EXCHANGE RATE VOLATILITY AND OUTPUT VOLATILITY: A THEORETICAL APPROACH

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ABSTRACT

In this paper we make an attempt to determine exchange rate and exchange rate uncertainty as well as output and output variability. In the context of an ad hoc model of a small open economy under flexible exchange rates regime we assume that prices are affected by unanticipated shocks, rational expectations hold and there is capital mobility. We find that the level both of exchange rate and output is affected by monetary and inflationary shocks, as well as shocks in government spending, output and trade balance. The most interesting result is that the uncertainty of exchange rate and output has been proved to be associated positively with the uncertainty of all shocks while the contemporaneous occurrence of selected shocks imposes either positive or negative impact on exchange rate and output volatility. Moreover, it is shown that the effect of the determinants either of exchange rate volatility or output volatility is very sensitive to the parameter values. Some of the findings of exchange rate uncertainty are similar to those obtained by the optimizing behaviour framework of Obstfeld and Rogoff (1995, 1996).

Keywords: exchange rate volatility, output volatility, open-economy models

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I. INTRODUCTION

An important issue in the theoretical and empirical macro literature concerns the relationship between macroeconomic uncertainty and macroeconomic performance indicators such as the output growth rate. Macro uncertainty obtains from unanticipated changes of important macro variables which affect either the growth process of the economy or the country’s economic partners, in the short run and long run. Lately, there has been an increasing focus on the analysis of volatility of macro variables, in particular volatility of output growth (Stock and Watson, 2002). This emphasis is motivated by the stylized fact of falling output growth volatility in the G7, a result dubbed the Great Moderation.

Following the collapse of Bretton Woods system of fixed exchange rates (in the late 1960s), the main country participants in global trade (USA, Japan, European countries) adopted a floating exchange rates regime early in 1973 based on the forces of supply and demand in each economy. What should be taken into consideration is the impact of this new regime on economies’ welfare. Therefore, a very interesting matter concerns the effect of exchange rate variability on the real economy, and more specifically, how exchange rate uncertainty affects the growth of a real economy.

As impending international currency crises erupted in the late 1960s, many economists began advocating the flexibility of exchange rates. Many of them argued that a floating exchange rates regime would be beneficial for the world economy because: (a) monetary policy autonomy is retained. Governments would be able to attain internal and external balance through monetary policy without the intervention of central banks. (b) Symmetry. Under a system of flexible exchange rates the United States would have the same opportunity as other countries to influence its exchange rate. (c) Exchange rates as automatic stabilizers. Countries would maintain (through exchange rate changes) internal and external balance according to the changes in aggregate demand.

A much discussed consequence of the flexible exchange rate regime is the resulting exchange rate uncertainty. This type of uncertainty is considered to be an important hindrance for the trade between countries and for their growth in the long run. Fluctuating currencies make importers uncertain about the prices they will have to pay for goods in the future, and make exporters more uncertain about the prices they will receive. Exchange rate uncertainty is claimed to make trade
volumes worsening and gains from trade shrinking. In other words, if there is no growth in trade volumes, then there will be welfare failure globally. Although, a priori, there is a negative impact of exchange rate uncertainty on growth and welfare, there is some empirical evidence of positive effects of exchange rate uncertainty on trade and growth.

The literature to date on macro uncertainty focuses on the impact of this uncertainty on macro variables, such as output growth, inflation, trade volumes and investment. However, there seems to be a lack of studies examining the determinants of macro uncertainty from both a theoretical and an empirical point of view. To fill this gap, at least partially, in the theoretical literature, this study purports to examine the determinants of exchange rate and output uncertainty in an open-economy macro model. The paper, which is stimulated by Driskill and McCafferty (1980), develops a model of a small, open economy under floating exchange rates regime that incorporates uncertainty and seeks to define the determinants of exchange rate volatility and how fluctuations of these factors affect exchange rate uncertainty. We assume an exogenous price level which is determined by inflationary shocks and an aggregate demand function which depends on relative prices, interest rate and government spending which is also defined by shocks on government expenditures. In addition, we try to model output volatility and analyze its determinants. It is assumed that agents have rational expectations; there is capital mobility; the domestic output, which is dependent on government expenditure, affects trade.

In our model we find that the determinants of exchange rate volatility is the volatility of money supply shocks, price volatility, government spending volatility, output volatility and trade balance volatility. In addition, we assume that exchange rate volatility is determined by the existence of contemporaneous shocks in (i) money supply and prices, (ii) money supply and government spending, (iii) prices and trade balance, (iv) government spending and trade balance, and, (v) output and trade balance. We also find that the previous types of volatility and the covariance of selected shocks determine output volatility as well. Furthermore, the construction of the model is based on an ad hoc monetary framework and not on an optimizing behaviour model like the model of Obstfeld and Rogoff (1995, 1996). However, as shown below, the two models lead to some common findings.
Finally, we illustrate the comparative-statics results for exchange rate volatility and output volatility using an IS-LM-BP model with flexible exchange rates.

In section II we appose the relevant studies to exchange rate volatility and in section III we state the theoretical open-economy macro model for the analysis of exchange rate volatility. In section IV we solve the model for the exchange rate process under rational expectations. Section V sets out the comparative-statics results between exchange rate variance and the variance of its determinants. In section VI we review the empirical literature on the relationship between output volatility and other macro variables such as inflation and growth. In section VII we solve the model for output volatility process and we compute the comparative-statics between output volatility and the variance of its determinants. Section VIII infers the main results.

II. THE DETERMINANTS OF EXCHANGE RATE VOLATILITY

(i) THEORETICAL LITERATURE

Driskill and McCafferty (1980) examine the exchange rate model in terms of uncertainty in a small open economy and under flexible exchange rate regime. They assume that rational expectations hold and they shed light on the capital mobility role. They claim that high capital mobility: a) increases the portfolio variability when changes in anticipated assets relative returns take place; b) decreases the exchange rate volatility which is caused by real shocks; c) increases the exchange rate volatility when unanticipated shocks occur within the economy. In other words, the exchange rate uncertainty depends on the source of the shock that hits the economy. Moreover, Driskill and McCafferty find that there is a correlation among trade balance, changes in relative prices and changes in spot and forward rates. The inclusion of uncertainty permits to search the sources of exchange rate variability to the interaction of real and monetary shocks.

Later, Turnovsky and Bhandari (1982) extending the Driskill-McCafferty analysis focus not only on the short-run effects of structural shocks of domestic and foreign variables on domestic economy, but also they examine the impact of the degree of capital mobility on the determinants’ variance. Under the assumptions of flexible exchange rates, rational expectations and imperfect capital mobility, they conclude that the balance of payments determines greatly the exchange rate.
The major inference they draw is that an increase in capital mobility increases the variance of the trade balance irrespectively of the source of the shock. The existence of a shock in the foreign price level and the foreign nominal interest rate in combination with the increasing capital mobility leads to an increase in the variance of domestic real output, prices, interest rate and exchange rate while the variance of foreign prices causes a decrease in the exchange rate. Furthermore, the occurrence of a supply disturbance affects positively the variance of real domestic output and negatively the variance of the domestic price level as well as the exchange rate. It is found that a monetary shock imposes a vague impact on the variance of domestic interest rate. A random demand shock is shown to affect negatively the variance of the domestic variables (apart from trade balance) while a monetary shock affects positively the variance of all the domestic variables with exception to the variance of the interest rate.

More recently, Driskill and McCafferty (1987) determine the exchange rate (and therefore exchange rate volatility) under a flexible regime extending their previous model. In this paper a classical equilibrium model is developed including the assumption of risk-aversion and the considerations about goods-market. They assume that domestic price is not constant and thus may affect the money market equilibrium. Also, the asset demand equation is derived from optimizing behaviour rather than ad hoc theory. From this perspective, Driskill and McCafferty may investigate the changes in preferences and technology and may clarify the existence of multiple equilibria.

