Non-linearities in the dynamics of the demand for euro area M1

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PRELIMINARY DRAFT

Abstract

The paper investigates whether there is evidence of non-linearity in the short-run dynamics of the euro area money demand. It first estimates a long-run money demand relationship over a sample period comprising the last three decades. Although the parameters of the long-run relationship are stable, there are indications of non-linearity in the residuals of the linear error-correction model. This non-linearity is explicitly modelled with satisfactory results using a fairly general Markov-switching error-correction model. The empirical findings of the paper are consistent with theoretical predictions and with recent empirical evidence for some European countries and the US.

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Keywords: Euro area, cointegration, non-linear error correction, demand for money.

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1 Introduction

Linear models embodying error-correction mechanisms have become the standard macroeconometric tool in the empirical literature on money demand.¹ These models combine a theoretically-grounded description of the behavior of money demand in equilibrium with a data-driven specification of the (linear) dynamics of disequilibrium correction in the short-run. One of the main reason for their popularity is that these models have been able to provide a statistically meaningful representation of the sluggishness in the portfolio allocation behavior of economic agents.

Yet, such sluggishness derives from the existence of market rigidities, such as portfolio adjustment costs, which may also translate into non-linearities in the dynamics of adjustment to equilibrium. For instance, market rigidities may potentially give rise to non-linear correction processes where the speed of adjustment varies with the size of the deviation from equilibrium.

“Buffer stock” and “target-threshold” models are among the most prominent classes of theoretical models postulating asymmetric correction of disequilibria in money markets.² These models are based on the observation that, due to shocks of various nature, the monetary holdings of individual agents may depart from their desired or “target” levels. However, in the presence of adjustment costs, it may not be optimal for agents to bring their balances back to the target straight away. Only when the deviations of monetary holdings from desired levels become relatively large (and, in the case of target-threshold models, exceed some specified thresholds), agents engage in those transactions needed to bring their balances back to target levels. At the aggregate level, this may result in persistent deviations of long-run money demand from the equilibrium level and in non-linearities in the short-run dynamics of money.

Consistent with the predictions of these theoretical models, some authors have found evidence of non-linearities in the short-run dynamics of monetary aggregates (see Hendry and Ericsson, 1991). In recent years, several authors have modelled such non-linearities using regime-dependent models of money demand in Europe (see, for instance, Escribano, 2004, Lütkepohl et al., 1999, Ordóñez, 2003, Sarno, 1999, and Teräsvirta and Eliasson, 2001) and in the

²For a discussion of the notion of buffer stock in monetary economics see Laidler (1984). Mizen (1994) is a comprehensive study of buffer stock money demand models, also including target-threshold models as a special type.
US (e.g. Sarno et al., 2003).

The purpose of this note is to investigate empirically whether there is similar evidence of non-linearity in the short-run dynamics of euro area M1 demand. In this paper non-linearity is characterised in terms of state-dependency in the dynamic behavior of money, i.e. allowing for the possibility that the short-run dynamics of real money varies across different states of the economy governed by an unobservable first-order Markov process.

The structure of the paper is as follows. Section 2 discusses the theoretical foundations of non-linearities in short-run monetary dynamics, Section 3 proposes a long-run relationship for the euro area M1 demand, while Section 4 deals with the econometrics of non-linear dynamics. Section 5 concludes.

2 Theoretical background

Non-linearities in the dynamics of money demand are typically rationalised on the basis of target-threshold and buffer stock models. As noted by Mizen (1994) in his comprehensive survey, these types of money demand form a fairly broad category sharing the principle that, in the presence of costly asset portfolio adjustment in the short-run, monetary holdings are used to "absorb" the impact of shocks. Thus, the cost-minimising response by an individual to a shock is not to re-adjust his/her asset portfolios immediately to the change in conditions, but rather to let his/her monetary balances fluctuate as a temporary buffer stock until the other assets can be adjusted. This implies that, between adjustments, the actual monetary holdings of individuals may deviate from their desired or target levels. At the macroeconomic level, this may translate into deviations of aggregate money demand from the equilibrium level.

