}_T)$$

where $\theta_{-\tilde{R}_T}$ refers to all the parameters of the model other than $\tilde{R}_T$ (which is treated here as a vector of parameters). It follows from the Markov property of $S_t$ that the joint conditional density can be factorized as follows:

$$g(\tilde{R}_T|\theta_{-\tilde{R}_T}, \tilde{y}_T) = g(\tilde{R}_T|\tilde{y}_T) \prod_{t=1}^{T-1} g(R_t|R_{t+1}, \tilde{y}_t)$$  \hspace{1cm} (5)

Equation (5) shows that $g(\tilde{R}_T|\theta_{-\tilde{R}_T}, \tilde{y}_T)$ can be evaluated once $g(\tilde{R}_T|\tilde{y}_T)$ as well as $g(R_t|R_{t+1}, \tilde{y}_t)$, $t = T - 1, T - 2, \ldots, 1$, are known. This suggests employing a two-step procedure in order to generate these conditional densities.

**Step 1** Run Hamilton’s (1989) basic filter to get $g(R_t|\tilde{y}_t)$, $t = 1, 2, \ldots, T$. That is, iterate on the following pair of equations:

$$g(R_t = i|\tilde{y}_t) = \frac{g(R_t = i, y_{t-1}|\tilde{y}_{t-1})}{g(y_{t-1}|\tilde{y}_{t-1})} = \frac{g(y_{t-1}|R_t = i, \tilde{y}_{t-1}) g(R_t = i|\tilde{y}_{t-1})}{\sum_{j=1}^{2} g(y_{t-1}|R_t = j, \tilde{y}_{t-1}) g(R_t = j|\tilde{y}_{t-1})}, \quad i = 0, 1$$

$$\begin{bmatrix} g(R_{t+1} = 0|\tilde{y}_t) \\ g(R_{t+1} = 1|\tilde{y}_t) \end{bmatrix} = \begin{bmatrix} q & 1 - p \\ 1 - q & p \end{bmatrix} \begin{bmatrix} g(R_t = 0|\tilde{y}_t) \\ g(R_t = 1|\tilde{y}_t) \end{bmatrix}$$

The last iteration of the filter provides us with $g(R_T|\tilde{y}_T)$, from which $\tilde{R}_T$ is generated.

**Step 2** The following result, which follows from the Markov property of $R_t$, provides us with the smoothed conditional densities $g(R_t|R_{t+1}, \tilde{y}_t)$:

$$g(R_t|R_{t+1}, \tilde{y}_t) = \frac{g(R_{t+1}|R_t, \tilde{y}_t) g(R_t|\tilde{y}_t)}{g(R_{t+1}|\tilde{y}_t)} = \frac{g(R_{t+1}|R_t) g(R_t|\tilde{y}_t)}{g(R_{t+1}|\tilde{y}_t)} \propto g(R_{t+1}|R_t) g(R_t|\tilde{y}_t)$$  \hspace{1cm} (6)

where $g(R_{t+1}|R_t)$ is the transition probability and $g(R_T|\tilde{y}_T)$ is saved from step 1. The result in equation (6), can be used to recursively generate $\tilde{R}_t$ from $g(R_t|R_{t+1}, \tilde{y}_t)$, for $t = T - 1, T - 2, \ldots, 1$. 

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A.2 Generating the transition probabilities, $p$ and $q$

Conditional on $\tilde{R}_T$, the transition probabilities $p$ and $q$ are independent of the data set, $\tilde{y}_T$, and the model’s other parameters. The conditional distribution $p, q|\tilde{R}_T$ can be obtained from standard Bayesian results on Markov chains. Given, $\tilde{R}_T$, the transitions $n_{ij}$ from state $i$ to $j$, with $i, j = 0, 1$, provide sufficient statistics for $p$ and $q$. The likelihood function for $p$ and $q$ is given by:

$$L(p, q|\tilde{R}_T) = p^{n_{11}}(1 - p)^{n_{10}}q^{n_{00}}(1 - q)^{n_{01}}$$

The form of the likelihood suggests the use of the beta distribution as a conjugate prior for the transition probabilities.