Later, Manuelli and Peck (1990) consider an overlapping generation model with stochastic endowments in their attempt to determine the exchange rate and its uncertainty. They assume that: a) the exchange rate depends on the entire history of endowments; b) there is only one commodity and they focus on the role of international currencies. Exchange rate volatility is measured in terms of variance, taking into account the whole available information. Although they find that there are many exchange rate processes that are equilibrium processes (indeterminacy of exchange rate equilibrium), they prove: i) the existence of equilibria under flexible exchange rates regime despite of the volatility in fundamentals; ii) the consumption process occurs to be that of a constant exchange rate regime.

Furthermore, Betts and Devereux (1996) seek to determine the exchange rate in a pricing-to-market model. They assume stickiness of prices as Obstfeld and Rogoff (1995) introduce and they
analyze an optimizing behaviour model. The exchange rate is found to be a function of consumption differentials, the elasticity of demand for consumption goods, the elasticity of demand for money, and the portion of goods under pricing-to-market regime. Their findings reveal that the greater the portion of goods under pricing-to-market regime the higher the relative exchange rate variance. In other words, the model of pricing-to-market imposes a great impact on the uncertainty of exchange rate.

(ii) EMPIRICAL LITERATURE

In empirical literature, exchange rate volatility is usually found as a deterministic factor of other important macroeconomic variables. Although our purpose is to appose those studies in which exchange rate volatility is the dependent variable, we find more evidence for the determination of the level of exchange rate.

Bayoumi and Eichengreen (1998) review the effect of Optimum Currency Areas (OCA)\(^1\) variables on nominal exchange rate volatility including the pressure of government intervention on exchange market. It is found that the terms of trade, the capital control and the money supply impose a negative impact on exchange rate volatility. Later, Canales-Kriljenko and Habermeier (2004) show that the volatility of exchange rate is determined by structural macroeconomic variables such as: inflation, GDP growth, fiscal deficit and trade openness. What they also find is that countries under flexible exchange rate regime are characterized from greater volatility on the exchange rates. In addition, researchers conclude that the volatility in terms of trade does not affect significantly the exchange rate volatility.

Karras (2006) uses two different data sets\(^2\) and three different methods\(^3\) in order to provide evidence for the linkage between exchange rate variability and trade openness. Computing the standard deviation of exchange rate he infers that openness imposes a positive but insignificant impact on exchange rate variability but this effect appears to be positive and significant in the second

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\(^1\) OCA variables include: a) output volatility (in terms of standard deviation); b) export heterogeneity; c) terms of trade; d) size of economy; e) capital control; f) money supply; g) interest rate volatility.

\(^2\) The first one consists of 56 countries over the period 1951-1998 and the other of 105 countries covering the time span 1960-1997.

\(^3\) The first is referred to the calculation of growth rates of real GDP, consumption and investment. The second is the Hodrick-Prescott (HP) filter and the third is the Band-Pass (BP) filter.
data set when applying HP and BD filters. Moreover, he concludes that the higher the trade openness the lower the depreciation rate of exchange rate but not significantly. Additionally, his findings show that exchange rate variability is inversely related to trade openness and economic size for both samples and according to methods that are implemented.

Apart from the already apposed studies there are others in which exchange rate volatility is found to affect exports volume either negatively according to Pozo (1992), Ozbay (1999), Cho, Sheldon and McCorriston (2002), Grier and Smallwood (2003) and Agolli (2004) or positively as proposed by McKenzie and Brooks (1997) and Doyle (2001) while Tenreyro (2004) supports that nominal exchange rate volatility does not affect trade flows. Moreover, there is evidence in favor of the significant impact of exchange rate volatility on foreign direct investment and private investment according to Esquivel and Larrain (2002), Byrne and Davis (2003), Alaba (2003), Becker and Hall (2003), Lemi and Asefa (2003), Kiyota and Urata (2004) and Iannizzotto and Miller.

Singh (2002) in an attempt to examine the determinants of export and trade weighted real effective exchange rate as well as unweighted official and black market real exchange rate focuses the interest on India for the time periods 1975-1996 and 1960-1996 respectively. The results reveal that exchange rate volatility, which is measured as the conditional variance using ARCH and GARCH processes, imposes a strong negative effect on both types of real exchange rate. Furthermore, a study of Supaat et al (2003) is indicative of the impact of exchange rate volatility as well as volatility of bilateral nominal exchange rate on volatility of output, 3-month interbank rate, exports, imports, money. In a sample of 8 countries (Singapore, Hong Kong, Korea, Taiwan, Malaysia, Indonesia, Thailand, Philippines) over the period 1980-2002, researchers find evidence that exchange rate volatility does not impose a strong impact on the above macroeconomic variables in terms of uncertainty, while they find that an increase in the volatility of bilateral nominal exchange rate between Singapore and other country cause a decrease in the volatility of the dependent variables. Extending their results, they find a weak negative relationship between exchange rate volatility and bilateral trade.
III. THE MODEL

The model is of a small, open economy with a perfectly elastic supply of imports at a fixed world price. The main difference with Driskill and McCafferty (1980) lies to the domestic output which is not exogenous. We propose an aggregate demand specification depending on the level of real exchange rate (relative prices) and the government spending volume. Domestic residents can hold either bonds (domestic or foreign) or domestic money. Foreigners are assumed to hold bonds and foreign money.

The building blocks in the model are: (1) a standard money market equilibrium condition (2) the goods market where the output is demand determined, and, (3) the foreign exchange market represented by the Balance of Payments (BOP) equilibrium condition. Furthermore, we assume rational expectations for exchange rate. All equations are presented in their log-linear form.

A. The Money Market

Money demand is assumed to be:

\[ m^d_t = p_t - \lambda r_t, \quad \lambda > 0 \]  

where \( m^d_t \) is the log of domestic money demand, \( p_t \), \( r_t \) reflect the domestic price level and the domestic interest rate, respectively and \( \lambda \) is a positive coefficient indicating that money demand is a decreasing function of interest rate. Money supply is assumed to have the form:

\[ m^s_t = \bar{m} + \epsilon_t = \epsilon_t \]  

where \( \bar{m} \) is a constant around which the domestic money supply fluctuates and is set at zero (for simplicity) and \( \epsilon_t \) is a serially uncorrelated random variable with zero mean and known variance \( \sigma^2_{\epsilon} \).

The domestic price level is given by:

\[ p_t = \bar{p} + z_t = z_t \]  

where \( \bar{p} \) is assumed to be a constant which is set equal to zero and \( z_t \) is a shock in prices (inflationary shock) that determines the price level. \( z_t \) is also a serially uncorrelated random variable with zero mean and known variance \( \sigma^2_{z} \). Solving equations (1)-(3) we get the money market equilibrium condition which can be written as:
\( r_t = -\left(\frac{1}{2}\right)e_t + \left(\frac{1}{2}\right)z_t. \)

B. Demand for Domestic Output

The output volume is assumed to be a decreasing function of domestic interest rate and an increasing function of relative prices and the domestic government expenditure:

\[ y_t = \phi_1 r_t + \phi_2 \left( e_t + p'_t - p_t \right) + \gamma g_t + v_t, \quad \phi_1 < 0, \quad \phi_2 > 0, \quad \gamma > 0 \]

where \( e_t \) is the exchange rate between domestic and foreign economy defined as units of domestic currency per units of foreign currency, \( p'_t \) is the exogenous foreign price level and is set at zero (for simplicity), \( g_t \) represents the domestic government spending factor and \( v_t \) is assumed to reflect unanticipated shocks in output process. In addition, \( v_t \) is a serially uncorrelated random variable with zero mean and known variance \( \sigma_v^2 \). The assumption \( \phi_1 < 0 \) holds because of the inverse relationship between output and interest rate; \( \phi_2 > 0 \) reflects the exchange rate specification and \( \gamma > 0 \) indicates the positive relationship between government expenditure and output. Moreover, the government spending volume is defined as:

\[ g_t = \bar{g} + \omega_t = \omega_t \]

where \( \bar{g} \) is assumed to be a constant which is set equal to zero and \( \omega_t \) is an unexpected shock in the government spending volume. \( \omega_t \) is also a serially uncorrelated random variable with zero mean and known variance \( \sigma_\omega^2 \).