The mechanism through which portfolio adjustment costs lead to non-linear monetary dynamics can be briefly illustrated by the main characteristics of the microfounded target-threshold money demand model by Miller and Orr (1966). These authors extend the Baumol-Tobin inventory theoretic model of the demand for transactions balances by households to the analysis of the optimal management of cash balances by firms. The main assumptions of Miller and Orr’s (1966) model are that: (1) there are only two assets: the firm’s non-remunerated cash balance and a portfolio of liquid assets bearing a daily interest v; (2) transfers between these two assets may take place at

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3See for instance Ordóñez (2003), Sarno (1999) and Sarno et al. (2003).
any point in time and with no delay, at the fixed cost of $\gamma$ per transfer; (3) a firm’s cash balance cannot fall below zero; and (4) the net cash flows are stochastic.

Because of the introduction of uncertainty about future cash flows, Miller and Orr (1966) is generally viewed as providing microfoundations to the demand for money for precautionary motive. In particular, these authors assume for expositional convenience that the cash balances follow a Bernoulli distribution: in each fraction of a working day ($1/t$) there is a probability $p$ that the cash balances increase by $m$ and a probability $q$ (with $q = 1 - p$) that they decrease by the same amount. Over an interval of $n$ days, the distribution of changes in the cash balances will have mean $\mu_n = ntm(p - q)$ and variance $\sigma^2_n = 4ntpqm^2$. In the special case of symmetry ($p = q = \frac{1}{2}$) the moments of the distribution become $\mu_n = 0$ and $\sigma^2_n = ntm^2$.

Consistently with standard inventory management theory, the objective of the firm is to minimise the long-run average cost of managing its cash-balances using a simple $(S, s)$ or target-threshold ”rule”. Under this rule, the firm refrains from engaging in frequent transfers and allows its cash balance to fluctuate unimpeded between a zero lower bound and a $h$ upper bound. Only once a bound is hit, the firm undertakes a transfer at the cost $\gamma$ to bring its balance back to a specific level $z$.

The firm’s optimisation problem consists of choosing $h$ and $z$ in order to minimise its expected daily cash management cost $E(c)$ over a finite planning horizon of $T$ days:

$$E_t(c) = \gamma \frac{E_t(N)}{T} + vE_t(M).$$

The first term of the LHS is given by the marginal cost per transfer, $\gamma$, times the expected number of transfers $E(N)$ over $T$ (i.e. the probability of daily transfers), the second term denotes the opportunity cost of holding the expected average daily cash balance $E(M)$. The optimal solution to the problem is then given by:

$$z^* = \left(\frac{3\gamma m^2 t}{4v}\right)^{1/3}; h^* = 3z^*,$$

from which it is evident that in equilibrium the demand for cash balances is also a function of the variance of cash flows, reflecting its use for precautionary purposes.
It should be noted that in the case of Miller and Orr’s (1966) and other target-threshold models of money demand, the non-linearity in the monetary dynamics derives from the assumption that the optimising firm follows a \((S, s)\) rule. However, as noted by Sarno (1999), portfolio adjustment costs can result in non-linearities in dynamic money demand also in buffer stock models, even if the behavior of firms is not rule-determined in these models. The forward-looking rational expectations model by Cuthbertson and Taylor (1987) is representative of this category. In this model the problem for the individual economic agent is to choose short-run monetary balances in order to minimise the expected discounted present value of an inter-temporal quadratic loss function:

\[
L = E_t \sum_{i=0}^{\infty} D^i \left[ a_o \left( M_{t+i} - M^*_{t+i} \right)^2 + a_1 \left( M_{t+i} - M_{t+i-1} \right)^2 \right],
\]

where \(D \in (0,1)\) is the discount factor, \(M\) denotes actual nominal monetary balances and \(M^*\) indicates the desired level of balances implied by a conventional long-run money demand function. The first term of the loss function measures the cost of being out of long-run equilibrium, while the second term represents the cost of adjustment in monetary balances. The rational-expectations solution to the minimisation problem is a money demand equation also including forward-looking components

\[
M_t = \lambda M_{t-1} + (1 - \lambda)(1 - \lambda D) \sum_{i=1}^{\infty} (\lambda D^i)E_{t-1} M^*_t + M^u_t + e_t,
\]

where \(\lambda \in (0,1)\), and \(M^u\) denotes monetary shocks. Deviations of money demand from equilibrium may now be determined not only by shocks but also by changes in expectations about the desired level of monetary balances in the future. The fact that the adjustment costs are significantly more important than the cost of being out of equilibrium implies that agents will react only slowly to changes in expectations about fundamentals that determine relatively small monetary disequilibria.\(^4\)

It is worth dedicating few words to the choice of functional form for the long-run money demand relationship describing the behavior of \(M^*\). This is