Prior Assuming independent beta distributions for the priors of $p$ and $q$, the prior is given by:

$$p \sim \text{beta}(u_{11}, u_{10})$$
$$q \sim \text{beta}(u_{00}, u_{01})$$

with

$$g(p, q) \propto p^{u_{11} - 1}(1 - p)^{u_{10} - 1}q^{u_{00} - 1}(1 - q)^{u_{01} - 1}$$

where $u_{ij}$, $i, j = 0, 1$, are the hyperparameters of the prior.

Combining the prior and the likelihood, the following posterior is obtained:

Posterior

$$g(p, q|\tilde{R}_T) = g(p, q)L(p, q|\tilde{R}_T)$$
$$\propto p^{u_{11} - 1}(1 - p)^{u_{10} - 1}q^{u_{00} - 1}(1 - q)^{u_{01} - 1}$$
$$p^{n_{11}}(1 - p)^{n_{10}}q^{n_{00}}(1 - q)^{n_{01}}$$

(7)

I have estimated the model using a noninformative, flat, prior as well as with an informative prior. For the noninformative prior, I set $u_{11} = u_{10} = u_{01} = u_{00} = 1$.

Using the formulas for the expected value and variance of a beta random variable, I obtain:

$$E(p) = E(q) = \frac{1}{2}, \quad \text{Var}(p) = \text{Var}(q) = \frac{1}{12}$$

Based on data prior to our sample, it appears that the upswings and downswings of the Japanese current account have an average duration of roughly ten to twelve quarters. This suggests choosing an informative prior for the transition probabilities with the following first and second moments:

$$E(p) = E(q) = \frac{10}{11}, \quad \text{Var}(p) = \text{Var}(q) = \frac{1}{2000}$$

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Notice that the expected duration of regime 1, say, equals $1/(1 - p)$. The implied hyperparameters of the prior are:

$$u_{11} = u_{00} = 149, \quad u_{10} = u_{01} = 14.9$$

### A.3 Generating $\nu$ and $\Psi$

Conditional on all other parameters, $\psi_{z,1}$, $\psi_{z,2}$, $\psi_{z,3}$ and $\nu_z$ may be generated as follows. Define $C$ to be the Choleski decomposition of $\Sigma$. Since $\Sigma$ is given, $C$ may be evaluated and the equation (2) may be multiplied through with the inverse of $C$.

After rearranging, I obtain:

$$y^* = \Psi^* d_t + \nu^* R_t + \varepsilon_t^*$$

where

$$y^* \equiv C^{-1} \left( \Delta y_t - \pi_0 - \sum_{i=1}^{h-1} \Pi_{t-i} \Delta y_{t-i} - \Pi_y_{t-1} \right)$$

$$\Psi^* \equiv C^{-1} \Psi, \quad \nu^* \equiv C^{-1} \nu, \quad \varepsilon_t^* \equiv C^{-1} \varepsilon_t$$

Notice that as $\varepsilon_t^* \sim N(0, I)$, equation (8) represents a system of two independent equations. With all elements of $y^*$ given, the second equation—which contains the parameters that are of interest to us—may be estimated in isolation. Consider therefore the Bayesian estimation of the following linear regression model:

$$Y^* \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [D \ R] \begin{bmatrix} \psi_{z,1} \\ \psi_{z,2} \\ \psi_{z,3} \\ \nu \end{bmatrix} + E^* \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where $E^* [0, 1]' \sim N(0, I_T)$.

The natural conjugate prior for $[\psi_{z,1}, \psi_{z,2}, \psi_{z,3}, \nu]'$ is the normal density:

**Prior of** $[\psi_{z,1}, \psi_{z,2}, \psi_{z,3}, \nu]'$

$$[\psi_{z,1} \ \psi_{z,2} \ \psi_{z,3} \ \nu]' \sim N(\gamma_0, \Omega_0)$$

where $\gamma_0$ and $\Omega_0$ can be chosen appropriately (a noninformative prior may be obtained by choosing $\Omega_0$ very large).