C. The Foreign Exchange Market

The supply of foreign exchange comes from the transactions on the trade account. Therefore, the net supply of foreign exchange is supposed to be an increasing function of the log of relative prices and a decreasing function of the log of domestic output volume.

\[ T_t = \alpha \left( e_t + p'_t - p_t \right) + \beta y_t + u_t, \quad \alpha > 0, \quad \beta < 0 \]

where \( T_t \) reflects the trade balance in foreign currency units, \( e_t \) is the exchange rate in logarithmic form, \( p'_t \) and \( p_t \) are the logs of foreign and domestic price level, respectively, \( y \) presents the
domestic product and \( u \) is a serially uncorrelated random variable with zero mean and known variance \( \sigma_u^2 \). The assumption that \( a > 0 \) means that the sum of price elasticity of imports and exports (in absolute value) is greater than 1. Moreover, the restriction that \( \beta < 0 \) denotes the negative relationship between output and trade balance. For simplicity, \( p' \) is set at zero.

The net demand for foreign assets \( B_t \) is indicated by the capital flows function which is assumed to be:

\[
B_t = \eta \left[ E_t e_{t+1} - e_t + r'_{t+1} - r_t \right], \quad \eta > 0
\]

where \( E_t e_{t+1} \) is the expected value of exchange rate at \( t + 1 \) given the available information at time \( t \), \( r'_{t+1} \) reflects the foreign interest rate which is set equal to zero and \( \eta \) measures the degree of capital mobility\(^5\) which is considered as given.

The foreign exchange market clearing condition is

\[
\Delta B_t = T_t
\]

which implies that the exchange rate at time \( t \) adjusts until the change in the domestic demand for foreign assets equals the supply of foreign exchange from the trade balance.

**IV. THE PROCESS OF EXCHANGE RATE DETERMINATION**

Equations (1)-(9) constitute a system of equations that can be solved for \( e_t \) under rational expectations. We substitute equations (1)-(8) into equation (9) and we get the following difference equation of \( e_t \):

\[
e_t = \frac{\eta}{\eta + \alpha + \beta \phi_2} e_{t-1} + \frac{\eta}{\eta + \alpha + \beta \phi_2} E_t e_{t+1} - \frac{\eta}{\eta + \alpha + \beta \phi_2} E_{t-1} e_t
\]

\[
+ \frac{\eta + \beta \phi_1}{\lambda (\eta + \alpha + \beta \phi_2)} e_t - \frac{\eta}{\lambda (\eta + \alpha + \beta \phi_2)} e_{t-1} - \left( \frac{\eta - \alpha \lambda + \beta \phi_1 - \beta \phi_2 \lambda}{\lambda (\eta + \alpha + \beta \phi_2)} \right) z_t
\]

\(^4\) This assumption is known as Marshall-Lerner condition where \( |Ex| + |Em| \geq 1 \) and \( Ex, Em \) corresponds to export elasticity and import elasticity, respectively.

\(^5\) In our model it can be proved that the degree of capital mobility does not affect the final results.
The solution of this equation takes the form:

\[(11)\quad e_t = \pi_0 e_{t-1} + \pi_1 e_{t-1} + \pi_2 z_{t-1} + \pi_3 \epsilon_t + \pi_4 z_t + \pi_5 \omega_t + \pi_6 v_t + \pi_7 u_t\]

where the \(\pi_s\) \((s = 0, \ldots, 6)\) are functions of the model parameters to be determined below. The terms \(E_t e_{t+1}\) and \(E_z e_t\) can be computed by taking expectations of equation (11). Substituting the resulting expressions into (10) yields the equation:

\[(12)\quad e_t = \frac{\eta(1 - \pi_0)}{\eta(1 - \pi_0)} e_{t-1} + \frac{\eta(\lambda \pi_1 + 1) + \beta \phi_2}{\eta(1 - \pi_0)} e_{t-1} - \frac{\eta(\lambda \pi_2 - 1)}{\eta(1 - \pi_0)} z_{t-1} - \frac{\eta(\lambda \pi_3 + \alpha + \beta \phi_2) - \eta - \beta \phi_2}{\eta(1 - \pi_0)} \epsilon_t - \frac{\beta \gamma}{\eta(1 - \pi_0) \alpha + \beta \phi_2} \omega_t - \frac{1}{\eta(1 - \pi_0)} u_t.

Equating the coefficients of equations (11) and (12) we get:

\[(13)\quad \pi_0 = \frac{\eta(1 - \pi_0)}{\eta(1 - \pi_0) + \alpha + \beta \phi_2}\]

\[(14)\quad \pi_1 = \frac{-\eta(\lambda \pi_1 + 1)}{\eta(1 - \pi_0) + \alpha + \beta \phi_2}\]

\[(15)\quad \pi_2 = \frac{-\eta(\lambda \pi_2 - 1)}{\eta(1 - \pi_0) + \alpha + \beta \phi_2}\]

\[(16)\quad \pi_3 = \frac{\eta(\lambda \pi_3 + \alpha + \beta \phi_2) - \eta - \beta \phi_2}{\eta(1 - \pi_0) + \alpha + \beta \phi_2}\]

\[(17)\quad \pi_4 = \frac{\lambda (\eta \pi_3 + \alpha + \beta \phi_2) - \eta - \beta \phi_2}{\eta(1 - \pi_0)} + \alpha + \beta \phi_2\]

\[(18)\quad \pi_5 = \frac{-\beta \gamma}{\eta(1 - \pi_0) + \alpha + \beta \phi_2}\]
Equation (13) is quadratic in $\pi_0$. It can be shown that the smaller positive root is

$$\pi_0 = 1 + \frac{\alpha}{2\eta} + \frac{\beta \phi_2}{2\eta} - \frac{\sqrt{(\alpha + \beta \phi_2)(\alpha + 4\eta + \beta \phi_2)}}{2\eta}$$

and ranges from zero to one. In order to solve for $\pi_1$, we rearrange equation (13) to get:

$$\eta(1 - \pi_0) + \alpha + \beta \phi_2 = \frac{\eta(1 - \pi_0)}{\pi_0}$$

Therefore, equations (14) through (20) become:

$$\pi_1 = -\frac{\pi_0}{\lambda}, \quad \pi_2 = \frac{\pi_0}{\lambda}, \quad \pi_3 = \frac{\pi_0}{\lambda}\left(1 + \frac{\beta \phi_1}{\eta(1 - \pi_0)}\right),$$

$$\pi_4 = \frac{\pi_0}{(1 - \pi_0)}\left(\frac{\eta(\pi_0 - 1) + \lambda(\alpha + \beta \phi_2) - \beta \phi_2}{\lambda \eta}\right),$$

$$\pi_5 = \frac{-\beta \gamma \pi_0}{\eta(1 - \pi_0)}, \quad \pi_6 = \frac{-\beta \pi_0}{\eta(1 - \pi_0)}, \quad \pi_7 = \frac{-\pi_0}{\eta(1 - \pi_0)}.$$ Substituting (23) into (11) we obtain the following equation:

