Effects of Banking in a Small Open Economy
Model with Financial Distress

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Abstract

This paper estimates a small open economy model with banking sector to assess the evidence for the existence and importance of financial frictions in the amplification and propagation of the effects of transitory shocks, the production technology shock, the monetary shock and one other shock emanating from the banking sector itself: a shock that alters the collateral value of capital. The analysis is based on a New Keynesian model where banks supply loans to transaction constrained consumers.

Keyword: Small open economy, Financial shock, Banking, Monetary Policy.

1 Introduction

The possible interactions between credit markets and the real economy are a customary part of the overall assessment on the policy stance. Past episodes like the U.S Savings and Loans crises in 1980s, the prolonged recession in Finland and Japan in the 1990s stand as compelling empirical evidence that the banking sector can considerably affect the developments of the real economy. For example, in a speech at "The credit channel of Monetary Policy in the Twenty-first Century" conference held on 15 June 2007 at the Federal Reserve Bank of Atlanta, Chairman Bernanke Stated that "Just as a healthy financial system promotes growth, adverse financial conditions may prevent an economy from reaching its potential. A weak banking system grappling with nonperforming loans and insufficient capital, or firms, whose credit worthiness has eroded because of high leverage or declining asset values, are examples of financial condition that could undermine growth."

An extensive literature has discussed the interaction between financial and credit markets on the one hand, and the rest of the economy on the other hand. It has argued that credit market frictions may amplify and propagate conventional interest rate effects (for example Bernanke and Gertler (1989 and 1995) and Carlstrom and Fuerst (1997)). A growing number of studies have used Dynamic Stochastic General Equilibrium (DSGE) models to analyze the role of
credit market frictions in economic fluctuations. Almost all of these studies use calibrated rather than estimated models. Bernanke, Gertler and Gilchrist’s (1999) model is the basic model underpinning much of the research on the role of financial frictions in the business cycles. In their model, they introduce loans and collateral requirements to study how macroeconomic shocks are transmitted in the presence of these financial elements. Their framework links the cost of firms’ external finance to the quality of their balance sheet. A number of studies have used this type of model to account for macro-economic developments at times of financial crises. Cespedes, Chang and Velasco (2004), Gertler, Gilchrist and Natalucci (2003) and Tovar (2005) consider the case of open economies in emerging markets. Using a sticky-price model calibrated to U.S data during normal times, BGG show that the financial accelerator amplifies the impact of shocks and provides a quantitatively important mechanism that propagates shocks at business cycle frequencies.

Much recent work in macroeconomics has involved the development and evaluation of monetary models that bring imperfect competition and nominal rigidities into the dynamic stochastic general equilibrium structure that for a long time had been the hallmark of real business cycles (RBC) theory.

In the present paper we lay out a small open economy version of a model with Calvo-type “staggered” price setting and financial frictions, and use it as a framework for analyzing the properties and macroeconomic implications of alternative transitory shocks: the production technology shock, the monetary shock and one other shock emanating from the banking sector itself: a shock that alters the collateral value of capital.

The use of a staggered price setting structure allows for richer dynamic effects of monetary policy than those found in the models with one-period advanced price-setting that are common in the open economy literature (see e.g. Obstfeld and Rogoff (1995, 1999), Bacchetta and Van Vincoop (2000) and Corsetti and Pesenti (2001)).

Most importantly, and in contrast with most of the existing literature- where monetary policy is introduced by assuming that some monetary aggregate follows an exogenous stochastic process- we model monetary policy as endogenous. Our approach accords much better with the practice of modern central banks, and provides a more suitable framework for policy analysis than the traditional one. Financial shocks can have a relevance of their own for real activity, and banks play a major role in their origin and, probably, in their propagation. The financial turmoil that started in summer 2007 was characterized by a gradual deterioration of banks liquidity and capital positions. Banks report that they reacted by tightening credit standards for lending to the private sector and by increasing both collateral requirements and margins on loans.

The coefficients in the open economy’s equilibrium condition also depend on parameters that are specific to the open economy (in our case, the degree of openness, the elasticity of substitution between domestic and foreign goods).

The remainder of the paper is organized as follows. In section 2 we lay out the basic model. Section 3 derives the equilibrium in log-linearised form. Section 4 analyses the macroeconomic implications of three alternative shocks.
2 Framework

We lay out a small open economy version of the Calvo sticky price model. We use the resulting framework to analyse the macroeconomic implications of a banking sector through three alternative shocks: the production technology shock $A_t$, the monetary shock $\varepsilon_t$, and one other shock emanating from the banking sector itself: a shock that alters the collateral value of capital $\gamma_{1,t}$.

Compared to the closed economy, the presence of open economy transmission channels and shocks not only produces new trade-offs for monetary policy, but also introduces additional sources of variability. It may be more important for central banks in small open economies to understand the nature of price setting and the effects of exchange rate movements on the economy than the determination of the exchange rate itself.

Our model is based on the New Keynesian small-open-economy model developed by Gali (2007). In this model the world economy is considered as a continuum of small open economies represented by the unit interval. Since each economy is of measure zero, its domestic policy decision do not have any impact on the rest of the world. We assume that different economies share identical preferences, technology and market structure. Our focus is on the behaviour of a single economy and its interaction with the world economy. Households choose how much to consume and how much labour to supply. Firms choose prices and then produce enough goods to meet demand. A fraction of the domestically produced goods are exported and a fraction of the domestically consumed goods are imported, with the size of the fractions determined by the relative price of goods produced at home and abroad. This is the minimal structure needed to capture the open economy dimension.

2.1 Households

A continuum of households populate the economy, consume goods and supply labour to firms and banks. A typical small open economy is inhabited by a representative household who consumes goods produced both domestically and abroad and seeks to maximize the discounted sum of its expected utility,

$$E_t \sum_{t=0}^{\infty} \beta^t U (c_t, n_t, m_t^s)$$
where \(n_t\) and \(m^*_t\) denote, respectively, labor in firms and in banks. \(c_t\) is a composite consumption index. In what follows we specialize the period utility function to take the form

\[
U(c_t, n_t, m^*_t) = \frac{c_t^{1-\sigma}}{1 - \sigma} - \frac{\psi(n_t + m^*_t)^{1+\kappa}}{1 + \kappa}
\]

First, the maximisation of the utility is subject to a sequence of budget constraints of the form

\[
\int_0^1 c_{H,t}(j)dj + \int_0^1 \int_0^1 c_{i,t}(j)djdi + w_t m^d_t + q_t(K_{t+1} - (1 - \delta)K_t)
\]
\[+ \frac{q_t}{2}(K_{t+1} - K_t)^2 + b_{t+1} + T_t\]
\[= w_t(n_t + m^*_t) + qt(r^k_t)K_t + (1 + r^B_t)h_t
\]

"Notice that money does not appear in either the budget constraint or the utility function: throughout we specify monetary policy in terms of an interest rate rule. (That modelling strategy has been adopted in much recent research on monetary policy. In it money can be thought of as playing the role of unit of account only)."

Households’ preferences are specified over a continuum of differentiated goods that enter the households’ utility function. Households thus prefer to consume a mixture of differentiated goods rather than consuming just one variety. The consumption bundle \(c_t\) is a constant elasticity of substitution (CES) aggregated index of domestically produced and imported sub-bundles \(c_{H,t}\) and \(C_{F,t}\). The composite consumption index is defined by

\[
c_t \equiv \left[(1 - \alpha)^{\frac{1}{s}} (c_{H,t})^{\frac{s-1}{s}} + \alpha^{\frac{1}{s}} (C_{F,t})^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}}
\]

where \(c_{H,t}\) is an index of consumption of domestic goods given by the CES function

\[
c_{H,t} \equiv \left(\int_0^1 c_{H,t}(j)^{\frac{s-1}{s}} dj\right)^{\frac{s}{s-1}}
\]
where \( j \in [0, 1] \) denotes the good variety (each country produces a continuum of differentiated goods, represented by the unit interval). \( c_{F,t} \) is an index of imported goods given by

\[
c_{F,t} = \left( \int_0^1 c_{F,t}(j)^{1-\epsilon} \, dj \right)^{1/\epsilon}
\]

\( \epsilon > 1 \) denotes the elasticity of substitution between varieties. \( \alpha \in [0, 1] \) is related to the degree of home bias in preferences and is, thus, a natural index of openness. Parameter \( \mu \) measures the substituability between domestic consumer.