\(^4\)In their empirical analysis for the UK, Cuthbertson and Taylor (1995) estimate that the costs of adjustment are about 30 times more important than those of being out of equilibrium.
an issue that has gained significant attention in the debate on the welfare costs of inflation at times of low interest rates, such as the present. Lucas (2000) argues that a log-log functional form (i.e. taking the interest rate in natural logarithms) provides a superior description of the historical behavior of US money demand and a more precise calculation of the welfare costs of inflation at low interest rates. In addition, in the framework of the shopping-time model of money demand determination by McCallum and Goodfriend (1987), Lucas (2000) notes that, for reasonable estimates of the interest rate elasticity, the log-log money demand equation is more consistent with inventory-theoretic models, such as that by Miller and Orr (1966).

Chadha et al. (1998) concur on the theoretical superiority of the log-log form. They also use McCallum and Goodfriend’s (1987) model to show that the choice of any well-behaved utility function and transactions technology (e.g. Cobb-Douglas, CES and translog functions) is likely to result in a log-log specification of long-run money demand. However, using UK data, Chadha et al. (1998) find that the empirical advantages of the log-log specification may be more relevant for the dynamics of money demand than for its equilibrium behavior. More relevantly for this study, Stracca (2003) investigates empirically the issue of the choice of functional form for the long-run demand for M1 in the euro area and provides empirical evidence in support of using the log-log specification. On the basis of these considerations, in the empirical analysis we use a log-log functional form of the equilibrium money demand relationship.

3 The long-run equilibrium in the money demand

The empirical analysis of M1 demand in the euro area follows the general-to-specific approach for the cointegration analysis of vector autoregression (VAR) models with Markovian regime-shifts proposed by Krolzig (1997). This approach consists of two stages. In the first stage (the object of the next sub-section), the standard multivariate cointegration analysis procedure by Johansen (1995) is applied to a system of variables to determine the cointegrating rank and estimate the identified long-run money demand relation-

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5We also experimented with smooth-transition models. However, it was not possible to estimate the parameters governing the regime transition with precision.
ship. Since the focus of this exercise is on the identification and estimation of the equilibrium money demand relationship, evidence of weak exogeneity is used to reduce the original system to a smaller, conditional model. In the second stage (dealt with in the next Section), a Markov-switching model of the dynamics of monetary balances is selected and estimated, conditional on the cointegrating matrix estimated in the first stage. The outcome is an error-correction model of real M1 characterised by a non-linear dynamics of adjustment to monetary disequilibria.

The study is based on quarterly data for the euro area – defined according to the principle of changing composition (the 11 original countries up to 2000Q4; these plus Greece, thereafter) - over the period 1971Q4 to 2003Q3. The variables modelled consist of the narrow monetary aggregate M1 ($M_t$) deflated by the GDP deflator ($P_t$), real GDP ($Y_t$) and the short-term market interest rate ($R_t$). Nominal M1 is the period average of the end-of-month seasonally-adjusted (s.a.) notional stock compiled by the ECB. The GDP data are based on the aggregation of s.a. national accounts data (ESA95 whenever available) up to 1998Q4; hereafter, on area-wide Eurostat statistics. The national data on M1 and GDP prior to the introduction of the euro have been aggregated using the irrevocable conversion rates announced on 31 December 1998 (19 June 2000 for Greece).\textsuperscript{6} The interest rate is a weighted average (based on GDP weights at 2002 purchasing power parities) of national 3-month interbank interest rates up to 1998Q4; thereafter, it corresponds to the three-month EURIBOR.

The long-run money demand function is specified in a log-log form:

\begin{equation}
(m - p)_t = \beta_1 y_t - \beta_2 r_t
\end{equation}

where variables in italics indicate natural logarithms. As noted above, Lucas (2000) argues that this functional form presents significant advantages over alternative specifications in terms of sounder micro-foundations and of closer fit to the US data. The empirical investigation by Stracca (2003) confirms that this may also be the case for the euro area.

As a preliminary step, the statistical properties of the data are examined using standard unit root tests (augmented Dickey-Fuller and Phillips-Perron) as well as the KPSS stationarity test. The results - not reported for the sake

\textsuperscript{6}The choice of aggregation method for the historical national data has received a significant amount of attention in the literature on euro area money demand (see, for instance, Fagan and Henry, 1998 and Beyer, Doornik and Hendry, 2001).
of brevity - suggest that over the sample period considered all the variables in the system should be modelled as \( I(1) \) in levels.