$$e_t = \pi_0 e_{t-1} + \frac{\pi_0}{\lambda}\left(1 + \frac{\beta \phi_1}{\eta(1 - \pi_0)}\right)e_t - \frac{\pi_0}{\lambda}(1 - \pi_0)\left(\eta(\pi_0 - 1) + \frac{\lambda(\alpha + \beta \phi_2) - \beta \phi_2}{\lambda \eta}\right)\pi_t$$

$$+ \frac{\pi_0}{\lambda}z_{t-1} - \frac{\beta \gamma \pi_0}{\eta(1 - \pi_0)}\alpha_t - \frac{\beta \pi_0}{\eta(1 - \pi_0)}v_t - \frac{\pi_0}{\eta(1 - \pi_0)}u_t,$$

which can be solved iteratively yielding:

$$e_t = \frac{\pi_0}{\lambda \eta(1 - \pi_0)}\left[\eta(1 - \pi_0) + \beta \phi_1\right]e_t + \frac{\pi_0}{\lambda \eta(1 - \pi_0)}\left[\eta(\pi_0 - 1)\right]e_{t-1} + \frac{\pi_0}{\lambda \eta(1 - \pi_0)}\left[\eta(\pi_0 - 1)^2 + \beta \phi_2 e_{t-2} + \pi_0^2 e_{t-3} + \ldots\right]$$

$$+ \frac{\pi_0}{\lambda \eta(1 - \pi_0)}\left[\eta(\pi_0 - 1) + \lambda(\alpha + \beta \phi_2) - \beta \phi_2\right]z_t + \frac{\pi_0}{\lambda \eta(1 - \pi_0)}\left[\eta(\pi_0 - 1)^3 + \lambda \pi_0(\alpha + \beta \phi_2) - \beta \phi_2 \pi_0\right]$$

$$+ \frac{\pi_0}{\lambda \eta(1 - \pi_0)}\left[\eta(\pi_0 - 1)^4 + \lambda \pi_0(\alpha + \beta \phi_2) - \beta \phi_2 \pi_0\right]$$
Equation (25) indicates that the exchange rate is determined by contemporaneous unanticipated shocks in money supply, price level, government spending, output and the trade balance account, as well as by the past values of these shocks.

In order to examine the determinants of exchange rate uncertainty, we compute the unconditional variance of equation (25). It is implied that the unconditional mean of \( e_t \) is \( E(e_t) = 0 \) for all \( t \). We assume that the various shocks that affect the exchange rate are (i) permanent, thus including the contemporaneous value and the lagged values are identical, and (ii) pairwise correlated.

Under these assumptions, the exchange rate variance is formed by the following equation:

\[
\sigma^2_e = \frac{\pi_0^2}{\beta^2 \gamma^2 \lambda^2 \eta^2 (1 - \pi_0)^2} \left\{ \left[ \eta (1 - \pi_0) + \beta \phi \right]^2 + \frac{1}{1 - \pi_0} \left[ -\eta (\pi_0 - 1)^2 + \beta \phi \pi_0 \right]^2 \right\} \sigma^2_\epsilon + \\
+ \frac{\pi_0^2}{\lambda^2 \eta^2 (1 - \pi_0)^2} \left[ \eta (\pi_0 - 1) + \lambda (\alpha + \beta \phi_2) - \beta \phi \right]^2 + \frac{1}{1 - \pi_0} \left[ -\eta (\pi_0 - 1)^2 + \beta \phi \pi_0 \right]^2 + \\
+ \frac{\beta^2 \gamma^2 \pi_0^2}{\eta^2 (1 - \pi_0)^4 (1 + \pi_0)} \sigma^2_\epsilon + \\
+ \frac{\pi_0^2}{\lambda^2 \eta^2 (1 - \pi_0)^4} \sigma^2_\epsilon + \\
+ \frac{2 \pi_0^2}{\lambda^2 \eta^2 (1 - \pi_0)^4} \left[ \lambda (\alpha + \beta \phi_2) \beta \phi_2 - \beta \phi_2 \right] \text{Cov}(\epsilon_t, \epsilon_t) - \frac{2 \pi_0^2 \beta \gamma}{\lambda \eta^2 (1 - \pi_0)^4} \left[ -2 \eta (\pi_0 - 1)^2 + \\
+ \beta \phi \right] \text{Cov}(\epsilon_t, \epsilon_t) - \frac{2 \pi_0^2 \beta \gamma}{\lambda \eta^2 (1 - \pi_0)^4} \left[ \lambda (\alpha + \beta \phi_2) - \beta \phi_2 \right] \text{Cov}(\epsilon_t, \epsilon_t) - \\
- \frac{2 \pi_0^2 \beta \gamma}{\eta^2 (1 - \pi_0)^4} \text{Cov}(\epsilon_t, \epsilon_t) - \frac{2 \beta \pi_0^2}{\eta^2 (1 - \pi_0)^4} \text{Cov}(\epsilon_t, \epsilon_t).
\]
V. COMPARATIVE STATICS OF EXCHANGE RATE AND ITS VARIANCE

Having specified the equation of exchange rate we compute the comparative-statics results in order to investigate the conjunction between the level of exchange rate and its determinants. We provide evidence that money supply and government shocks and unanticipated changes in output are associated positively with exchange rate level, as given below:

\[
\frac{\partial e}{\partial \pi_o} = \frac{\beta \phi \pi_o}{\lambda \eta (\pi_o - 1)^2} > 0, \quad \frac{\partial e}{\partial \omega} = -\frac{\beta \gamma \pi_o}{\eta (1 - \pi_o)^2} > 0, \quad \frac{\partial e}{\partial v} = -\frac{\beta \pi_o}{\eta (1 - \pi_o)^2} > 0
\]

On the other hand, trade balance shocks affect negatively the level of exchange rate while inflationary shocks are found to have an ambiguous effect on exchange rate, as expressed in the following equations, respectively:

\[
\frac{\partial e}{\partial u} = -\frac{\pi_o}{\eta (1 - \pi_o)^2} < 0, \quad \frac{\partial e}{\partial z} = \frac{\pi_o (\beta \phi_1 - \alpha \lambda - \beta \phi_2 \lambda)}{\lambda \eta (1 - \pi_o)^2}
\]

Having derived the equation for the exchange rate variance, we can compute the comparative statics results in order to determine the relationship between the exchange rate variance and its various determinants.

First, we consider the effect of changes of money supply uncertainty:

\[
\frac{\partial \sigma_e^2}{\partial \sigma_e^2} = \frac{\pi_o^2}{\lambda \sigma_e^2 (1 - \pi_o)^2} \left[ \frac{\eta (1 - \pi_o) + \beta \phi_1}{\eta (1 - \pi_o)^2} + \frac{1}{1 - \pi_o} \left[ -\eta (\pi_o - 1)^2 + \beta \phi_2 \pi_o \right] \right] > 0.
\]

An increase in the money supply uncertainty increases the exchange rate variance, depicted in Figure 1. Using a variant of the Mundell-Fleming model under flexible exchange rates we develop the IS, LM and BP curves in the \((r,e)\) space. LM curve appears to be horizontal because Central Bank targets the interest rate level. In order to justify the slope of the BP curve we consider that a depreciation of the exchange rate is associated with a fall in the interest rate for the maintenance of equilibrium in foreign exchange market, implying that BP curve is downward-sloping. Moreover, as the domestic interest rate rises, investment volume falls and so does output. Furthermore, a depreciation of exchange rate \((e\text{ increases})\) leads to an increase in output. Thus the IS curve is upward-sloping, but flatter than the BP under the assumption that a change in domestic interest rate imposes a greater effect on domestic demand than on foreign demand. The initial equilibrium point is
A. We assume that a small increase in money supply occurs. Such increase causes a fall to interest rates leading to a downward shift of LM and the economy moves from point A to point B. Point B reflects a deficit in the Balance of Payments because a fall in interest rates causes an increase in output through the increase of capital outflows leading to a CA deficit. In order to restore the equilibrium, IS shifts to the right and BP to the left and the new equilibrium is reached at point C, where the economy exhibits an increase in exchange rate (depreciation of exchange rate) from $e_0$ to $e_1$. If money supply increases more then interest rates will fall more and the exchange rate will depreciate more (point $C'$).