The posterior distribution for $[\psi_{z,1}, \psi_{z,2}, \psi_{z,3}, \nu]'$ is given by the following normal distribution (Kim and Nelson, 1999, page 174):

**Posterior of** $[\psi_{z,1}, \psi_{z,2}, \psi_{z,3}, \nu]'$

$$[\psi_{z,1} \ \psi_{z,2} \ \psi_{z,3} \ \nu]' \sim N(\gamma_1, \Omega_1)$$
where
\[
\gamma_1 = \left[ \Omega_0^{-1} + R'D'DR \right]^{-1} \left[ \Omega_0^{-1}\gamma_0 + R'D'Y^* \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right]
\]
\[
\Omega_1 = \left[ \Omega_0^{-1} + R'D'DR \right]^{-1}
\]

### A.4 Generating $\Sigma$ and $B$

Consider now the estimation of $\Sigma$ and $B$ conditional on all the other parameters. The system in equation (2) then becomes:

\[
y_t^{**} \equiv y_t - \Psi d_t - \nu R_t
\]
\[
= \pi_0 + \sum_{i=1}^{h-1} \Pi_{t-i}\Delta y_{t-i} + \alpha\beta' y_{t-1} + \varepsilon_t
\]
\[
\varepsilon_t \sim \text{i.i.d. N}(0, \Sigma)
\]

Since $\beta$ as well as all the parameters contained in $y_t^{**}$ are given, I can treat equation (9) as a multivariate linear regression model. Recalling the notation of equation (3), this model becomes:

\[
Y^{**} \equiv Y - D\Psi' - R\nu'
\]
\[
= WB + E
\]
\[
\text{vec}(E) \sim N_{TN}(0, \Sigma \otimes I_T)
\]

This regression can be estimated using Theorem 9.1 (or Theorem 9.3) in Bauwens, Lubrano and Richard (1999).

**Prior of $B$ and $\Sigma$**

A noninformative prior is applied:

\[
g(B, \Sigma) \propto |\Sigma|^{-(n+1)/2}
\]

**Posterior of $B$ and $\Sigma$**

Let $k$ equal the number of columns of $W$. The posterior densities of $B$ and $\Sigma$ are given by:

\[
B \sim \text{Mt}_{k \times n}(\hat{B}, W'W, S, T - k)
\]
\[
\Sigma \sim \text{IW}_n(S, T)
\]

where

\[
\hat{B} = (W'W)^{-1}W'Y
\]
\[
S = (Y - W\hat{B})(Y - W\hat{B})
\]

Mt and IW refer to the matricvariate Student and inverted Wishart distributions respectively.
A.5 Generating $\beta$ using griddy Gibbs-sampling

I now turn to the problem of estimating the cointegrating vector $\beta$. Let $g(\beta)$ be the prior for $\beta$. Following Bauwens et al. (1999, Theorem 9.3), the kernel of $\beta$ is given by:

$$g(\beta|\tilde{y}_T) \propto g(\beta')|\beta'V_0\beta'|^{l_0}/|\beta'V_1\beta'|^{l_1}$$

(10)

where

$$V_0 = Z'M_XZ$$
$$V_1 = Z'M_Y[I_T - X(X'M_YX)^{-1}X']M_YZ$$

$$M_X = I_T - X(X'X)^{-1}X', \quad M_Y = I_T - Y(Y'Y)^{-1}Y'$$

$$l_0 = (T - k - n)/2, \quad l_1 = (T - k)/2$$

Note that when the first element of $\beta$ is normalized to one, the task reduces to sampling from the univariate conditional posterior distribution of $\beta_2$. Some analytical results are available. Bauwens et al. (1999, Corollary 9.4) show that under a noninformative prior for $\beta$—which is the simplest case—, the posterior density of $\beta$ is a 1-1 poly-t density. However, the simulation of a 1-1 poly-t density is by no means easy and involves a substantial fixed cost in terms of programming (for an algorithm, see Bauwens and Richard, 1985).

For this reason, I apply the griddy Gibbs-sampler as proposed by Ritter and Tanner (1992). The griddy Gibbs-sampler is an attractive device whenever it is difficult to directly sample from $g(\theta_i|\tilde{y}_T, \theta_{-i})$, the posterior density of a parameter $\theta_i$ conditional on the data and all other parameters of the model. The idea is to form a simple approximation to the inverse cdf of this density based on the evaluation of $g(\theta_i|\tilde{y}_T, \theta_{-i})$ on a grid of points. The procedure consists of the following steps (letting $j$ denote the current Gibbs iteration):

Step 1 Specify a grid for $\theta_i$, say $\theta_{i,1}, \theta_{i,2}, \ldots, \theta_{i,n}$.

Step 2 Evaluate $g(\theta_i|\tilde{y}_T, \theta_{-i})$, using the most recently simulated values for $\theta_{-i}$, to obtain $w_1, w_2, \ldots, w_n$.