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

\[
c_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} c_{H,t} ; \quad c_{F,t}(j) = \left( \frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\epsilon} c_{F,t} \tag{2}
\]

for all \( j \in [0, 1] \) where \( P_{H,t} = \left( \int_0^1 P_{H,t}(j) \, df(1-\epsilon) \right)^{1/\epsilon} \) is the domestic price index (an index of prices of domestically produced goods) and \( P_{F,t} = \left( \int_0^1 P_{F,t}(j) \, df(1-\epsilon) \right)^{1/\epsilon} \) is a price index of imported goods (expressed in domestic currency).

We can write total expenditures on domestic goods as \( \int_0^1 P_{H,t}(j)c_{H,t}(j) \, dj = P_{H,t}c_{H,t} \), and total expenditures on imported goods as \( \int_0^1 P_{F,t}(j)c_{F,t}(j) \, dj = P_{F,t}c_{F,t} \).

Finally, the optimal allocation of expenditures between domestic and imported goods is given by

\[
c_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_{c,t}} \right)^{-\mu} c_t ; \quad c_{F,t} = \alpha \left( \frac{P_{F,t}}{P_{c,t}} \right)^{-\mu} c_t \tag{3}
\]

where \( P_{c,t} = \left[ (1 - \alpha) (P_{H,t})^{1-\mu} + \alpha (P_{F,t})^{1-\mu} \right]^{1/\mu} \) is the consumer price index (CPI).

Notice that when the price indexes for domestic and foreign goods are equal, parameter \( \alpha \) corresponds to the share of domestic consumption allocated to imported goods. Thus \( \alpha \) represents a natural index of openness.
Total consumption by domestic households is given by

\[ c_t = P_{H,t} c_{H,t} + P_{F,t} c_{F,t} \]

Thus, the budget constraint can be rewritten as

\[ c_t + w_t m^d_t + q_t (K_{t+1} - (1 - \delta) K_t) + \frac{q_t}{2} (K_{t+1} - K_t)^2 + b_{t+1} + T_t = w_t (n_t + m^s_t) + q_t r^K_t K_t + (1 + r_t^{B_t}) b_t \]

where \( b_{t+1} \) are bonds in real terms bought in period \( t \) and to be reimbursed in \( t+1 \) with a real interest rate \( r_t^{B_t} \). Household earns the hourly market-clearing real wage \( w_t \) and a rent on the capital used by the firm, \( r_t^K K_t \). Income is spent on purchases of investment goods, \( I_t \), on purchase of government bonds \( b_{t+1} \) and on real lump-sum tax payments, \( T_t \).

In addition, the model includes one more constraint. Since we require that the model must pertain to a monetary economy, we need to incorporate the transaction facilitating properties of money—the medium of exchange. The most concrete way possible is via a constraint requiring the household to pay for consumption spending during period \( t \) with money held in that period. The medium of exchange composed entirely of bank deposits and will require that consumption during \( t \) must rigidly relate to deposits held at the end of \( t \).

\[ P_t c_t = v D_t \]

Bank’s balance sheet consists of high-powered (base) money \( H_t \) and loans \( L_t \) as assets and households’ deposits \( D_t \) as liabilities.

\[ H_t + L_t = D_t \] (5)

The bank’s ratio of reserves over deposits is assumed to be constant, \( rr = \frac{H_t}{D_t} \), which allows us to rewrite the deposit-in-advance constraint in real terms as

\[ c_t = \frac{v}{1 - rr} \frac{L_t}{P_t} \] (6)

The bank’s production of loans in real terms is constrained by the following technology
\[
\frac{L_t}{P_t} = F(b_{t+1} + \gamma_{1,t} \tau K_{t+1})^\xi (m^d_t)^{1-\xi}
\]

with \(F\) the productivity coefficient and \(\tau\) a parameter determining the relative efficiency of capital as collateral. \(m^d_t\) denotes the demand for labour at the bank. \(b_{t+1} + \gamma_{1,t} \tau K_{t+1}\) denotes the collateral. \(\gamma_{1,t}\) is a shock term that affects the collateral relative capacity of the stock of capital.

The partial derivatives relating the change in the amount of loans to the change in the factors of loans production are:

\[
\frac{\partial L_t}{\partial b_{t+1}} = \frac{\xi c_t}{b_{t+1} + \gamma_{1,t} \tau K_{t+1}} \frac{1 - rr}{v}
\]

\[
\frac{\partial L_t}{\partial K_{t+1}} = \frac{\xi \gamma_{1,t} \tau c_t}{b_{t+1} + \gamma_{1,t} \tau K_{t+1}} \frac{1 - rr}{v}
\]

\[
\frac{\partial L_t}{\partial m^d_t} = \frac{(1 - \xi)c_t}{m^d_t} \frac{1 - rr}{v}
\]

The lagrangian multiplier on (4) is denoted \(\lambda_t\) and for (6) is \(\varphi_t\). Then we can write the remaining optimality condition for the household’s problem as follows:

\[
c_t^{\sigma} - \lambda_t + \varphi_t = 0
\]

\[-\psi (n_t + m^s_t)^{\kappa} + \lambda_t w_t = 0\]

\[-\lambda_t w_t - \varphi_t \frac{(1 - \xi)c_t}{m^d_t} = 0\]

\[-\lambda_t + \beta E_t \lambda_{t+1} (1 + r_{t+1}) - \varphi_t \frac{\xi c_t}{b_{t+1} + \gamma_{1,t} \tau K_{t+1}} = 0\]

\[-\lambda_t q_t (1 - (K_{t+1} - K_t)) + \beta E_t \lambda_{t+1} q_{t+1} (r^K_{t+1} + (1 - \delta) + (K_{t+2} - K_{t+1}))\]

\[-\varphi_t \frac{\xi \gamma_{1,t} \tau c_t}{b_{t+1} + \gamma_{1,t} \tau K_{t+1}} = 0\]
By rearranging the FOC with respect to $m_t^d$ we can derive a relationship between the two lagrangian multipliers.

$$\varphi_t = -\lambda_t w_t \frac{m_t^d}{(1-\xi)c_t} = -\lambda_t w_t \frac{\partial m_t^d}{\partial L_t} \frac{\partial L_t}{\partial c_t}$$ (7)

The relationship can be interpreted as the value of the consumption that would be enjoyed with the labour income obtained with the work time required to produce the amount of loan needed to fund purchase of consumption. If we substitute this expression in the FOC with respect to consumption we obtain an expression of $\lambda_t$

$$\lambda_t = \frac{c_t^{-\sigma}}{1 + w_t \frac{m_t^d}{(1-\xi)c_t}}$$ (8)

We define $\phi_t = \frac{w_t m_t^d}{(1-\xi)c_t}$, the marginal financial cost of consumption.

By combining the first order conditions on bonds and consumption, the Euler equation of the intertemporal allocations of consumption is the following

$$\beta E_t \lambda_{t+1} (1 + r_{t+1}^B) = \lambda_t + \varphi_t \frac{\xi c_t}{b_{t+1} + \gamma_1 t \tau K_{t+1}}$$

The introduction of banking features gives collateral services to bonds which can be used to produce more loans. By using the expression of $\varphi_t$ we obtain

$$\beta E_t \lambda_{t+1} (1 + r_{t+1}^B) = \lambda_t [1 - w_t \frac{\xi m_t^d}{(1-\xi)(b_{t+1} + \gamma_1 t \tau K_{t+1})}]$$ (9)

As Goodfriend and McCallum (2007), we define $LSY_t^B = w_t \frac{\partial m_t^d}{\partial L_t} \frac{\partial L_t}{\partial c_t}$, the "liquidity service yield on bonds", the income value of the amount of bank labour that can be saved by the used collateral services of the bond.