The cointegrating properties of the system \( z_t = [(m - p)_t, y_t, r_t] \) are tested by means of the multivariate cointegration procedure by Johansen (1995):

\[
\Delta z_t = v + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + \alpha \beta' z_{t-1} + \Psi D_t + u_t \tag{2}
\]

where the parameters of the model are represented by the vector \( v \) of deterministic components, the matrices \( \Gamma \) and \( \Psi \) of short-run coefficients, the vector \( \alpha \) of loading factors and the vector \( \beta \) of long-run coefficients. \( \beta' z_{t-1} \) denotes the one-period lagged money-demand error correction term implied by the cointegrating vector; \( D_t \) is a vector of \( I(0) \) exogenous variables; and \( u_t \) is the error vector (assumed to be serially uncorrelated with zero mean and constant covariance matrix). Consistent with Stracca (2003), \( D_t \) includes two impulse dummies (\( ID99Q1 \) and \( ID00Q1 \)) taking the value 1 in the first quarter of 1999 and 2000, respectively, and zero elsewhere, as exogenous variables.\(^7\)

The application of the Johansen (1995) procedure enables to determine the number of cointegrating vectors and, subject to appropriate specification testing, allows to identify and estimate such vectors. On the basis of the Akaike, Hanna-Quinn and Schwartz information criteria, the lag order \( p \) of the testing VAR (including linear trends in the levels of the data and an unrestricted intercept in the cointegrating vector) is set at 2. Panel A of Table 1 reports the Johansen’s trace (\( \lambda_{\text{trace}} \)) and maximum eigenvalue (\( \lambda_{\text{max}} \)) cointegrating tests. Both tests reject the hypothesis of no cointegration at the conventional significance levels, while accepting that of at most one cointegrating relationship. The evidence of cointegration is robust to the use of test statistics adjusted for degrees of freedom (as suggested in Reimers, 1992) in order to control for potential small-sample bias.

\(^7\)The first dummy is introduced in order to control for the exceptionally large rise in the demand for M1 holdings (especially for overnight deposits) recorded in correspondence with the start of Stage Three of European Monetary Union in January 1999. This large rise probably reflected temporary uncertainty regarding the new monetary policy regime as well as the associated institutional novelties (e.g. the introduction of a new reserve requirements system). The second dummy controls for the temporary rise in demand for currency at the time of the “millennium bug” scare, when concerns about possible disruptions to retail payment systems and cash dispensing machines became widespread in many euro area countries.
The results of the long-run exclusion tests in Panel B show that none of the variables can be excluded from the cointegrating vector at the conventional significance levels. Furthermore, the tests for weak exogeneity reveal that $y$ and $r$ can be treated as weakly exogenous to the system, both individually and jointly. This finding is important since it implies that, in the spirit of the general-to-specific approach, the system can be reduced to a single equation without incurring a loss of information from not modelling the determinants of $\Delta y$ and $\Delta r$.

The estimated cointegrating vector (normalised with respect to real $M_1$ and to zero mean) conditional on joint weak exogeneity of real GDP and the short-term interest rate is presented in Panel C. The estimated income elasticity is 0.740. This estimate is consistent with theoretical predictions as it falls between the value of 0.5 anticipated by the Baumol-Tobin inventory-theoretic model of transaction demand for money and the unitary elasticity implied by the quantity theory.8 The interest rate elasticity of the demand for real $M_1$ is estimated at −0.391. Because of the relatively low and sluggish average remuneration of the deposits included in $M_1$ (which also includes zero-remunerated currency in circulation), this interest rate can be interpreted as approximating the opportunity cost of holding this monetary aggregate. Given the functional form, the interest rate elasticity is constant across interest rates and measures the percentage change in the demand for money in response to a one percent change in the short-term interest rate.

On the basis of the magnitude and sign of the coefficients, this cointegrating vector can be interpreted as representing a long-run demand function for real $M_1$.

Given the relatively broad time span covered by the sample period, which comprises periods of both high and low interest rates, it is important to test for the stability of the coefficients of the equilibrium money demand relationship. For this purpose, we apply two types of Nyblom tests for parameter constancy of the cointegrating vector as extended to cointegrated VARs by Hansen and Johansen (1999). The null hypothesis of these tests - which are respectively based on the maximum (Sup) and the mean (Mean) of a weighted LM-type statistics over the sample period - is the joint stability of the parameters of the cointegrating vector. The supremum and mean test statistics

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8In the conditional model, the hypothesis of a unitary income elasticity is rejected by the data ($\chi^2=8.70 \quad [p\text{-value}=0.04]$).
yield 1.60 \[p\text{-}value=0.53\] and 0.98 \[p\text{-}value=0.20\], respectively. The high level of the \(p\)-values indicates that the null hypothesis cannot be rejected at the conventional significance levels, suggesting that the long-run parameters are jointly-stable over a sample period covering the last three decades.

