Optimal fiscal and monetary policy action in a closed economy*

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Abstract

We study optimized monetary and fiscal feedback policy rules. The setup is a New Keynesian DSGE model of a closed economy which is solved numerically using common parameter values and fiscal data from the euro area. Our aim is to welfare rank alternative tax-spending policy instruments used for shock stabilization and/or debt consolidation when, at the same time, the monetary authorities can follow a Taylor rule for the nominal interest rate.

Keywords: Feedback policy rules, New Keynesian.

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1 Introduction

Policymakers use their instruments to react to economic conditions. For instance, central banks may respond to inflation, the fiscal authorities to the state of public finances, and both of them to real economic activity. It is nevertheless believed that the use of fiscal policy is more complex and controversial than the use of monetary policy (see e.g. Leeper, 2010). The debate over the use of fiscal policy has been intensified since 2009 when most European governments embarked on the difficult task of reducing their public debts at a time of stagnant or negative growth.\textsuperscript{1} What is the best policy reaction to economic conditions within this environment?

In this paper, we search for the best mix of monetary and fiscal policy actions in a closed economy, when the policy role is twofold: to stabilize the economy against shocks and to improve resource allocation by gradually reducing the public debt burden over time. In order to do so, we welfare rank various fiscal policy instruments used jointly with nominal interest rate policy.

Following most of the related literature (see below), we work with feedback policy rules. In particular, we specify feedback rules for public spending, the tax rate on labor income, the tax rate on capital income and the tax rate on consumption, when these fiscal policy instruments are allowed to respond to a number of macroeconomic variables used as indicators, while, at the same time, monetary policy can be used in a standard Taylor-type fashion. We optimally choose the indicators that the fiscal and monetary authorities react to, as well as the magnitude of feedback policy reaction to those indicators. The welfare criterion is household’s expected discounted lifetime utility. This type of policy is known as "optimized" feedback policy rules (see Schmitt-Grohé and Uribe, 2005, 2007, and many others). This enables us to welfare rank alternative policies in a stochastic setup, without our results - and, in particular, our welfare ranking of alternative policies - being driven by ad hoc differences in feedback policy coefficients, as it happens in most of the related literature on debt consolidation (see below). We work within two policy environments. In the first, used as a benchmark, the authorities just stabilize the economy from shocks. In the second, the fiscal authorities also aim at gradually reducing the output share of public debt over time, which means that now we combine shock stabilization with resource allocation policy.

\textsuperscript{1}Even in normal times, fiscal imbalances can jeopardize the stability of the whole euro area. Hence the arguments for the restrictions of the Stability and Growth Pact. After the global financial crisis of 2007 and the sharp deterioration of public finances (in the euro area, the output share of public debt was around 69% in 2008 and increased to around 95% in 2013), in view of growing concerns about fiscal sustainability, many countries have been forced to initiate substantial fiscal adjustments. However, the resulting spending cuts, and in particular the reliance on tax revenue measures, are believed to cause a significant burden on real activity. See e.g. European Commission (2013) and CEifo (2014).
The setup is a rather standard New Keynesian DSGE model of a closed economy featuring imperfect competition, Calvo-type price fixities and real wage rigidities. The model is solved numerically using common parameter values and fiscal data from the euro area over 1995-2010. To solve the model and, in particular, to solve for welfare-maximizing policy rules, we adopt the methodology of Schmitt-Grohé and Uribe (2004, 2007), in the sense that we take a second-order approximation to both the equilibrium conditions and the welfare criterion. In turn, we compute the welfare-maximizing values of various feedback policy rules and the associated social welfare under various scenarios.

Our main results are as follows. First, in all cases studied, the monetary authorities should aggressively react to price inflation and the fiscal authorities should react to public debt. Also, in all cases studied, interest rate reaction to the output gap should be smaller in magnitude than reaction to inflation (this is the case even if the policy target for output is the so-called natural level of output, which is a rather ambitious target). In other words, price stability should be the key concern of monetary authorities. On the other hand, the degree of fiscal reaction to the output gap (the so-called fiscal activism) relative to reaction to public debt, and hence what should be the key concern of fiscal authorities, depend crucially on the distorting effects of each fiscal instrument and the degree of rigidities in the labor market. In particular, the more distorting a fiscal policy instrument is, the less it should be used for debt consolidation and the more it should be used to support the real economy. This applies in particular to labor taxes all the time and to capital taxes in the medium and long term. Rigidities in the labor market provide further arguments for fiscal activism. All this means that, under optimized rules, the final, or net, change in fiscal policy instruments is determined by the reconciliation of two typically conflicting aims: to reduce public debt and to stimulate the economy. The final, or net, effect is a quantitative matter (see our fourth result below).