We can rewrite the Euler equation

$$\beta E_t \left( \frac{c_t^{-\sigma}}{1 + \phi_{t+1}} \right) (1 + E_t r_{t+1}^B) = \frac{c_t^{-\sigma}}{1 + \phi_t} \left[1 - LSY_t^B \right]$$ (10)
For capital accumulation we use the first order condition of the stock of capital

\[-\lambda_t q_t (1 + (K_{t+1} - K_t)) + \beta E_t \lambda_{t+1} q_{t+1} [r_{t+1}^K + (1 - \delta) + (K_{t+2} - K_{t+1})] \]

\[+ \lambda_t w_t \frac{m^d_t}{(1 - \xi) c_t} \frac{\xi \gamma_{1,t+1} c_t}{b_{t+1} + \gamma_{1,t+1} K_{t+1}} = 0 \] (11)

We define \( LSY_t^K = w_t \frac{\partial m^d_t}{\partial K_{t+1}} \frac{\partial L_t}{\partial K_{t+1}} \), the "liquidity service yield on capital", the income value of the amount of bank labour that can be saved by the used collateral services of the capital. The equation of capital dynamics is then,

\[\beta E_t \left( \frac{c_{t+1}^{-\sigma}}{1 + \phi_{t+1}} \right) q_{t+1} [r_{t+1}^K + (1 - \delta) + (K_{t+2} - K_{t+1})] \]

\[= \frac{c_t^{-\sigma}}{1 + \phi_t} [q_t + q_t (K_{t+1} - K_t) - LSY_t^K] \] (12)

2.1.1 Domestic inflation, CPI inflation, Real exchange rate

Before proceeding with our analysis of equilibrium we introduce several assumptions and identities. We start by defining the bilateral terms of trade between the domestic economy and country \( i \) as \( TOT_{i,t} = \frac{P_{F,t}}{P_{H,t}} \), the price of country \( i \)'s goods in terms of home goods. The effective terms of trade are, thus, given by

\[TOT_t = \frac{P_{F,t}}{P_{H,t}} = \left( \int_0^1 (TOT_{i,t})^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \] (13)

The loglinear expression \( tot_t = \int_0^1 tot_{i,t} di \), where \( i \in [0, 1] \).

Loglinearization of the CPI around a symmetric steady-state satisfying the purchasing power parity (PPP) condition \( P_{F,t} = P_{H,t} \) yields

\[p_{c,t} = (1 - \alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha tot_t \] (14)

where \( tot_t = \log TOT_t = p_{F,t} - p_{H,t} \), denotes the (log) effective terms of trade (the price of foreign goods in terms of domestic goods).
It follows that domestic inflation, $\pi_{H,t} = P_{H,t+1} - P_{H,t}$ and CPI inflation are linked according

$$\pi_{c,t} = \pi_{H,t} + \alpha \Delta \text{tot}_t \quad (15)$$

which makes the gap between our two measures of inflation proportional to the per cent change in the terms of trade with the coefficient of proportionality given by the index of openness $\alpha$.

We assume that the law of one price holds for individual goods at all times (both for import and export prices) implying that $P_{i,t}(j) = e_{i,t} P_{i,t}^j$ for all $i,j \in [0,1]$, where $e_{i,t}$ is the bilateral nominal exchange rate (the price of country $i$’s currency in terms of the domestic currency), and $P_{i,t}^j$ is the price of country $i$’s good $j$ expressed in the producer’s (country $i$’s) currency. Using the definition of $P_{i,t}$ we obtain

$$P_{i,t} = e_{i,t} P_{i,t}^j \quad (16)$$

where $P_{i,t}^j \equiv \left( \int_0^1 P_{i,t}(j)^{1-t} \, dj \right)^{\frac{1}{1-t}}.$

By using the expression of $P_{F,t}$ and after log-linearizing it yields

$$p_{F,t} = \int_0^1 \left( \text{ner}_{i,t} + p_{i,t}^j \right) \, di = \text{ner}_t + p^*$$

where $\text{ner}_t$ is the (log) nominal effective exchange rate and $p^*$ is the world price. We note that for the world as a whole there is no distinction between CPI and domestic price level nor for their corresponding inflation rates. The rest of the world is assumed to be unaffected by the economy.

Combining the previous result with the definition of the terms of trade we obtain the following expression

$$\text{tot}_t = p_{F,t} - p_{H,t} = \text{ner}_t + p^* - p_{H,t}$$

Now we turn to derive a relationship between the terms of trade and the real exchange rate. We note $RER_{i,t} \equiv \frac{e_{i,t} P_{i,t}^j}{P_{F,t}}$ the bilateral real exchange rate with country $i$ (the ratio of the two countries’ CPIs both expressed in domestic currency). Let $rer_{i,t} \equiv \int_0^1 rer_{i,t} \, di$ be the (log) effective real exchange rate (where $rer_{i,t} = \log RER_{i,t}$).
\[ rer_t = \int_0^1 (ner_{i,t} + p_{i,t}^t - p_{c,t})di \\
= ner_t + p_t^* - p_{c,t} \\
= tot_t + p_{H,t} - p_{c,t} \\
= (1 - \alpha)tot_t \]

The last equality holds only up to a first order approximation when $\mu \neq 1$.

\section*{2.2 Firms}

\subsection*{2.2.1 Technology}

A typical firm in the home country produces a differentiated good with a technology represented by the production function.

\[ Y_t(j) = A_tK_t(j)^{w}n_t(j)^{1-w} \]  

(17)

where $A_t$ is the labour productivity evolving according to the following autoregressive structure $a_t = \log A_t = \rho_a a_{t-1} + \zeta_t$, $0 < \varpi < 1$ is the capital share in production function, $n_t(j)$ is the labour demand, $K_t(j)$ is the stock of capital and where $j \in [0,1]$ is a firm-specific index. Hence the real marginal cost (expressed in terms of domestic prices) will be common across domestic firms.

Let $Y_t = \left[ \int_0^1 Y_t(j)^{1-\frac{1}{\varpi}}dj \right]^{\frac{1}{1-\varpi}}$ represent an index for aggregate domestic output. $n_t = \int_0^1 n_t(j) dj$ is an index to aggregate employment, and $K_t = \int_0^1 K_t(j) dj$ is an index to aggregate capital. Thus, up to a first order approximation we have an aggregate relationship

\[ \hat{y}_t = \varpi \hat{k}_t + (1 - \varpi) \hat{n}_t + \hat{a}_t \]

\subsection*{2.2.2 Price-setting}

Firms have some market power over the price of the goods that they are selling since consumers prefer a mixture of differentiated goods rather than consuming just one variety. Unlike the case when all goods are perfect substitutes, this means that consumers will not switch consumption away completely from a lightly more expensive goods. In this monopolistically competitive environment firms charge a mark-up over marginal cost. Quantities sold in a given period are
demanded-determined in the sense that firms are assumed to set prices in domestic currency terms and then supply the amount of goods that are demanded by consumers at that price.

There are, of course, many ways to specify stickiness in price-setting. Prior to the rational expectations revolution of the 1970s, expectations of future inflation were backward-looking, usually following an error-correction process in which the change in expected inflation was a fraction of the difference between last period’s expected inflation and the actual inflation. In this section we allow the firms to adopt Calvo-style price-setting behavior. This structure allows the introduction of nominal rigidities while maintaining a constant returns-to-scale assumption which is necessary for aggregation when financial market imperfections are introduced.

We model nominal rigidities by means of the Calvo pricing assumption: a given producer is free to change his price in a given period only with probability \((1 - \theta)\). Following Calvo (1983), we introduce nominal rigidities on price setting by assuming that firms have a chance to price optimally only when receiving a market signal that comes up with a constant probability \((1 - \theta)\). By contrast, there is probability \(\theta\) that the price cannot be changed. The price fixed by these firms is different from the flexible price (reoptimized in each period) because they anticipate the realization of a period during which they will be incapable to modify their price again.