Conditional on the findings of joint weak exogeneity for \(y\) and \(r\), the dynamic model is specified as a single equation error-correction model. The estimated equation is reported in Panel D. In particular, the coefficient of the error correction term is negative and statistically significant, supporting the interpretation of the cointegrating vector as a long-run money demand function. Yet, the relatively small size of the coefficient (-0.051) reveals a rather sluggish adjustment to equilibrium in case of deviation. This is illustrated in Figure 1 by the slow rate at which monetary disequilibria are corrected.

{Insert Figure 1}

Finally, the statistical properties of the residuals of the model are evaluated by means of several standard mis-specification tests for autocorrelation, non-normality and heteroscedasticity. Their results are satisfactory suggesting that the model is adequately specified. However, we fail to reject the null-hypothesis of no mis-specification of the RESET test. Originally developed to test for omitted regressors, a significant value of the RESET statistic may often be indicative of non-linearity in the residuals (see Granger and Teräsvirta, 1993). This suggests that the specification of the equation may be improved by modelling explicitly such non-linearity. The next section formally investigates this issue.

4 Modelling the non-linear dynamics of M1

The analysis of the residuals of the linear error-correction model suggests that a standard model with time invariant parameters may not provide an appropriate representation of the short-run dynamics of M1. Such dynamics may be better captured by a model allowing for some form of regime-dependent behavior. In particular, if the non-linear process is time-invariant conditional on an (unobservable) regime variable \(s_t\), a Markov-Switching model may be considered as a general framework. The idea behind this class of model is

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9The distributions of the tests are bootstrapped using 1,000 replications. The computations are performed using the program Structural VAR, version 0.19, by Anders Warne (downloadable from www.texlips.hypermart.net/svar).
that the parameters of the underlying data generating process of the observed time series vector $z_t$ depend upon an unobservable regime variable $s_t$, representing the probability of being in a certain state of the world.

Letting $s_t \in \{1, \ldots, M\}$ indicate the regime prevailing at time $t$ and $p$ the lag-length, the properties of a generic MS($M$)-VECM($p$) model can be analysed depending on the realization of the regime:

$$
\Delta z_t = v(s_t) + \sum_{i=1}^{p-1} \Gamma_i(s_t) \Delta z_{t-i} + \Pi(s_t) z_{t-p} + u_t,
$$

where $v(s_t)$ is a $k$-dimensional vector of regime-dependent intercept terms, $\Gamma_i(s_t)$ are the $k \times k$ matrices of short-run parameters, $\Pi(s_t)$ are the long-run impact matrices and $u_t$ is the vector of disturbances. In particular, the $\Pi(s_t)$’s are the state-dependent matrices defined by the $r \times k$ matrix of cointegrating vectors ($\beta'$), and the $k \times r$ state-dependent matrices of adjustment coefficients $a(s_t)$. Thus we have that $\Pi(s_t) = a(s_t) \beta'$. The underlying hypothesis is that the equilibrium relationship among the variables (in levels) does not vary across regimes, it is only the speed of the adjustment to the error correction term that is allowed to vary. Finally, also the vector $u_t$ depends on the realization of the regimes since $u_t \sim NID(0, \Sigma(s_t))$.\(^{10}\)

Since the parameters depend on a regime which is assumed to be stochastic and unobservable, a generating process for the states $s_t$ has to be formulated. Then, using this law, the evolution of regimes can be inferred from the data. In particular, the stochastic process generating the unobservable regimes is assumed to be an ergodic Markov chain defined by the transition probabilities:

$$
p_{ij} = \Pr(s_{t+1} = j \mid s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1 \quad \forall i, j \in \{1, \ldots, M\}.
$$

Thus, by inferring the probabilities of the unobservable regimes conditional on the available information set, it is possible to reconstruct the regimes’ evolution. In particular, the MS-VECM can be estimated using a two-stage

\(^{10}\)The general model in which the intercept term, the autoregressive parameters (including the adjustment coefficient), and the disturbances’ variance are allowed to vary with the regime switch is labelled as MSIAH.
maximum likelihood procedure. As shown by Saikkonen (1992) and Saikkonen and Luukkonen (1997), in the first stage it is not necessary to model the Markovian regime shifts explicitly in order to derive the equilibrium relationships. So in the first step the usual Johansen (1995) procedure is applied to determine the cointegration rank and estimate the cointegration matrix. In the second stage the expectation-maximization (EM) algorithm is applied to obtain the estimates of the remaining parameters, considering the ECT as an exogenous variable (Dempster et al.; 1997, Krolzig; 1997).