Step 3 Use $w_1, w_2, \ldots, w_n$ to obtain an approximation to the empirical inverse cdf of $g(\theta_i|\tilde{y}_T, \theta_{-i})$. Denote the approximate inverse cdf as $\hat{F}^{-1}(\cdot)$.

Step 4 Draw $\zeta^{(j)}$, a uniformly distributed random variable on $[0, 1]$, to obtain $\theta_i^{(j)} = \hat{F}^{-1}(\zeta^{(j)})$.
To simulate $\beta$, I use a piecewise linear approximation to the empirical inverse cdf based on the posterior density of $\beta$ given in equation (10). Ritter and Tanner (1992) discuss a number of possible enhancements to the procedure, regarding for example the adjustment of the grid or the approximation to the inverse cdf. However, I find that already a simple approximation and a uniformly spaced, stable grid do a satisfactory job.

A.6 Design of the Gibbs-sampler

The Gibbs sampling scheme is run over 5000 iterations, of which the first 2500 simulations are discarded.

To start the sampling algorithm, initial values for the parameters are specified. As an starting value for the cointegrating vector, $\beta$, I take the estimate from a standard vector error correction model with constant, non-switching intercepts. I further choose 0.5 as the initial value for the transition probabilities. $\Sigma$ is set to equal the identity matrix. All other parameters are set to zero. The estimation results appear robust to changes in these specifications.

B Additional exchange rate determinants

Tables 4 and 5, which are being referred to in Section 5.3, show that there exists a cointegrating relationship between the Japanese nominal effective exchange rate, current account, equity securities balance and changes in reserves.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$r$</th>
<th>Trace test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$&gt; 0$</td>
<td>58.784</td>
<td>[0.003] **</td>
</tr>
<tr>
<td>$\leq 1$</td>
<td>$&gt; 1$</td>
<td>27.692</td>
<td>[0.087]</td>
</tr>
<tr>
<td>$\leq 2$</td>
<td>$&gt; 1$</td>
<td>12.659</td>
<td>[0.128]</td>
</tr>
<tr>
<td>$\leq 3$</td>
<td>$&gt; 1$</td>
<td>1.1952</td>
<td>[0.274]</td>
</tr>
</tbody>
</table>

Table 4: Testing for cointegration, with additional exchange rate fundamentals. Testing for the number of distinct cointegrating vectors, using 6 lags (based on AIC criterion and likelihood-ratio test). Double asterisks (**) mark significance at the 99% level.

The Markov-switching vector error-correction model discussed in this paper allows only for two variables. With more than two variables, the number of parameters increases rapidly. However, as an extension of the model of this paper, I have introduced the equity securities balance as a second exchange rate determinant in addition to the current account. I find that the estimation procedure works
<table>
<thead>
<tr>
<th>Nominal effective exchange rate</th>
<th>1.0000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current account</td>
<td>-0.28410</td>
</tr>
<tr>
<td>Equity securities balance</td>
<td>-0.15879</td>
</tr>
<tr>
<td>Reserve changes</td>
<td>-0.15499</td>
</tr>
</tbody>
</table>

Table 5: **Cointegration between Japanese exchange rate and economic fundamentals.** Cointegrating vector of Japanese nominal effective exchange rate and three balance-of-payments fundamentals. Equity outflows and rising reserves are defined as negatively signed flows here, that is, they tend to put downward pressure on the exchange rate.

still very well, producing comparable results as in the two-variable setup. The estimated regime probabilities (not shown) look very similar to the original ones in Figure 8.

C Data

Unless indicated otherwise, all data used for this paper are from the IMF’s International Financial Statistics. Data on the Japanese balance-of-payment components has been converted into yen for the estimations, using the IMF’s quarterly yen-dollar exchange rate.

D Software

The computations for this paper were carried out using Ox, version 3.0 (see Doornik and Ooms, 2001), and PcFiml (see Doornik and Henry, 1997). The programs are available from the author upon request. Separate code was written for generating random numbers from the matrix-variate Student and inverted Wishart distributions. In order to sample from the matrix-variate Student distribution, I used the algorithm in Bauwens et al. (1999, Appendix B.4.5). As regards the inverted Wishart distribution, I translated to Ox a Gauss code that I kindly received from Luc Bauwens.

References