Producers maximize the anticipated path of profits per units of wealth:

\[
E_t \sum_{k=0}^{\infty} \beta^k \theta^k \left( \frac{P_{H,t+k}(j)}{P_{H,t+k}} \right)^{1-\epsilon} Y_{t+k} - w_{t+k} n_{t+k}(j) - r_{t+k} K_{t+k}(j)
\]

in which \(\epsilon\) is the elasticity of substitution between differentiated goods. With staggered prices and fixed probabilities, the FOC for domestic selling price is

\[
(1 - \epsilon) E_t \sum_{k=0}^{\infty} (\beta \theta)^k \left( \frac{P_{H,t}(j)}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k} - \frac{m_{t+k}(j)}{P_{H,t+k}} = 0
\]

\[
m_{t+k}(j)(-\epsilon) E_t \sum_{k=0}^{\infty} (\beta \theta)^k \left[ \frac{P_{H,t}(j)}{P_{H,t+k}} \right]^{-\epsilon-1} Y_{t+k} = 0
\]

We define \(m_{t}(j) = \frac{w_{t}}{f_{n_t(j)}}\) the real marginal cost as the ratio between the real wage and the marginal product of labour, where \(\frac{\partial m_{t}(j)}{\partial Y_t(j)} = \frac{1}{f_{n_t(j)}}\) is the inverse of the marginal product of labour. By substitution we can obtain \(P_{H,t}(j) = \frac{e}{1-\epsilon} P_{H,t} m_{t}
\]
The Calvo Pricing behavior gives rise to the following Dixit-Stiglitz aggregate price level

\[ P_{H,t} = \left( 1 - \theta \right) \left[ \tilde{P}_{H,t}(j) \right]^{1 - \epsilon} + \theta \left[ P_{H,t-1} \right]^{1 - \epsilon} \]

where \( \tilde{P}_{H,t}(j) \) is the domestic optimal selling price during period \( t \).

The FOC can be rearranged to be solved out for \( P_{H,t}(j) \) as follows

\[ P_{H,t}(j) = \frac{\epsilon}{1 - \epsilon} \frac{E_t \sum_{k=0}^{\infty} (\beta \theta)^k m_{ct+k}(j) (P_{H,t+k})^\epsilon Y_{t+k}}{E_t \sum_{k=0}^{\infty} (\beta \theta)^k (P_{H,t+k})^{\epsilon-1} Y_{t+k}} \]

After linearizing the optimal price given by the previous equation we obtain

\[ \bar{p}_{H,t} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \left( p_{H,t+k} + \tilde{m}_{ct+k} \right) \]

(18)

where \( \bar{p}_{H,t} \) denotes the (log) of newly set domestic prices. The log of the optimal price set depends positively on the expected future evolution of both the log of the aggregate domestic price level and the log of the real marginal cost.

The calvo-type aggregation scheme implies a proportional relationship between the rate of inflation and the optimal relative price

\[ \pi_{H,t} = \frac{1 - \theta}{\theta} (\bar{p}_{H,t} (j) - p_{H,t}) \]

(19)

with the definition of inflation \( \pi_{H,t} = p_{H,t} - p_{H,t-1} \).

In the small open economy, the dynamics of domestic inflation in terms of real marginal cost are described by an equation analogous to the one associated with a closed economy. Hence

\[ \pi_{H,t} = \beta E_t \pi_{H,t+1} + \frac{(1 - \beta \theta)(1 - \theta)}{\theta} m_{ct} \]

(20)
The inflation equation obtained does not depend on any of the parameters that characterize the open economy. It implies that current inflation depends upon its expected value for the next period and the current real marginal cost under optimal pricing. The determination of the real marginal cost as function of domestic output in the small open economy differs somewhat from that in the closed economy, due to the existence of a wedge between output and consumption and between domestic and consumer prices

\[
m_{t} = w - p_{H,t} - y + n_t = (w_t - p_{c,t}) + (p_{c,t} - p_{H,t}) - y_t + n_t
\]

\[
= w_t - p_{c,t} + \alpha t o t_t - y_t + n_t
\]

### 2.3 Interest rates

As in Goodfriend and McCallum (2007), we introduce a nominal (fictitious) security to provide a benchmark interest rate (for uncollateralised loans), \( R^T_t \). In contrast to bonds, this security does not provide any collateral. The interest rate differential between these two rates can be obtained by substituting the Euler equation for the nominal security in the Euler equation for bonds

\[
\frac{1 + r^B_t}{1 + r_t} = 1 - LSY^B_t
\]  

(21)

If we take the log we obtain

\[
r^T_t = r^B_t + LSY^B_t
\]  

(22)

The nominal interest rate on the fictitious security is \( R^T_t = r^T_t + E_t \tau p_{H,t+1} \), then the difference in nominal returns is the liquidity service yield on bonds

\[
LSY^B_t \simeq R^T_t - R^B_t
\]  

(23)

Capital will serve as collateral less well than bonds, by the factor \( \tau \), therefore the liquidity service yield on capital is smaller, \( LSY^K_t = \tau LSY^B_t \).

Households can borrow funds from the central bank at a rate \( R^{IB}_t \). Households loan these funds to other households at the rate \( R^T_t \) as the nominal security pays the same benchmark interest, reflecting a no-arbitrage condition between
loan and asset markets. However, loan production requires monitoring as well as collateral provided by the households as factor inputs. At the cost-minimising optimum the real marginal cost loan production equals the factor price divided by the factor’s marginal product (for each factor of production). Thus, marginal cost can be calculated by dividing the real wage by the partial derivative of $L_t / P_t$ with respect to $m^d_t$, $w_t / \partial L_t / \partial m^d_t = w_t / (1 - \xi) c_t v (1 - r r_t) = \frac{m^d_t}{c_t} v = \frac{v}{1 - r r_t} \phi_t$.

Because of the no-arbitrage condition between the loan market and the asset market, banks would provide uncollateralised loans to households at the rate $R^T_t$, implying a differential between the interbank rate $R^{IB}_t$ and the benchmark rate $R^T_t$ given by

$$(1 + R^{IB}_t) (1 + \frac{v}{1 - r r_t} \phi_t) = (1 + R^T_t)$$

which can be approximated as

$$R^T_t - R^{IB}_t = \frac{v}{1 - r r_t} \phi_t$$

Finally, as loans are collateralised in equilibrium and since $(1 - \xi)$ is the factor share for monitoring in the loan production function, the marginal cost of collateralised loans are $(1 - \xi) \frac{v}{1 - r r_t} \phi_t$. Profit maximisation by banks implies then

$$(1 + R^{IB}_t) (1 + (1 - \xi) \frac{v}{1 - r r_t} \phi_t) = (1 + R^L_t)$$

a log approximation gives

$$R^L_t - R^{IB}_t = \frac{(1 - \xi) v}{1 - r r_t} \phi_t$$

The nominal interest rate on collateralised loans, $R^L_t$ is lower than $R^T_t$ because the borrowers come with bonds and capital to provide the collateral service, then they receive a deduction on the loan rate equal to the share $\xi$ of collateral in loan costs.

We suppose that there is a concept of country risk so that the interest rate a citizen of a country pays on international borrowing is an increasing function of the country’s total international debt (and the interest rate its citizen get for international savings declines as a function of total savings)

$$r^B_t = r^* - \alpha b_{t+1}$$
where \( r^* \) is a constant added so that international savings can occur at a positive interest rate. The expression represent that as a country accumulates foreign debt, the international interest rate it must pay rises. A highly indebted country will have a higher equilibrium interest rate.