In order to select the specification of the model we first run a battery of tests of linearity against various types of Markov-switching models to assess the relevance of Markov-Switching non-linearity. We subsequently use various statistics to select among the various possible Markov-switching specifications.

{Insert Table 2}

The first column of Table 2 reports the p-value of the upper-bound of the Likelihood-Ratio (LR) statistic, testing the null-hypothesis of linearity against the alternative of a specific type of Markov-Switching non-linearity. On the basis of the LR test, the data fails to reject the null of linearity for the models specifying regime switching behavior only for the intercept term (MSI), the variance-covariance matrix (MSH) or the autoregressive components of the error-correction model (MSA). By contrast, the null of linearity is easily rejected at the conventional significance levels for the specifications combining different types of regime-dependence behavior: in the intercept and autoregressive component (MSIA), in the intercept and variance-covariance matrix (MSIH) and in the intercept, variance-covariance matrix and autoregressive component (MSIAH). These results suggest that in order to identify and describe the regimes in the data it is necessary to use models specifying a quite general form of regime-switching.

The second column shows the p-values of the restriction-testing procedure. In particular, the null hypothesis of no autoregressive components shifting (MSIAH versus MSIH) is strongly rejected by the data. By contrast, the null of shifting in the variance-covariance matrix (MSIAH versus MSIA) can not be rejected.

However, there are some indications that the more general MSIAH specification is to be preferred. In particular, the Regime Classification Measure

11 Only for the MSAH model, in which the autoregressive component and variance-covariance matrix are regime-dependent, the null of linearity can not be rejected.
(RCM) proposed by Ang and Bekaet to discriminate among different types of Markov-Switching models suggests a better fit of the MSIAH (fourth column of Table 2). The RCM is a summary point statistic of the degree of accuracy with which a model identifies regime switching behavior over the sample period. The statistic ranges between 0 and 100, with 0 denoting a perfect regime classification performance and 100 indicating that the model fails to provide any information on regime-dependence. The values of the RCM statistic recorded for the MSIAH specification is fairly low, and significantly smaller that the RCM for the MSIA model. In addition, the MSIAH model seems to fit better the data as can be evinced from the higher value of the adjusted Coefficient of determination: 0.66 versus 0.63 (fifth column of Table 2). Finally, the dating cycle identified by the MSIA model is relatively more volatile and harder to relate to the economic developments in the euro area over the sample period.

On the basis of these considerations, in the rest of this section we restrict our attention to the MSIAH specification. However, before presenting the results, we finally test whether it may be statistically more appropriate to use a model allowing for 3 instead of 2 regimes. The null of a two-regime MSIAH model versus a three-regime model can be rejected only at the 10% significance level (third column of Table 2). This suggests that it may preferable to retain the more parsimonious specification specifying only two regimes.

Table 3 reports the estimation results from the MSIAH(2)-ECM(1) specification. In each regime there are a large enough number of observations for robust statistical inference. The regimes are fairly persistent, with the conditional probabilities ($p_{11} = 0.94, p_{22} = 0.90$) implying average duration of around 4.5 years and 2.5 years, respectively, for the first and second regime.

Standard mis-specification tests (not reported) fail to reveal signs of autocorrelation, non-normality or heteroscedasticity for both the standardised residuals and the one-step prediction errors, suggesting that the model is satisfactorily specified.\footnote{However, the results of these tests should be interpreted with caution given that their asymptotic distributions may not be valid for residuals from Markov-switching models.}

Figure 2 depicts the smoothed probabilities of being in Regime 1 together with the annual growth rate of real M1. Regime 1 includes the periods of highest volatility in real monetary growth over the last thirty years. In par-
ticular, it comprises a protracted period of relatively low but volatile growth in real M1 throughout most of the 1970s and beginning of the 1980s as well as a long time span of relatively high and volatile monetary growth throughout the 1990s. By contrast, the probabilities of Regime 2 are associated with periods of relatively stable and moderate money growth.\textsuperscript{13} Consistent with these observations, the standard error of the residuals is higher in Regime 1 (0.62\%) than in Regime 2 (0.41\%).