Second, when we focus on lifetime utility only, welfare differences between debt consolidation and no debt consolidation look to be small. However, this happens only because short-term effects work in opposite direction from medium- and long-term effects, so that the net, or lifetime, effects are small. In particular, the comparison of outcomes with consolidation to outcomes without consolidation implies that, in most cases, consolidation is costly in the short run and that these costs are not trivial. By contrast, in the medium- and long-term, debt consolidation becomes superior across all cases and this more than offsets its short-term costs, so that eventually lifetime, or net, utility is higher with debt consolidation.\(^2\)

\(^2\)This intertemporal tradeoff also holds in most open economy models (see e.g. Philippopoulos et al., 2013, and the references therein).
Third, in the case of debt consolidation, the choice of the fiscal policy instrument matters for how quickly public debt should be brought down. For instance, in our baseline experiments, public debt reduction from 85%, which is its average value in the recent euro data, to the 60% target level, which is the reference level of the Maastricht Treaty, should be achieved within 5 to 12.5 years depending on how distorting the fiscal instrument is (5 years if we use public spending or consumption taxes, and 12.5 years if we use capital taxes). This pace should be slower if there are labor market rigidities since, in the presence of such rigidities, fiscal policy should be mainly concerned about the real economy. On the other hand, if we use labor taxes, which are a particularly distorting instrument at any time, the pace of public debt reduction should be very slow, following an almost unit root process, and this is irrespectively of the degree of labour market rigidities.

Fourth, the choice of the fiscal policy instrument matters for welfare too. If there are no rigidities in the labor market, the concern for public debt should dominate the concern for output and, in this case, it is better to use public spending along with interest rate policy. Practically, this means that, if there are no labor market rigidities, the best fiscal policy is to cut public spending initially so as to bring public debt down. On the other hand, if there are rigidities in the labor market, the concern for output should dominate the concern for public debt and, in this case, it is better to use income (labor or capital) taxes on the side of fiscal policy. Practically, this means that, if there are labor market rigidities, the best fiscal policy is to cut labor and capital taxes initially so as to stimulate the real economy and only in turn raise them to bring public debt down gradually over time.3

How does our work differ? Although there has been a rich literature on the interaction between fiscal and monetary policy,4 as well as on public debt consolidation,5 there has not been a welfare comparison of the main tax-spending policy instruments in a unified framework of a closed economy, and how this comparison depends on policy goals (shock stabilization only, or shock stabilization plus debt consolidation) as well as on the presence of labor market rigidities. Also, as said above, our results are based on optimized policy rules.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the data, parameter values and the steady state solution. Section 4 explains how we work. The main results are in Sections 5 and 6. Various robustness checks are in Section 7.

3These results should be contrasted to those in an open economy facing sovereign risk premia and a non-zero probability of default, where fiscal policy instruments should be earmarked to debt consolidation almost in all cases (see Philippopoulos et al., 2013).
5See e.g. Coenen et al. (2008), Forni et al. (2010a, 2010b), Bi et al. (2012), Cantore et al. (2012), Cogan et al. (2013), Erceg and Lindé (2013) and Philippopoulos et al. (2013, 2014).
Section 8 closes the paper. Details are in an Appendix.

2 Model

The model is a standard New Keynesian model featuring imperfect competition and Calvo-type nominal rigidities, which is extended to include a relatively rich menu of state-contingent policy rules.

2.1 Households

There are $i = 1, 2, \ldots, N$ identical households. The objective of each $i$ is to maximize expected discounted lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, m_{i,t}, g_t)$$

where $c_{i,t}$ is $i$’s consumption bundle (defined below), $n_{i,t}$ is $i$’s hours of work, $m_{i,t}$ is $i$’s real money balances, $g_t$ is per capita public spending, $0 < \beta < 1$ is the time discount rate, and $E_0$ is the rational expectations operator.

In our numerical solutions, we use a utility function of the form (see also e.g. Gali, 2008):

$$u_{i,t}(c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \frac{n_{i,t}^{1+\eta}}{1+\eta} + \frac{m_{i,t}^{1-\mu}}{1-\mu} + \frac{g_t^{1-\zeta}}{1-\zeta}$$

where $\chi_n, \chi_m, \chi_g, \sigma, \eta, \mu, \zeta$ are preference parameters.