### 2.4 Equilibrium

#### 2.4.1 Consumption and output in the small open economy

Goods market clearing in the representative small open economy requires

\[
Y_t(j) = c_{H,t}(j) + \int_0^1 c_{H,t}^i(j) di + I_t
\]

\[
= \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} c_{H,t} + \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} \int_0^1 c_{H,t}^i di + I_t
\]

for all \( j \in [0,1] \) and all \( t \), where \( c_{H,t}^i(j) \) denotes the country \( i \)’s demand for good \( j \) produced in the home country.

\[
Y_t = \left[ \int_0^1 Y_t(j)^{\frac{\epsilon}{1-\epsilon}} dj \right]^{1-\epsilon} \text{ is the aggregate domestic output}
\]

\[
Y_t = c_{H,t} + \frac{P_{F,t}}{P_{H,t}} c_{F,t} + I_t
\]

in the loglinearized version we have

\[
\hat{y}_t = (1 - \alpha) c_{H,t} + \frac{\alpha c}{c + \delta k} \hat{c}_{F,t} + \frac{\delta k}{c + \delta k} \hat{\epsilon}_t + \frac{\alpha c}{c + \delta k} (\hat{p}_{F,t} - \hat{p}_{H,t})
\]

By aggregating overall countries we can derive a world market clearing condition

\[
y_t^* = \int_0^1 y_t^* di = \int_0^1 c_t^* di = c_t^*
\]

where \( y_t^* \) is the world output (in log terms) and \( c_t^* \) is the world consumption (in log terms).
2.4.2 Trade balance

Let \( nx_t = \left( \frac{1}{\tau} \right) \left( Y_t - \frac{P_{ct,c_t}}{P_{xt,t}} \right) \) denotes net exports in terms of domestic output, expressed as a function of steady-state output \( Y_t \). In the particular case of \( \sigma = \mu = \gamma = 1 \) it follows \( P_{xt,t}Y_t = P_{ct,c_t} \) for all \( t \), thus implying a balanced trade at all times. More generally, a first order approximation yields

\[ nx_t = y_t - c_t - \alpha dt \]

2.5 Calibration and steady state solution

The strategy of the paper is to use observed average historical values of interest rates and observations on banking and macroeconomic aggregates, to calibrate parameters and to determine the steady-state solution of the model with realistic trend productivity growth in the production of goods and loans. The objective is to determine the extent to which the introduction of financial frictions and the openness of the economy can account for observable interest rate differentials and how much it matters quantitatively for aggregates.

2.5.1 Calibration of parameters

The calibration of parameters of the model is used to explore the implications of the openness of the economy and of introducing banking sector in an otherwise conventional benchmark New Keynesian model. The model has 18 parameters that need to be specified. We choose a standard value of the discount factor \( \beta = 0.99 \). We set the quarterly depreciation rate \( \delta = 0.025 \) as assumed in the real business cycle literature. The parameters that determine the elasticities of marginal utility of consumption and labor in the utility function are set at \( \sigma = 2 \) and \( \kappa = 1.5 \). The parameter that determines the weight of labor disutility is set at \( \psi = 0.82 \) to imply that total labor (labor at non-banking firms and labor at banking firms) is equal to one in steady-state, \( n + m = 1 \). This is a way for normalizing total labor in a way that leaves the value of both \( n \) and \( m \) in steady-state as the fraction that represent with respect to total labor. We choose \( \omega = 0.36 \) to reflect relative shares of capital and labor in goods production function. For the parameter that determines the Dixit-Stiglitz elasticity in the demand curves is \( \epsilon = 11 \). In steady-state, the value assigned to \( \epsilon \) conveys a mark-up of prices over marginal costs equal to \( \frac{1}{\epsilon - 1} = 1.1 \), which means that firms would charge a 10% higher price relative to marginal costs. For Calvo price setting, the Calvo probability is set, as in Taylor (1999) and in the literature, at \( \theta = 0.75 \) which implies that the average frequency of optimal pricing is \( \frac{1}{\theta} = 4 \) quarters. We set a baseline value for \( \alpha \) (or degree of openness) of 0.4. The latter
corresponds roughly to the import/GDP ratio in Canada, represent a prototype small open economy.

For the banking parameter, the labor share in loan production $\xi = 0.65$, and the capital collateral inferiority parameter $\tau = 0.2$. The latter parameter reflects the inferiority of capital to bonds for collateral purposes, resulting because banks specialize in information-intensive lending and capital goods require substantial monitoring to verify their physical condition and market value. By using values of US public debt, the stock of bonds in steady state represents 42% of GDP, as of the third quarter of 2005. We calibrate the reserve ratio as the average of required reserves over total deposits in the US, $rr = 0.0218$. As in Goodfriend and McCallum (2007), we calibrate velocity as the ratio of US GDP to M3 for the fourth quarter of 2005, then $v = 0.31$.

The monetary policy rule is parameterized as in Taylor (1993), the sensitivity of nominal rate to inflation $\mu_1 = 1.5$, the sensitivity of nominal rate to output gap $\mu_2 = 0.5$. For the smoothing parameter, $\mu_3 = 0.8$, which implies enough inertial behavior of nominal interest rates.

Table 1. Calibration of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption elasticity</td>
<td>$\sigma = 2$</td>
</tr>
<tr>
<td>Labor elasticity</td>
<td>$\kappa = 1.5$</td>
</tr>
<tr>
<td>Labor weight</td>
<td>$\psi = 0.82$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.99$</td>
</tr>
<tr>
<td>Capital share in production function</td>
<td>$\omega = 0.36$</td>
</tr>
<tr>
<td>Rate of capital depreciation</td>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td>Calvo sticky prices</td>
<td>$\theta = 0.75$</td>
</tr>
<tr>
<td>Dixit-Stiglitz demand elasticity</td>
<td>$\epsilon = 11$</td>
</tr>
<tr>
<td>Sensitivity of nominal rate to inflation</td>
<td>$\mu_1 = 1.5$</td>
</tr>
<tr>
<td>Sensitivity of nominal rate to output gap</td>
<td>$\mu_2 = 0.5$</td>
</tr>
<tr>
<td>Smoothing parameter</td>
<td>$\mu_3 = 0.8$</td>
</tr>
<tr>
<td>Labor share in loan production</td>
<td>$\xi = 0.65$</td>
</tr>
<tr>
<td>Reserve coefficient</td>
<td>$rr = 0.0218$</td>
</tr>
<tr>
<td>Deposit velocity</td>
<td>$v = 0.31$</td>
</tr>
<tr>
<td>Capital collateral inferiority parameter</td>
<td>$\tau = 0.2$</td>
</tr>
<tr>
<td>Degree of openness</td>
<td>$\alpha = 0.4$</td>
</tr>
<tr>
<td>Substituability between domestic and foreign goods</td>
<td>$\mu = 1$</td>
</tr>
<tr>
<td>Substituability between goods produced in different foreign countries</td>
<td>$\gamma = 1$</td>
</tr>
</tbody>
</table>

2.5.2 Steady state solution

The steady-state is characterized by zero inflation and all variables growing (or shrinking) along a deterministic growth path, all the exogenous sources of variability are fixed at their expected value. Hence the exogenous processes are
at zero on steady-state $A_t = \varepsilon_t = \gamma_{1,t} = 0$. The steady-state is calculated numerically starting with bonds over output calibrated to 0.42. By substituting out certain variables we can express the core steady-state system in terms of fifteen equations involving fifteen variables $y, c, K, w, m, n, l, mc, \phi, LSY^B, LSY^K, r^K, R^B, R^T, R^{IB}$.

\[ \beta \left(1 + r^B\right) = 1 - LSY^B \]

\[ \beta \left( r^K + (1 - \delta) \right) = 1 - LSY^K \]

\[ \frac{1}{1 + R^T} = \frac{1}{1 + R^B} - LSY^B \]

\[ (1 + R^{IB}) \left(1 + \frac{v}{1 - \frac{\phi}{1 - r^T}}\right) = (1 + R^T) \]

\[ mc = \frac{\epsilon - 1}{\epsilon} \]

\[ w = mc \frac{(1 - \omega) Y}{n} \]

\[ r^K = mc \frac{\omega Y}{K} \]

\[ Y = c + \delta K \]

\[ Y = K^\omega n^{1-\omega} \]

\[ \phi = w \frac{m}{(1 - \xi) c} \]

\[ LSY^B = \frac{\xi \cdot wm}{1 - \xi \cdot b + \tau K} \]
\[ LSY^K = \tau.LSY^B \]

\[ n + m = 1 \]

\[ m = 0.05 \]

\[ L = \frac{1 - rr}{v}c \]

Thus, the steady-state solution of the model can be reached by solving a non-linear system of fifteen equations.