The theoretical models surveyed in the previous section would lead to predict that the process of adjustment to equilibrium should be more effective during the first regime - characterised by more extreme developments in monetary balances - than in the second regime. Indeed, buffer stock models would suggest that in the periods when the behavior of money deviates significantly from its norm, agents should adjust to the “desired” level at a higher speed than in tranquil periods. The regime-dependent coefficient of adjustment provide some support to this hypothesis. In both regimes the coefficient of adjustment has the expected negative sign and is significantly different from zero. However, in Regime 1 the estimated coefficient is larger than in Regime 2 (0.073 and 0.053, respectively), confirming the hypothesis of differences in the speed of monetary disequilibria adjustment depending on the prevailing monetary conditions regime.

It should be noted that, while the value of the coefficient of adjustment in Regime 2 is fairly close to the estimate for the linear model, the estimated loading factor in Regime 1 implies a faster adjustment to equilibrium. Ceteris paribus, the process of monetary disequilibrium adjustment should be about 1\textfrac{1}{4} years shorter in the first regime than in the second regime.

These stylized facts find further confirmation in the behavior of the error-correction term in Figure 1. High probabilities of being in Regime 1 - the regime in which the coefficient of adjustment of the error-correction term is higher - are typically associated to periods in which the deviations from equilibrium are large. By contrast, the probabilities of being in Regime 2 - in which the adjustment to equilibrium is slower - are usually higher in correspondence with periods of relatively small deviations from equilibrium. To sum up, our empirically findings seem to provide evidence that euro area agents asymmetrically react to the deviations in their holdings of monetary

\textsuperscript{13}As for the last period, after the cash-changeover of January 2001 – event associated with a significative drop in the holdings of currency – the demand for M1 increased constantly over a relatively long time span to recover to level consistent with the average growth rates recorded before the changeover.
5 Conclusion

This paper investigates empirically whether is evidence of non-linearities in the dynamic behavior of the demand for real M1 in the euro area. Using a fairly general Markov-switching error-correction model, the paper provides some evidence that in the euro area agents asymmetrically react to deviations of their holdings of monetary balances from the desired level. When these balances deviate significantly from the desired level, agents engage more intensively in costly transactions to bring them back to target, than when the deviations are relatively small. These empirical findings are consistent with theoretical predictions by buffer stock and target-threshold models and with analogous results for several European countries and the US recently reported in the empirical literature.
References


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<th>Table 1 Johansen procedure</th>
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<td><strong>A. Cointegration tests</strong></td>
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<tr>
<td>0.09891</td>
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**B. $\chi^2$ restriction tests (conditional on unitary rank)**

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<th>(m - p)</th>
<th>y</th>
<th>r</th>
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<tbody>
<tr>
<td>Exclusion</td>
<td>$\chi^2_1 = 17.54$ [0.00]</td>
<td>$\chi^2_1 = 13.00$ [0.00]</td>
</tr>
<tr>
<td>Weak exogeneity</td>
<td>$\chi^2_1 = 13.44$ [0.00]</td>
<td>$\chi^2_1 = 2.43$ [0.12]</td>
</tr>
<tr>
<td>Joint weak exogeneity (y and r)</td>
<td>$\chi^2_2 = 2.99$ [0.22]</td>
<td></td>
</tr>
</tbody>
</table>

**C. Estimated cointegrating vector**

(conditional on weak exogeneity of y and r)

$$ (m - p) = 0.744 y - 0.392 r $$

(0.07) (0.04)

**D. Dynamic money demand equation**

$$ \Delta(m - p)_t = -0.150 - 0.051 ECT_{t-1} + 0.178 \Delta(m - p)_{t-1} - 0.084 \Delta y_t $$

$$ + 0.013 \Delta y_{t-1} - 0.024 \Delta r_t - 0.022 \Delta r_{t-1} + 0.027 I^D99Q1_t $$

$$ + 0.031 I^D99Q1_t + \varepsilon_t $$

T = 128; $R^2 = 0.67$; s.e.(ε_t) = 0.68%; $LM(1) : F(1,118) = 1.52[0.22]$; $LM(1-5) : F(5,114) = 0.79[0.56]$; $ARCH(1-4) : F(4,111) = 0.63[0.64]$; $NORM : \chi^2_2 = 1.58[0.45]$; $HET : F(14,104) = 1.21[0.28]$; $RESET : F(1,118) = 6.01[0.02]$ |