![Figure 1: The positive effect of the variance of a money supply shock on exchange rate variance](image)

Following the previous result, the variance of price shocks, government spending shocks, output demand shocks and trade balance shocks is linked positively with exchange rate variance:

---

6 We focus on the effect of uncertainty round a money supply shock and thus we refer to small and large fluctuations of such shock.

7 The diagrammatic analysis of these comparative-statics is not reported because of its simplicity.
An interesting aspect of exchange rate volatility is how it is affected by the covariance of various shocks. For instance, if we take into consideration that there are contemporaneous shocks in money supply and prices then the effect on exchange rate volatility is given by:

\[
\frac{\partial \sigma^2_e}{\partial \sigma^2_i} = \frac{2\pi^2}{\lambda^2 \eta^2 (1-\pi_o)^4} \left[ \lambda (\alpha + \beta \phi_i) - \beta \phi_i \right] > 0
\]

The sign of equation (34) is ambiguous meaning that when a price shock and a money supply shock coexist then the impact on the variance of exchange rate is either positive or negative. Moreover, if government decides to increase the money supply in order to finance the volume of expenditures, the impact on exchange rate volatility will be:

\[
\frac{\partial \sigma^2_e}{\partial \text{Cov}(\varepsilon, \omega)} = -\frac{2\pi^2 \beta \gamma}{\lambda^2 \eta^2 (1-\pi_o)^4} \left[ -2\eta (\pi_o - 1)^2 + \beta \phi_i \right]
\]

Just as in the previous case the sign of (35) can be either positive or negative. Furthermore, the impact of the coexistence of shocks in prices and trade balance on the variance of exchange rate is given by the equation:

\[
\frac{\partial \sigma^2_e}{\partial \text{Cov}(z, u)} = -\frac{2\pi^2}{\lambda \eta^2 (1-\pi_o)^4} \left[ \lambda (\alpha + \beta \phi_i) - \beta \phi_i \right]
\]

8 For instance, if an increase in interest rates occurs then deposits will rise and thus the money supply increases.
which has ambiguous sign. Contrary to the already reported findings, the covariance of government spending shocks and trade balance shocks as well as the covariance of output and trade balance unanticipated changes, impose a positive impact on exchange rate volatility given by the following equations, respectively:

\[
\frac{\partial \sigma_e^2}{\partial \text{Cov}(\omega, u)} = -\frac{2\pi_0^2 \beta \gamma}{\eta^2 (1 - \pi_0)^4} > 0
\]

\[
\frac{\partial \sigma_e^2}{\partial \text{Cov}(v, u)} = -\frac{2\pi_0^2 \beta}{\eta^2 (1 - \pi_0)^4} > 0
\]

In an attempt to clarify which variance or covariance impinges greater on the exchange rate variance, we compute the pairwise relative effect of selected variances and covariances. More specifically, we compare (i) equation (29) with equations (30) and (31) respectively, (ii) (31) with (32), (iii) (32) with (33), (iv) (34) with (35) and (36) respectively, (v) (35) with (37), and (vi) (37) with (38). We find that either the one impact or the other affects more the variance of exchange rate according to the restrictions that are imposed for each parameter. For instance, the variance of a government spending shock has a greater impact on exchange rate variance than the variance of a shock in output, \( \frac{\partial \sigma_e^2}{\partial \sigma_u^2} > \frac{\partial \sigma_e^2}{\partial \sigma_v^2} \), under the condition \( \gamma > 1 \). A plausible explanation for this result is that a large increase in government spending affects the output more than an output shock. In terms of volatility, the variance of a government spending shock will affect the variance of output more than the variance of an output shock. Therefore, the variance of a government spending shock impinges greater impact on the variance of exchange rate via the channel of output variance. If \( 0 < \gamma < 1 \), then the inverse result holds, \( \frac{\partial \sigma_e^2}{\partial \sigma_u^2} < \frac{\partial \sigma_e^2}{\partial \sigma_v^2} \). Moreover, the impact of the variance of an output shock on the exchange rate variance is greater than that of the variance of a trade balance shock, \( \frac{\partial \sigma_e^2}{\partial \sigma_v^2} > \frac{\partial \sigma_e^2}{\partial \sigma_u^2} \), under the constraint \( \beta < -1 \). In this case a large increase in output will affect greater the trade balance (negatively) rather than a shock in trade balance. In terms of volatility, the variance of an output shock affects the variance of exchange rate more than the variance of a trade balance shock.
via the variance of trade balance. If $-1 < \beta < 0$, then the inverse result holds, $\frac{\partial \sigma_2^2}{\partial \sigma_v^2} < \frac{\partial \sigma_1^2}{\partial \sigma_v^2}$. The same inference is drawn for the rest relative effects but for their computation more than one restriction for each parameter should be satisfied.

At this point we can try a comparison of exchange rate volatility and its determinants, which are also expressed in terms of volatility, of our model with the optimizing behaviour model of Obstfeld and Rogoff (1995, 1996).

Obstfeld and Rogoff (1995, 1996) assume that: a) there is a continuum of individual monopolistic producers indexed by $z \in [0,1]$, each of whom produces a single differentiated good $z$; b) All producers reside in two countries, Home and Foreign. Home producers produce goods indexed by $z \in [0,\eta]$ whereas Foreign producers produce goods indicated by $z \in (\eta,1]$; c) There is no capital or investment and the economy exhibits elastic labor supply; d) the production of good $z$ depends on the marginal revenue of higher production, the disutility of effort and the marginal utility of consumption; e) all agents within a country have symmetric preferences and constraints; f) PPP holds. The utility function of a typical Home agent $j$ is given by:

$$U_j^i = \sum_{i=1}^{\infty} \beta^{i-1} \left[ \log C_j^i + \chi \log \frac{M_j^i}{P_j} - \frac{\kappa}{2} y_i^j (j)^2 \right]$$

Where $C$ is a real consumption index and $P$ is the Home money price level indexed respectively by:

$$C_j^i = \left[ \int_0^1 c_j^i (z)^{\theta-1} dz \right]^{\frac{1}{\theta-1}}, \quad P_j = \left[ \int_0^1 p(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$$

and $\frac{\kappa}{2} y_i^j (j)^2$, $\theta^9$ reflect the disutility of individual to produce more output and price elasticity of demand faced by each monopolist, respectively. The law of one price holds so $P = c P'$, where $P'$ denotes the foreign money price level. The individual budget constraint and the government budget constraint are:

$$P_j B_{t+1}^j + M_j^i = P_j (1 + r_i) B_{t+1}^j + M_{t+1}^j + p_i (j) y_i^j (j) - P_j C_j^i - P_j \tau_j$$

$^9 \quad \theta > 1$ because $MR < 0$ when the demand elasticity is less than 1 (CES).
where \( r_t \) reflects the interest rate on bonds between \( t-1 \) and \( t \), \( y_t(j) \) is output of good \( j \), \( p_t(j) \) is the domestic currency price for good \( j \) which need not be the same for all \( j \) because of product differentiation. The variable \( M_{t-1}^j \) indicates agents’ holdings of nominal money balances entering period \( t \) and \( \tau_t \) denotes lump-sum taxes.

\[
G_t = r_t + \frac{M_{t-1}^j - M_{t-1}}{P_t}
\]

Where \( G \) is government’s real consumption index being formed as:

\[
G = \int g(z)^{\theta-1} dz
\]

Governments act as price takers and their demand functions are:

\[
g(z) = \left( \frac{p(z)}{P} \right)^{\theta} G
\]

And therefore the demand function for representative agents is given by:

\[
y^d = \left( \frac{p(z)}{P} \right)^{\theta} \left( C^w + G^w \right), \text{ where } C^w = \eta C + (1-\eta)C^* \text{ and } G^w = \eta G + (1-\eta)G^* \text{. We denote with } ^* \text{ the magnitudes of foreign country.}
\]

Obstfeld and Rogoff maximizing (39) subject to budget constraints they find that the level of exchange rate is a function of money supply and government spending differentials. The mathematical form of this expression is:

\[
e = \frac{2\theta + \delta(1+\theta)}{2\theta + \theta\delta(\theta+1)}(m-m^*) + \frac{\delta(\theta^2-1)}{2\theta + \theta\delta(\theta+1)}(g-g^*)
\]

where \( \delta \) is rate of time preference. The variables \( m, m^* \) reflect the money supply of domestic and foreign country, respectively and \( g, g^* \) indicate the government spending volume in domestic and foreign country respectively.