3 Linearization

The model is solved by first taking linear approximations of the structural equations around the steady state and then finding the rational expectations equilibrium law of motion.

The dynamic equations of the New Keynesian model with banking activity in a small open economy are described by three sectors, the aggregate demand sector, the aggregate supply sector and the monetary sector.

**Aggregate Demand (AD) sector**

Consumption dynamic equation was described above as follows

\[ \hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma} \left[ \hat{\phi}_t - E_t\hat{\phi}_{t+1} \right] - \frac{1}{\sigma} E_t^r r^B_{t+1} - \frac{1}{\sigma} L\hat{SY}^B_t \]

Consumption dynamics are forward-looking and depend negatively on the real interest rate \( r^P_t \), the marginal collateral service of bonds \( L\hat{SY}^B_t \) and the current marginal financing cost of consumption \( \hat{\phi}_t \), the semi-loglinear approximation to \( \phi_t \) yields

\[ \hat{\phi}_t = \phi (\hat{w}_t + \hat{m}_t - \hat{c}_t) \]
where \( \phi \) denotes the steady-state level of \( \phi \). Domestic consumption is given by

\[
\hat{c}_{H,t} = \hat{c}_t - \mu \hat{q}_t
\]

Foreign consumption (imported goods) is given by

\[
\hat{c}_{F,t} = \hat{c}_t - \mu \hat{q}_t - \mu \hat{\omega}_t
\]

A semi-loglinear approximation to the liquidity service yield on bonds gives

\[
LSY^B_t = LSY^B \left( \hat{w}_t + \hat{m}_t - \frac{\tau K}{b + \tau K} \hat{k}_{t+1} - \frac{b}{b + \tau K} \hat{b}_{t+1} - \frac{\tau K}{b + \tau K} \gamma_{1,t} \right)
\]

where \( LSY^B \), \( b \) and \( K \) denote the the steady-state levels of the corresponding variables. A semi-loglinear approximation to the liquidity service yield on capital gives

\[
LSY^K_t = LSY^K \left( \hat{w}_t + \hat{m}_t - \frac{\tau K}{b + \tau K} \hat{k}_{t+1} - \frac{b}{b + \tau K} \hat{b}_{t+1} + \frac{b}{b + \tau K} \gamma_{1,t} \right)
\]

where \( LSY^K \) denotes the the steady-state levels of the liquidity service yield on capital. The capital accumulation dynamic equation was described above as follows

\[
K_{t+1} = \frac{1}{2 + r^K - \delta - LSY^K} E_t \hat{K}_{t+2} + \frac{r^K + (1 - \delta)}{2 + r^K - \delta - LSY^K} \hat{K}_t + \frac{1 - LSY^K}{qK(2 + r^K - \delta - LSY^K)} \left[ \hat{c}_t - E_t \hat{c}_{t+1} \right] + \frac{(1 - LSY^K)(r^K + (1 - \delta))}{qK(2 + r^K - \delta - LSY^K)} \left[ \hat{\phi}_t - E_t \hat{\phi}_{t+1} \right] + \frac{r^K(1 - LSY^K)}{qK(2 + r^K - \delta - LSY^K)} E_t \hat{r}^K_{t+1} + \frac{LSY^K(r^K + (1 - \delta))}{qK(2 + r^K - \delta - LSY^K)} LSY^K_t
\]
where $r_t^K$ is the rental rate of capital and is determined by the first order condition on the demand for capital, $r_t^K = mc_t.f_k(j)$, if we loglinearize the expression of $r_t^K$ we get

$$\hat{r}_t^k = \hat{mc}_t + \left(\hat{y}_t - \hat{k}_t\right)$$

The loglinear version of the investment dynamic yields

$$\hat{\dot{k}}_t = \frac{1}{\delta} \hat{k}_{t+1} - \frac{1 - \delta}{\delta} \hat{k}_t$$

Finally, the equilibrium in goods market is described by the overall resources constraint $Y_t = c_t + I_t$, which in loglinear terms becomes

$$\hat{y}_t = \frac{(1 - \alpha)c}{c + \delta k} \hat{c}_{H,t} + \frac{\alpha c}{c + \delta k} \hat{c}_{F,t} - \frac{\delta k}{c + \delta k} \hat{\dot{k}}_t + \frac{\alpha c}{c + \delta k} \left(\hat{\dot{p}}_{F,t} - \hat{\dot{p}}_{H,t}\right)$$

The loglinearized version of the net export is given by the following equation

$$\tilde{mx}_t = \hat{y}_t - \hat{\dot{c}}_t - \alpha \hat{\dot{a}}_t$$

**Aggregate Supply (AS) sector**

Assuming that prices are adjusted according to the Calvo (1983) mechanism, we get the following Phillips curve in log-linear terms

$$\pi_{H,t} = \beta \mathcal{E}_t(\pi_{H,t+1}) + \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \hat{mc}_t$$

where $\hat{mc}_t$ is the loglinearized definition of the real marginal cost of goods production with sticky prices $mc_t = \frac{\hat{w}_t}{\mathcal{E}_{nt}}$

$$\hat{mc}_t = \hat{w}_t - \hat{y}_t + \hat{n}_t - \hat{\dot{p}}_{c,t} + \alpha \hat{\dot{a}}_t$$

so marginal cost is not constant in this case. The loglinearized labor supply curve is described as follows

$$\kappa \frac{n}{n + m} \hat{n}_t + \kappa \frac{m}{n + m} \hat{m}_t = -\sigma \hat{c}_t - \phi_t + \hat{w}_t$$
The Cobb-Douglas production function in loglinear version yields

\[ y_t = \sigma \hat{k}_t + (1 - \sigma) \hat{n}_t + \hat{a}_t \]

Terms of trade are defined as

\[ \hat{\omega}_t = \hat{p}_{F,t} - \hat{p}_{H,t} \]

The relative price of capital is

\[ \hat{\eta}_t = \hat{p}_{H,t} - \hat{p}_{c,t} \]

The consumer price index (CPI) is given by the following equation

\[ \hat{p}_{c,t} = (1 - \alpha)\hat{p}_{H,t} + \alpha \hat{p}_{F,t} \]

where the price of imported goods is

\[ \hat{p}_{F,t} = \hat{w}e + \hat{p}^* \]

The consumer price index inflation is given by the following equation

\[ E_t \pi_{t+1}^c = (1 - \alpha) E_t \pi_{H,t+1} + \alpha E_t (\text{ner}_{t+1} - \text{ner}_t) \]

The current account in loglinearized terms yields

\[ \hat{b}_{t+1} = \frac{(1 + r^B)B}{Y - c - I + (1 + r^B)B} \hat{b}_t + \frac{r^B B}{Y - c - I + (1 + r^B)B} \hat{R}_{t+1}^B \]

\[ + \frac{Y}{Y - c - I + (1 + r^B)B} \hat{\nu}_{t} - \frac{c}{Y - c - I + (1 + r^B)B} \hat{c}_t \]

\[ - \frac{I}{Y - c - I + (1 + r^B)B} \hat{\eta}_{t} + \frac{(Y - I)}{Y - c - I + (1 + r^B)B} \hat{\eta}_{H,t} \]

\[ - \frac{c}{Y - c - I + (1 + r^B)B} \hat{\omega}_{c,t} \]
Monetary Sector

The loglinearized version of the loan-in-advance constraint is given by

\[
\hat{\ell}_t = \hat{\ell}_t
\]

where \(\hat{\ell}_t\) is the loan production function in loglinear terms

\[
\hat{\ell}_t = \frac{\xi \tau K}{b + \tau K} \hat{\ell}_{t+1} + (1 - \xi) \hat{m}_t + \frac{\xi b}{b + \tau K} \hat{b}_{t+1} + \frac{\xi \tau K}{b + \tau K} \hat{\gamma}_{1,t}
\]