Note: $\dagger$ denotes adjustment for degrees of freedom as in Reimers (1992);

**(*) rejection of the null hypothesis at 1% (5%) critical level.
P-values in square brackets; standard errors in parentheses.
### Table 2 Identification procedure

<table>
<thead>
<tr>
<th></th>
<th>LR-linearity</th>
<th>LR-restrictions</th>
<th>LR-regimes</th>
<th>RCM</th>
<th>Adj R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSI</td>
<td>0.143</td>
<td>0.000</td>
<td>0.182</td>
<td>36.6</td>
<td>0.64</td>
</tr>
<tr>
<td>MSA</td>
<td>0.367</td>
<td>0.179</td>
<td>0.013</td>
<td>63.2</td>
<td>0.65</td>
</tr>
<tr>
<td>MSH</td>
<td>0.999</td>
<td>0.000</td>
<td>0.999</td>
<td>99.8</td>
<td>0.62</td>
</tr>
<tr>
<td>MSAH</td>
<td>0.294</td>
<td>0.408</td>
<td>0.951</td>
<td>18.5</td>
<td>0.65</td>
</tr>
<tr>
<td>MSIA</td>
<td>0.048</td>
<td>0.981</td>
<td>0.132</td>
<td>31.3</td>
<td>0.63</td>
</tr>
<tr>
<td>MSIH</td>
<td>0.047</td>
<td>0.002</td>
<td>0.728</td>
<td>25.7</td>
<td>0.64</td>
</tr>
<tr>
<td>MSIAH</td>
<td>0.034</td>
<td>-</td>
<td>0.093</td>
<td>21.7</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Note: For the LR statistics only p-values are reported
Table 3 MSIAH(2)-ECT(1) estimation

<table>
<thead>
<tr>
<th>Transition probabilities</th>
<th>Regime properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reg 1</td>
</tr>
<tr>
<td>Reg 1</td>
<td>0.9429</td>
</tr>
<tr>
<td>Reg 2</td>
<td>0.0971</td>
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</tbody>
</table>

Regime 1

<table>
<thead>
<tr>
<th></th>
<th>Coef</th>
<th>t-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const</td>
<td>0.004</td>
<td>3.0498</td>
</tr>
<tr>
<td>DLM1R1</td>
<td>-0.036</td>
<td>-0.3831</td>
</tr>
<tr>
<td>DLYR</td>
<td>-0.31</td>
<td>-1.8731</td>
</tr>
<tr>
<td>DLYR1</td>
<td>0.0893</td>
<td>0.561</td>
</tr>
<tr>
<td>DLST</td>
<td>-0.0276</td>
<td>-2.5747</td>
</tr>
<tr>
<td>DLST1</td>
<td>-0.021</td>
<td>-1.6653</td>
</tr>
<tr>
<td>ECT1</td>
<td>-0.0733</td>
<td>-5.8973</td>
</tr>
<tr>
<td>ID00Q1</td>
<td>0.0334</td>
<td>4.7432</td>
</tr>
<tr>
<td>ID99Q1</td>
<td>0.0311</td>
<td>4.5974</td>
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</tbody>
</table>

Regime 2

<table>
<thead>
<tr>
<th></th>
<th>Coef</th>
<th>t-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const</td>
<td>0.0088</td>
<td>3.7789</td>
</tr>
<tr>
<td>DLM1R1</td>
<td>0.2675</td>
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</tr>
<tr>
<td>DLYR</td>
<td>0.0243</td>
<td>0.1888</td>
</tr>
<tr>
<td>DLYR1</td>
<td>-0.2431</td>
<td>-1.959</td>
</tr>
<tr>
<td>DLST</td>
<td>-0.0055</td>
<td>-0.5899</td>
</tr>
<tr>
<td>DLST1</td>
<td>-0.0239</td>
<td>-1.9263</td>
</tr>
<tr>
<td>ECT1</td>
<td>-0.053</td>
<td>-3.8165</td>
</tr>
<tr>
<td>ID00Q1</td>
<td>0.0272</td>
<td>0.6869</td>
</tr>
<tr>
<td>ID99Q1</td>
<td>0.0225</td>
<td>0.7645</td>
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</tbody>
</table>

Std error (Reg.1) 0.006177 Std error (Reg.2) 0.00409
Figure 1: Money demand error-correction term and the regime switching

Figure 2: Real money growth and Regime 1 smoothed probabilities