We compute the variance of equation (46) under the assumptions that monetary shocks in both countries are correlated as well as government spending shocks, and that changes in money supply and government spending occur contemporaneously in home and foreign country, respectively:
Computing the comparative statics of equation (47) we get:

\[
\sigma_e^2 = \left(\frac{2\theta + \delta(1 + \theta)}{2\theta + \delta\theta(\theta + 1)}\right)^2 \sigma_m^2 + \left(\frac{2\theta + \delta(1 + \theta)}{2\theta + \delta\theta(\theta + 1)}\right)^2 \sigma_g^2 + \delta \left(\frac{\theta^2 - 1}{2\theta + \delta\theta(\theta + 1)}\right)^2 \sigma_{eg}^2 - 2\left(\frac{2\theta + \delta(1 + \theta)}{2\theta + \delta\theta(\theta + 1)}\right)^2 Cov(m,m^*) - 2\left(\frac{\delta(\theta^2 - 1)}{2\theta + \delta\theta(\theta + 1)}\right)^2 Cov(g,g^*) + \left(\frac{\delta(1+\theta)^2(2\theta - 1)(\delta + 2\theta + \delta\theta)}{\theta^2(2 + \delta + \delta\theta)}\right) Cov(m,g) - \left(\frac{2\delta(1+\theta)^2(2\theta - 1)(\delta + 2\theta + \delta\theta)}{\theta^2(2 + \delta + \delta\theta)}\right) Cov(m^*,g^*)
\]

Equations (48), (49) and (52) show that an increase in the variance of money supply and government spending, as well as the coexistence of money supply and government spending shocks affect exchange rate volatility positively. This result is also inferred from the ad hoc model apart from the effect of the contemporaneous occurrence of money supply and government spending shocks which
can be either positive or negative. Equations (50), (51) and (53) are indicative of the negative relationship between the covariance of selected shocks and exchange rate volatility.

VI. THE DETERMINANTS OF OUTPUT VOLATILITY

Recently, an increasing number of empirical studies focuses on output growth volatility. Although most of the relevant studies seek to determine if growth is affected by growth volatility, our purpose is to appose the studies that present the determinants of output volatility.

Caporale and McKiernan (1996) estimate a GARCH–M model and they infer that there is a significant and affirmative relationship between volatility and growth for UK. The same result is also obtained by Grier and Perry (2000), differing the sample and the time span that are used. Grier and Perry employ US data for their study during the period 1948-1996 and they conclude that the link between volatility and growth is positive but it is not significant across all subsamples. Furthermore, Fountas, Karanasos and Kim (2002) investigate the link between average inflation and real growth as well as the nominal and real uncertainty focusing on the Japanese economy. They use a GARCH model in order to get estimates for uncertainty and their findings support that high rate of inflation (output growth) leads to high inflation (output growth) uncertainty.

Later, Fiaschi and Lavezzi (2003) draw the conclusion that growth volatility is associated negatively with total GDP, per capita GDP and trade openness, while there is a positive conjunction with the share of agricultural sector. The effect of per capita GDP on growth volatility vanishes when the other variables are taken into account. Moreover, Karras (2006) investigates the association between trade openness and economic growth in terms of GDP, aggregate consumption and aggregate investment using two different data sets and applying three different methods. In the case of GDP, trade openness is positively related to output volatility but this relationship is not statistically significant in both samples. The same result holds in the case of investment meaning that trade openness has a positive but insignificant impact on investment variability in both samples while consumption volatility seems to be affected positively and significantly only in the first sample. The preceding inferences change when the economic size is included in the model.
Contrary to other relevant studies for the relationship between output variability and growth, Fountas et al (2004) find no evidence of association between output variability and output growth. They draw such inference having based their research on quarterly data over the period 1961-2000 for the Japanese economy and having employed three different specifications of GARCH models. A previous study of Speight (1999) makes the inference that shocks to output growth affect significantly the output volatility.

Apart from the already reported studies there are others in which output volatility is found to affect growth. A representative study is that of Kormendi and Meguire (1985) who conclude that there is a positive and statistically significant link between volatility and growth, computing the standard deviation of real output growth as a measure of volatility. The same conclusion is reached a bit later by Grier and Tullock (1989). On the contrary, Ramey and Ramey (1995) demonstrate a strong negative relationship between growth and volatility in a panel of 92 countries over 1960-1985 while Fountas, Karanasos and Kim (2002) provide evidence of no impact of output growth uncertainty on both average inflation and output growth.

Kneller and Young (2001), modeling the business cycle volatility as the standard deviation of annual and quinquennial growth rates, they find that in the case of annual growth rates there is a significant positive relationship between volatility and growth contrary to five year period growth rates where growth is negatively correlated with volatility. Moreover, Kose et al (2006), applying the methodology of Ramey and Ramey (1995) infer that there is a positive (negative) conjunction between growth and volatility among industrial (developing) countries. This relationship differs across developing countries according to the financial integration. They provide evidence that the negative (positive) relationship between growth and volatility is strong when economies are characterized by less (more) financial integration.

In a more recent study Campos and Karanasos (2007) indicate that informal political instability, which is used as a proxy for growth volatility, has a direct and negative effect on economic growth contrary to formal political instability which imposes a negative but indirect effect on growth.

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10 The same conclusion is also reached by Martin and Rogers (2000) using different sample and focusing on expanded time period.
11 These results match those of Ramey and Ramey (1995).
Furthermore, Fountas and Karanasos (2007) reach the following conclusions: a) inflation determines positively inflation uncertainty; b) output growth uncertainty imposes a positive impact on output growth rate; c) inflation uncertainty is found to affect inflation and output growth diversely; d) there is not much evidence supportive of the positive effect of output uncertainty on inflation.