The nominal interest rate of bonds is represented by the Fisher equation

\[
R^B_t = r^B_t + E_t \pi_{H,t+1}
\]

A simple way to represent monetary policy that has been found to fit central bank behaviour quite well is to let the short interest follow a variant of the Taylor rule, letting the interest rate be determined by a reaction function of lagged interest rate, inflation and output gap, where \(\varepsilon_t\) is a transitory deviation from the rule.

\[
R^{IB}_t = \mu_1 R^{IB}_{t-1} + (1 - \mu_2) [\mu_1 \pi_{H,t} + \mu_2 (Y_t - Y)] + \varepsilon_t
\]

The fictitious nominal interest rate of bonds (with no collateral services) is given by the expression described in the section 3-3

\[
R^T_t = R^B_t + LSY^B_t
\]

Banking intermediation (borrowing from central bank at \(R^B_t\) and making a loan to another household at \(R^T_t\)). The marginal cost of loan production is

\[
\frac{\partial L_t}{\partial m_t} = \frac{\frac{w_t}{(1-\xi)\phi_t}}{\frac{1}{1-\xi} - \frac{1-\tau r}{\tau}} = \frac{w_t m_t^v}{(1-\xi)\phi_t} \frac{v}{1-\tau r} = \phi_t \frac{v}{1-\tau r}. \text{ Thus, we have } (1 + R^T_t) = (1 + R^{IB}_t) \left(1 + \frac{v}{1-\tau r} \phi_t\right) \text{ for an equilibrium condition. In semi-loglinear terms, it yields}
\]

\[
R^T_t = R^{IB}_t + \frac{v}{1-\tau r} \phi_t
\]
The uncovered interest rate parity

\[ r_t^B = r^* + E_t(\text{ner}_{t+1} - \text{ner}_t) \]

The endogenous variables that appear in this system are \( \hat{y}_t, \hat{c}_t, \hat{c}_{H,t}, \hat{c}_{F,t}, \hat{t}_t, \hat{k}_{t+1}, \hat{m}_t, \hat{w}_t, \hat{p}_t, \hat{b}_{t+1}, \hat{m}_{\text{w},t}, L\hat{S}Y^K_t, L\hat{S}Y^B_t, \hat{m}_{\text{w},t}, \pi_{c,t}, \pi_{H,t}, \pi_{F,t}, \text{tot}_t, q_t, \text{ner}_t, p_{c,t}, p_{H,t}, p_{F,t}, r_t^B, r_t^k, R_t^B, R_t^I, \) and \( R_t^T \). In addition, there are three predetermined variables: \( \hat{k}_{t+1}, b_t \) and \( R_{t-1}^I \) and exogenous variables: \( A_t \) and \( \gamma_{1,t} \).

The linearised equilibrium dynamics for the small open economy have a representation in terms of output gap and domestic inflation analogous to that of its closed economy counterpart. We see that the small open economy’s equilibrium is characterized by a forward-looking IS-type equation similar to that found in the closed economy. Two differences can be pointed out, however. First the degree of openness influences the sensitivity of the output gap to interest rate changes. In particular an increase in openness raises that sensitivity (through the stronger effects of the induced terms of trade changes on demand). Second, openness generally makes the natural interest rate depend on expected world output growth, in addition to domestic productivity. The solution of the model is found by solving the linearized versions of the relationships. Each endogenous variable would react to both predetermined variables and the shocks.

4 Impulse-response functions

Here we examine the impulse response functions by using the linearized model calibrated at the steady-state, to include Calvo-style staggered pricing and Taylor-policy rule. We study the effects of banking sector in a small open economy New Keynesian model by considering three sources of shocks: the production technology shock \( A_t \), the monetary shock \( \varepsilon_t \) and one other shock emanating from the banking sector itself: a shock that alters the collateral value of capital \( \gamma_{1,t} \). The third is meant to represent the recent financial crisis.

In the small open economy a change in domestic output has an effect on marginal cost, through its impact on employment and through the terms of trade (the degree of openness and the substitutability between domestic and foreign goods). In an open economy there is an additional factor that distorts the incentives of the monetary authorities (beyond the presence of market power): the possibility of influencing the terms of trade in a way beneficial to domestic consumers. This possibility is a consequence of the imperfect substitutability between domestic and foreign goods combined with sticky prices. As shown in Benigno and Benigno (2003) in the context of a two-country model, the introduction of an employment subsidy that exactly offsets the market power distortion is not sufficient to render the flexible price equilibrium allocation
optimal, for, at the margin the monetary authority would have an incentive to
deviate from it to improve the terms of trade.

A change in world output has an effect on marginal cost, through its effect
on consumption (and hence the real wage), and through its effect on the terms
of trade. An increase in openness reduces the impact of a change in domestic
output on marginal cost (and hence on inflation). An increase in openness raises
the positive impact of a change in world output on marginal cost by limiting the
size of the associated variation in the terms of trade, hence, its countervailing
effect. Openness, generally, makes the natural interest rate depend on expected
world output growth, in addition to domestic productivity.

How do the key endogenous variables behave in response to a temporary
change in the productivity index? Is the model stable, in the sense that endoge-
nous variables return to their steady state values after the one-off change?

In this section we will first use impulse response analysis to examine the
properties of the model. Impulse response analysis allows us to see which vari-
bles display more complicated (or more interesting) dynamics in their response
to one-period shock. For a given parameter configuration the type of simulation
allows us to see which variables display greater or lesser response and which
variables have more volatile or oscillatory dynamics.

To be sure, the “real world” has many recurring changes but these simple
impulse-responses help us to isolate key dynamics properties of the model that
we would not otherwise be able to do.

For the impulse-response analysis we work with the equation governing the
evolution of the logarithm of the productivity index. The productivity shock
follows a stochastic log-linear autoregressive process; with the disturbance term
normally distributed with mean zero and variance $\sigma^2$.

Starting from steady state, we work shock productivity by the value of one
standard deviation. Of course, we can specify $\sigma$ at arbitrarily large or small
value to see if there are different dynamic responses to the magnitude of the
exogenous change. Figure 1 shows the impulse-response paths for selected key
variables. The figure 1 shows that productivity has a positive effect on output
and a negative effect on price, which then encourages more consumption. The
improvement in productivity also results in a fall in labor and an increase in
the real wage. The interest rate falls with the fall in price. The shock leads
to persistent reduction in the domestic interest rate as it is needed in order to
support the transitory expansion in consumption and output.

The impulse responses to a unit shock to productivity show, as expected, that
GDP increases, inflation falls. A less obvious effect is that the consumption of
imported goods falls in spite of the appreciating exchange rate. This is because
domestic goods prices fall sufficiently so as to make imports relatively more
expensive.

Comparing the response of the real variables in the small open economy
with those in a closed economy, we see almost identical paths for output, con-
sumption, investment, real wages and labor, but the adjustment paths for these
variables are higher relative to the closed economy.

Figure 1 shows the impulse-response paths of key macroeconomic indicators
for a small open economy following a once only shock in the productivity index.

The model includes financial frictions. The behavior of the domestic debt reflects the need to engage in open market operations to support the interest rate policy.

In this section we characterize the equilibrium process for a number of variables of the small open economy. Thus we see that stabilization of the output gap and inflation requires a credible threat by the central bank to vary the interest rate sufficiently in response to any deviations of inflation and/or the output gap from the target yet, the very existence of that threat makes its effective application unnecessary.

Under strict domestic inflation targeting, the behavior of real variables in the small open economy corresponds to one we would observe in the absence of nominal frictions. We see that domestic output always increases in response to a positive technology shock.