VII. OUTPUT DETERMINATION AND OUTPUT VARIANCE

Having determined the level of exchange rate from equation (25) we can model the output level by substituting equations (3), (4), (6) and (25) into (5) yielding:

\[
\begin{align*}
\gamma_t &= \frac{1}{\lambda \eta (1-\pi_0)} \left[ \eta (1-\pi_0) \left( \pi_0 \phi_2 - \phi_1 \right) + \beta \phi_1 \pi_0 \phi_2 \right] \varepsilon_t + \frac{\pi_0 \phi_2}{\lambda \eta (1-\pi_0)} \left[ -\eta (1-\pi_0)^2 + \\
&+ \beta \phi_1 \pi_0 \left( \varepsilon_{t-1} + \pi_0 \varepsilon_{t-2} + \pi_0^2 \varepsilon_{t-3} + \ldots \right) + \frac{1}{\lambda \eta (1-\pi_0)} \phi_2 \left( \eta (1-\pi_0) - \lambda \eta (1-\pi_0) \right) \varepsilon_t - \\
&+ \lambda \left( \alpha + \beta \phi_2 \right) \pi_0 \left( \varepsilon_{t-1} + \pi_0 \varepsilon_{t-2} + \pi_0^2 \varepsilon_{t-3} + \ldots \right) - \frac{1}{\eta (1-\pi_0)} \phi_2 \left( \eta (1-\pi_0) \right) \varepsilon_t - \\
&- \eta (1-\pi_0) \omega_t - \frac{\beta \gamma \pi_0 \phi_2}{\eta (1-\pi_0)} \left( \omega_{t-1} + \pi_0 \omega_{t-2} + \ldots \right) - \frac{1}{\eta (1-\pi_0)} \left( \beta \pi_0 \phi_2 - \eta (1-\pi_0) \right) v_t - \\
&- \frac{\beta \pi_0^2 \phi_2}{\eta (1-\pi_0)} \left( \varepsilon_{t-1} + \pi_0 \varepsilon_{t-2} + \ldots \right) - \frac{\pi_0 \phi_2}{\eta (1-\pi_0)} \left( u_t + \pi_0 u_{t-1} + \pi_0^2 u_{t-2} + \ldots \right) \right].
\end{align*}
\]

Equation (54) indicates that the level of output is defined by the same factors as exchange rate level which seems to be reasonable since we have substituted the equation of exchange rate into the output equation. Thus, money supply shocks, inflationary shocks, unanticipated changes in government spending as well as in output process and trade balance affect the level of output. It is obvious that past values of these shocks play an important role in the determination of output level.

Furthermore, we compute the variance of equation (54) in order to make inference about the determinants of output uncertainty. It is implied that the unconditional mean of \( y_t \) is found to be \( E(y_t) = 0 \) for all \( t \). We assume that the various shocks that affect output are (i) permanent, thus
implying the contemporaneous value and the lagged values are identical, and, (ii) pairwise correlated.

Under these assumptions, the output variance is formed by the following equation:

\begin{equation}
\sigma^2 = \frac{1}{\lambda^2 \sigma^2 (1 - \pi_0)} \left\{ \eta (1 - \pi_0) \left( \pi_0 \phi_0 - \phi_0 \right) + \beta \phi_0 \pi_0 \left( \pi_0 - 1 \right) \right\} + \frac{\pi_0 \phi_0^2}{1 - \pi_0} \left[ -\eta \left( \pi_0 - 1 \right) \right] + 
\end{equation}

\begin{equation}
+ \frac{1}{\lambda^2 \sigma^2 (1 - \pi_0)} \left\{ \phi_0 \left[ \eta (1 - \pi_0) \left( \pi_0 + \lambda \right) + \lambda (\alpha + \beta \phi_0 \pi_0) \right] + 
\end{equation}

\begin{equation}
+ \frac{\gamma^2}{\pi^2 (1 - \pi_0)} \left[ \beta \pi_0 \phi_0 - \eta (1 - \pi_0) \right] + \frac{1}{\pi^2 (1 - \pi_0)^2} \left[ \beta \pi_0 \phi_0 - \eta (1 - \pi_0) \right] + \frac{1}{\pi^2 (1 - \pi_0)^2} \left[ \beta \pi_0 \phi_0 - \eta (1 - \pi_0) \right] + 
\end{equation}

\begin{equation}
+ \eta \left( \pi_0 - 1 \right) \left[ \beta \pi_0 \phi_0 - \eta (1 - \pi_0) \right] + \phi_0 \lambda \left[ -\eta \left( \pi_0 - 1 \right) \right] + 
\end{equation}

\begin{equation}
+ (\alpha + \beta \phi_0) \pi_0 \right\} \text{Cov} (\varepsilon, z_i) - \frac{2 \phi_0 \gamma}{\lambda \eta^2 (1 - \pi_0)^4} \left[ -\eta + \beta \phi_0 \pi_0 + 2 \eta \pi_0 - \eta \pi_0 \right] \text{Cov} (\varepsilon, \omega_i) - 
\end{equation}

II. COMPARATIVE STATICS OF OUTPUT AND ITS VARIANCE

In order to define, in the first place, the relationship between the output level and the various shocks that are found to affect it, we compute the comparative-statics analysis. More specifically, shocks in money supply, government spending and output impose a positive effect on the level of output, as shown below:
Contrary to these results, unanticipated changes in trade balance cause a negative effect on output level while price shocks are found to impose an ambiguous impact given by the following equations, respectively:

\[
\frac{\partial y}{\partial \omega} = -\frac{\gamma(-\eta + \beta \phi_2 \pi_o + 2\eta \pi_o - \eta \pi_o^2)}{\eta(\pi_o - 1)^2} > 0
\]

\[
\frac{\partial y}{\partial \omega} = -\frac{-\eta + \beta \phi_0 \pi_o + 2\eta \pi_o - \eta \pi_o^2}{\eta(\pi_o - 1)^2} > 0
\]

Given the variance of output we may compute the comparative-statics in order to provide evidence for the relationship between output variability and its determinants. We find that changes in the variance of money supply shock affects output volatility positively as expressed in the following equation:

\[
\frac{\partial \sigma_y^2}{\partial \sigma_e^2} = \frac{1}{\lambda^2 \eta^2 (1-\pi_o)^2} \left[ \eta(1-\pi_o)(\pi_o \phi_2 - \phi_1) + \beta \phi_0 \phi_2 \pi_o \right]^2 + \\
+ \frac{\pi_o^2 \phi_0^2}{1-\pi_o^2} \left[ \eta(\pi_o - 1)^2 + \beta \phi_0 \pi_o \right] > 0
\]

The effect of the variance of a money supply shock is illustrated in Figure 2. An increase (small) in money supply shifts the LM curve downwards and the economy goes from point A to point B. At B, there is an incipient deficit in the Balance of Payments because due to the lower interest rate the level of capital inflows is insufficient to offset the deficit in the CA. The deficit in the Balance of Payments means that the exchange rate has to depreciate. As the exchange rate depreciates (e increases), Net Exports increase because the relative price of domestic goods on international markets has fallen. The rise of NX has two effects: a) Total Expenditures increase therefore the IS
curve moves right, and, b) the Current Account improves so the BP curve moves to the right. The new equilibrium occurs at C, where the economy has had an increase in $y$ from $y_0$ to $y_1$. Following the same analysis for the case of a greater increase in the money supply, we find that the economy has had a greater increase in the output, $y$ (from $y_0$ to $y_2$). Thus, an increase in the variance of a monetary supply shock affects positively the output variance.

In addition, the variance of price level, government spending variance as well as output variance and trade balance variance impose a positive impact on output volatility as shown in equations (62) through (65), respectively.

\[
\frac{\partial \sigma_y^2}{\partial \sigma_f^2} = \frac{1}{\lambda^2 \eta^2 (1 - \pi_0)^2} \left\{ \left[ \phi_1 \left( \eta \pi_0 (1 - \pi_0) + \lambda (\alpha + \beta \phi, \pi_0) \right) + \phi_2 \left( \eta (1 - \pi_0) - \beta \phi, \pi_0 \right) \right] + \phi_3 \left[ \eta (\pi_0 - 1)^2 + \lambda \pi_0 (\alpha + \beta \phi, \pi_0)^2 \right] \right\} > 0
\]