Under domestic inflation targeting regime the nominal exchange rate moves one-for-one with the terms of trade. In the limiting case of constant world prices, the volatility of the nominal exchange rate under domestic inflation targeting will be proportional to the volatility of the gap between the natural level of domestic output (in turn related to productivity) and world output. A high positive (negative) correlation between domestic productivity and world output will tend to decrease (increase) the volatility of the nominal and real exchange rates.

Under domestic inflation targeting, the CPI level will vary with the (natural) terms of trade and will inherit its statistical properties. If the economy is very open, and if domestic productivity (and hence the natural level of domestic output) is not much synchronized with world output, CPI prices could potentially be highly volatile, even if the domestic price level is constant.

To illustrate how much a central bank could misjudge its interbank rate
policy we investigate the impulse response to a goods productivity shock by not taking into account the existence of financial sector. We try to answer the question: How much could a central bank that ignores banking misled the proper interest rate policy action to stabilize inflation in response to a goods productivity shock?

To answer this question we compare the responses of the interbank rate of interest to a goods productivity shock in the model with and without banking sector (Figure 2). If the central bank was costless, then there would be a single interest rate in the model $R^T$, and the associated real interest rate would move little in response to the highly persistent goods productivity shock. Hence, the output gap would be small and deflation could be averted with a relatively small cut in the interest rate.

Figure 3 and 4 report impulse responses of key endogenous variables to an annualized 1% monetary policy shock. Due to price stickiness, the higher nom-
inal interest rates neutralize much of the impact on consumption.

The fall in demand conditions stimulates a decrease in investment and production of goods and loans, the real marginal cost moves down which result in an inflation drop. A decrease in consumption generates a decrease in the demand of bank deposits that induce a lower demand for loans. The lower demand for loans results in a lower marginal financing cost that helps to contain the drop in consumption. The drop in the marginal financial cost is accompanied by a decline in consumption smaller in the model with banking sector. Bernanke et al (1989) neglect the direct relationship between consumption and the demand for bank deposits through the demand for money. The rise of real interest rates reduces the net present value of tomorrow’s capital holdings, causing banks to cut the amount of loans they are willing to supply.

Figure 5 displays impulse responses to a 1% negative shock that makes the capital less productive in securing and producing loans. The model is well-
suited to analyse the effects of a tightening in credit conditions on the real activity and to give indications on the appropriate response of a central bank following a Taylor-type monetary policy rule with lag interest rate.

The simulation indicates that hours worked in banking rise in response to the decline in effective collateral. This rise in banking labour together with implied changes in consumption and real wages, produces a rise in marginal financial cost, reflected in a fall in the interbank interest rate.

Central bank generally recognize the need to cut the interbank rate in response to widespread financial distress. But given the absence of banking in models of monetary policy, there is no way to judge how much to cut the interbank interest rate. (Goodfriend and McCallum, 2007). Financial distress is accompanied by a fall in production of goods and loans to accommodate downward demand conditions. Realistic interest rate policy produces realistic macroeconomic outcomes: there is a recession with a contraction in hours worked and a deflation. If banks were allowed to hold government securities outright in the model, then arbitrage between $R^{IB}$ and $R^B$ would induce banks to buy government securities in response to the financial distress. Such arbitrage would bring down $R^B$ and $R^T$, cushion the decline in $R^{IB}$, and improve the capacity of the model economy to respond to financial distress. To get the larger interest rate reductions that were observed in the financial crisis, some other demand shock (for example, a consumption preference shock) must be added to the model.

The shocks emanating from the financial sector can propagate abroad very quickly. The financial shock in the United States (2007) propagate to the rest of the world very quickly. The integration of the markets of capital operating by means of the financial flows, losses on credits and effects of valuation and the opening of exchange, were the main vectors of the speed and the strength of this transmission, amplified by the business inside subgroups of countries.

In particular, the small open economies, are more vulnerable in front of these shocks because their degree of openness is often a multiple of that of the large countries, while their capital markets are not very deep. Moreover, a high degree of synchronization can impose limits to the capacity of a country to stabilize its economy, requiring a bigger international coordination of policies.

If the macroeconomic stabilization is strongly recommended, its optimal degree depends on a certain number of factors. The macroeconomic policy should contribute to stabilize the production and the inflation, the strong fluctuations cannot be dreaded only by the households and the firms. Besides, the strong and prolonged periods of recession can reduce the productive capacity of the economy and justify a strong answer of public authorities to weaken the consequences. If the efforts of the stabilization are well established, the required degree is more difficult to determine. For that purpose, it is necessary to take into account the nature of the economy, the types of disturbances and its capacity to resist to the shock, having also an influence on the degree and the nature of the measures of stabilization. The small open economies can face more difficulties to stabilize their economy because of the fact that they are more exposed to the external shocks. They should turn to the political measures which allow
to strengthen the economy.

It is also necessary to take into account the predominance of the economic shocks on the supply or on the demand side. The macroeconomic measures of stabilization have, in general, a more direct action on an aggregate demand shock, but risk to hinder the required adjustment to a long-lasting supply shock.

If they are not carefully conceived, the measures of stabilization can damage the "stability of instruments". The attempts of adjustment of the economy can require strong public measures to neutralize the effects of the past decisions. This problem is important because it may compromise the credibility of the economic policy. The stabilization can put a challenge in the case of the small open economies. The policy of stabilization can face more serious difficulties in the case of the small open economy. A monetary policy which affects the exchange rate can be a powerful instrument of stabilization, but at the price of transfers of resources between the exposed sectors. For example, in New Zealand, the contraction of the monetary policy, motivated especially by anxieties concerning the evolution of the prices of assets, in particular housing assets, activated new entries of capital. Consequently, the long-term interest rates did not move, what weakened the expected effect on the domestic demand. In these conditions the appreciation of the exchange rate penalized, essentially the export sector and weakened the economy before the irruption of the financial crisis. Interventions on the exchange rate can, partially neutralize these effects, but these interventions should lean on an evaluation of misalignments, which are difficult to determine. Consequently, when the monetary policy leads to inconvenient fluctuations in exchange rate, the policy of stabilization requires more important supports of the fiscal policy. However, the efficiency of the fiscal policy is limited, in particular because a part of the effects is exported abroad by means of an increase of the imports. In these conditions the safety margins must be wider than usually to facilitate the stabilization in a small open economy.

An important lesson can be derived from the severity of the current recession, is that the policies will have to show more careful during the phases of recovery and increase safety margins to be able to react to large-scale negative shocks.

5 Conclusion

A growing literature has produced several alternative theoretical models in which financial-market imperfections are put forward as a central element of the transmission mechanism that triggers Sudden Stops and contagion of crises across markets. Dynamic results are based on impulse response functions implied by a linearized version of the model.

The present paper has developed and analyzed a tractable optimizing model of a small open economy with staggered price setting à la Calvo. We have shown that the equilibrium dynamics for the model economy have a canonical representation (in terms of domestic inflation and the output gap) analogous to that of its closed economy counterpart. More precisely, their representations differ only in two respects: first, some coefficients of the equilibrium dynamical
system for the small open economy depend on parameters that are specific to the latter (the degree of openness and the substitutability across goods produced in different countries), and second, the natural level of output and interest rates in the small open economy are generally a function of both domestic and foreign disturbances. In particular, the closed economy is nested in the small open economy model, as a limiting case. Comparing the response of the real variables in the small open economy with those in a closed economy, we see almost identical paths for output, consumption, investment, real wages and labor, but the adjustment paths for these variables are higher relative to the closed economy.

Given the simplicity of the model, the results of the paper hold promise for the usefulness of these types of open economy models as analytical tools.

A further interesting extension would involve the introduction along with sticky-prices, of sticky nominal wages in the small open economy. As pointed out by Erceg, Henderson, and Levine (2000), the simultaneous presence of the two forms of nominal rigidity introduces an additional trade-off that renders strict price inflation targeting policies suboptimal.

References


Gerke, Rafael., Hammermann, Felix and Lewis, Vivien. (2009). More or Less Aggressive? Robust Monetary Policy in a New Keynesian Model with Fi-