Figure 2: The positive effect of a change in the variance of a money supply shock on output variance
Contrary to the positive effect of the variance of various shocks on output volatility, the occurrence of contemporaneous shocks in money supply and prices as well as shocks in prices and trade balance affects either positively or negatively the variance of output as expressed in the following equations, respectively:

\[
\frac{\partial \sigma^2_y}{\partial \sigma^2_u} = \frac{\gamma^2}{\eta^2 (1 - \pi_0)^2} \left\{ \left[ \beta \pi_0 \phi_2 - \eta (1 - \pi_0) \right]^2 + \frac{\beta^2 \pi_0^4 \phi_2^2}{1 - \pi_0^2} \right\} > 0
\]

\[
\frac{\partial \sigma^2_y}{\partial \sigma^2_x} = \frac{1}{\eta^2 (1 - \pi_0)^2} \left\{ \left[ \beta \pi_0 \phi_2 - \eta (1 - \pi_0) \right]^2 + \frac{\beta^2 \pi_0^4 \phi_2^2}{1 - \pi_0^2} \right\} > 0
\]

\[
\frac{\partial \sigma^2_y}{\partial \sigma^2_{\omega}} = \frac{\pi_0^2 \phi_2^2}{\eta^2 (1 - \pi_0)^2 (1 + \pi_0)} > 0
\]

Furthermore, we provide evidence that the coexistence of (i) money supply shocks and government spending shocks, (ii) trade balance shocks and government spending shocks, as well as, (iii) the covariance of output shocks and trade balance shocks, impose a positive impact on output volatility, given by equations (68) through (70), respectively:

\[
\frac{\partial \sigma^2_y}{\partial \text{Cov}(\varepsilon, z)} = -\frac{2 \phi \gamma}{\lambda \eta^2 (\pi_0 - 1)} \left\{ \frac{\phi [\eta (\pi_0 - 1)^2 - \beta \phi \pi_0]}{\frac{\phi_2 \pi_0}{\lambda \eta^2 (\pi_0 - 1)^2}} \left\{ \phi \left[ (\pi_0 - 1)^2 - \beta \phi \pi_0 \right] + \phi_2 \phi \left[ -\eta (\pi_0 - 1)^2 + (\alpha + \beta \phi) \pi_0 \right] \right\}
\]

\[
\frac{\partial \sigma^2_y}{\partial \text{Cov}(z, u)} = -\frac{2 \phi_2 \pi_0}{\lambda \eta^2 (\pi_0 - 1)} \left\{ \frac{\phi [\eta (\pi_0 - 1)^2 - \beta \phi \pi_0]}{\frac{\phi_2 \pi_0}{\lambda \eta^2 (\pi_0 - 1)^2}} \left\{ \phi \left[ -\eta (\pi_0 - 1)^2 + (\alpha + \beta \phi) \pi_0 \right] \right\}
\]

\[
\frac{\partial \sigma^2_y}{\partial \text{Cov}(v, u)} = -\frac{2 \phi_2 \pi_0}{\lambda \eta^2 (\pi_0 - 1)^2} \left\{ \phi \left[ -\eta (\pi_0 - 1)^2 + (\alpha + \beta \phi) \pi_0 \right] \right\}
\]

26
At this point we compute the pairwise relative effect of selected variances and covariances in order to conclude which effect is greater on the output variance. Therefore, we compare (i) equation (61) with (62) and (63) respectively, (ii) (63) with (64), (iii) (64) with (65), (iv) (66) with (67) and (68) respectively, (v) (68) with (69) and (vi) (69) with (70). In all cases parameter values determine which effect is stronger on output variance. For instance, the impact of the variance of a government spending shock on the output variance is greater than that of the variance of an output shock, \( \frac{\partial \sigma_y^2}{\partial \sigma_v^2} > \frac{\partial \sigma_y^2}{\partial \sigma_o^2} \), under the condition \( \gamma > 1 \). The intuition of this relative effect lies to the greater impact of government spending shock on output than that of an output shock. In terms of volatility, a large increase in the variance of a government spending shock affects the variance of output more than the variance of an output shock. If \( 0 < \gamma < 1 \) then the inverse conclusion is drawn, \( \frac{\partial \sigma_y^2}{\partial \sigma_v^2} < \frac{\partial \sigma_y^2}{\partial \sigma_o^2} \).

Just in the case of the covariances effects we find that \( \frac{\partial \sigma_y^2}{\partial \text{Cov}(z_i, z_i)} \) < \( \frac{\partial \sigma_y^2}{\partial \text{Cov}(w_i, u_i)} \), meaning that the impact of contemporaneous money supply shocks and price shocks on the output variance is weaker than the effect of contemporaneous price and trade balance shocks. The rest comparisons of comparative statics require the satisfaction of several sets of parameter restrictions.

**VIII. CONCLUSIONS**

This study has analyzed the issue of volatility on exchange rates and output. Being stimulated by the paper of Driskill and McCafferty (1980), we have extended the ad hoc model by including an output demand equation and we define the determinants of exchange rate and output as well. Assuming that rational expectations hold for exchange rates, we conclude that the level of the exchange rate is affected by monetary and price shocks and shocks in government spending, output demand and the trade balance. These factors appear to determine the output volume too, since we have substituted the equation of exchange rate into the output equation.

\(^{12}\) We find that under this condition the covariance of a government spending shock and a trade balance shock imposes a greater impact on output variance than the coexistence of an output and a trade balance shock, \( \frac{\partial \sigma_y^2}{\partial \text{Cov}(w_i, u_i)} > \frac{\partial \sigma_y^2}{\partial \text{Cov}(v_i, u_i)} \). The inverse inference is drawn if \( 0 < \gamma < 1 \) holds.
The most important aspect of this paper is the determination of both exchange rate volatility and output volatility. Computing the variance of exchange rate and output and considering that the various shocks are pairwise correlated, we prove that the variance of both exchange rate and output are determined by the variance and covariance of the above shocks. It is of great interest the impact that the variance of each shock as well as the covariance of selected shocks impose on the variance of exchange rate and output. More specifically, the existence of uncertainty in shocks that affect money supply, price level, government spending, output and trade balance imposes a positive impact on the uncertainty of exchange rate and output as well. In the case of exchange rate volatility, the positive effect is also caused by the contemporaneous occurrence of shocks (i) in government spending and trade balance, and, (ii) in output and trade balance. Contrary to the previous results, the coexistence of (i) money supply shocks and price shocks, (ii) changes in money supply and government expenditure, and, (iii) unanticipated changes in price level and trade balance impose an ambiguous effect on exchange rate volatility.

In the case of output volatility, comparative-statics show that the covariance of money supply shocks and price shocks as well as the covariance of inflationary shocks and trade balance shocks may affect the variance of output in two opposite ways. On the other hand, the covariance of (i) money supply changes and government spending shocks, (ii) output shocks and trade balance shocks, and (iii) changes in government spending and trade balance affect output volatility positively. The conclusions for exchange rate volatility and output volatility are illustrated in the context of the IS-LM-BP model.

Another aspect of this paper is the examination of the relative effects of the determinants of exchange rate and output volatility. We show that in both cases the degree of impact of the determinants is very sensitive to the parameter values. Therefore we have ambiguous results apart from the case of output volatility where we clearly find that the covariance of a money supply shock and an inflationary shock is weaker than the coexistence of a shock in price level and the trade balance.

Finally, we have described briefly the optimizing behaviour model of Obstfeld and Rogoff (1995, 1996) and we compute the exchange rate volatility which is found to be affected by money
supply volatility, government spending volatility as well as by the covariance of money supply and government spending shocks in the home and foreign country. The comparative-statics are indicative of the positive relationship between exchange rate volatility and the volatility of money supply and government spending, as well as the coexistence of money supply and government spending shocks in the home country. Both models reach the same conclusions regarding these two determinants of exchange rate volatility apart from the effect of the covariance of money supply and government spending shocks, which is ambiguous in the ad hoc model. In addition, the contemporaneous occurrence of (i) money supply shocks in both countries, (ii) government spending shocks in both countries, and, (iii) changes in money supply and government spending in foreign country, is linked negatively with exchange rate volatility.

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