The budget constraint of each household $i$ (written in real terms) is:

$$(1 + \tau_{i,t}^c) c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} = (1 - \tau_{i,t}^k) \left( r_i^k k_{i,t-1} + d_{i,t} \right) + (1 - \tau_{i,t}^n) w_i n_{i,t} + R_{t-1} \frac{B_{i,t}}{P_t} b_{i,t-1} + \frac{B_{i,t-1}}{P_t} m_{i,t-1} - \tau_{i,t}^T$$

where $P_t$ is the general price index and small letters denote real variables, e.g. $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$, $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$, $w_t \equiv \frac{W_t}{P_t}$, $d_{i,t} \equiv \frac{D_{i,t}}{P_t}$, $\tau_{i,t}^k \equiv \frac{T_{i,t}^k}{P_t}$. Here, $x_{i,t}$ is $i$’s real investment at $t$, $B_{i,t}$ is $i$’s end-of-period nominal government bonds, $M_{i,t}$ is $i$’s end-of-period nominal money holdings, $r_i^k$ is the real return to inherited capital $k_{i,t-1}$, $D_{i,t}$ is $i$’s nominal dividends paid by firms, $W_t$ is the nominal wage rate, $R_{t-1} \geq 1$ is the gross nominal return to government bonds between $t-1$ and $t$, $T_{i,t}^l$ is nominal lump-sum taxes/transfers made to each $i$ from the government, and $0 \leq \tau_{i,t}^c, \tau_{i,t}^k, \tau_{i,t}^n < 1$ are respectively tax rates on consumption, capital income and labour income.
The motion of physical capital for each household $i$ is:

$$k_{i,t} = (1 - \delta)k_{i,t-1} + x_{i,t}$$  \hspace{1cm} (4)$$

where $0 < \delta < 1$ is the depreciation rate of capital.

The quantity of variety $h$, produced monopolistically by firm $h$, and consumed by household $i$, is denoted as $c_{i,t}(h)$. Using a Dixit-Stiglitz aggregator, the composite of goods consumed by household $i$ is given by:\footnote{As in e.g. Blanchard and Giavazzi (2003), we work with summations rather than with integrals.}

$$c_{i,t} = \left[ \sum_{h=1}^{N} \lambda[c_{i,t}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$  \hspace{1cm} (5)$$

where $\phi > 0$ is the elasticity of substitution across goods produced and $\sum_{h=1}^{N} \lambda = 1$ are weights (to avoid scale effects, we set $\lambda = 1/N$ in equilibrium). Household $i$’s total consumption expenditure is:

$$P_{t}c_{i,t} = \sum_{h=1}^{N} \lambda P_{t}(h)c_{i,t}(h)$$  \hspace{1cm} (6)$$

where $P_{t}(h)$ is the price of variety $h$.

Each household $h$ acts competitively taking prices and policy variables as given. Details and the solution of household’s problem are in Appendix 1.

### 2.2 Firms

There are $h = 1, 2, \ldots, N$ firms. Each firm $h$ produces a differentiated good of variety $h$ under monopolistic competition facing Calvo-type nominal fixities. The nominal profit of firm $h$ is defined as:

$$D_{t}(h) = P_{t}(h)y_{t}(h) - P_{t}k_{t}^{k}k_{t-1}(h) - W_{t}n_{t}(h)$$  \hspace{1cm} (7)$$

All firms use the same technology represented by the production function:

$$y_{t}(h) = A_{t}[k_{t-1}(h)]^{\alpha}[n_{t}(h)]^{1-\alpha}$$  \hspace{1cm} (8)$$

where $A_{t}$ is an exogenous stochastic TFP process whose motion is defined below.

Profit maximization by firm $h$ is also subject to the demand for its product (see Appendix
which says that demand for firm $h$’s product, $y_t(h)$, comes from households’ consumption and investment, $c_t(h)$ and $x_t(h)$, where $c_t(h) = \sum_{i=1}^{N} c_{i,t}(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_t$ and $x_t(h) = \sum_{i=1}^{N} x_{i,t}(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} x_t$, as well as from the government, $g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} g_t$.

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price. A firm $h$, which is able to reset its price, chooses its price $P^P_{t}(h)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed. This objective is given by:

\[ E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k} (h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P^P_{t}(h) y_{t+k} (h) - \Psi_{t+k} (y_{t+k} (h)) \right\} \]

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm, $y_{t+k} (h) = \left[ \frac{P^P_{t}(h)}{P_{t+k}} \right]^{-\phi} y_{t+k}$ and $\Psi_{t}(\cdot)$ denotes the minimum nominal cost function for producing $y^H_{t}(h)$ at $t$ so that $\Psi_{t}(\cdot)$ is the associated nominal marginal cost.

Details and the solution of firm’s problem are in Appendix 2.

2.3 Government budget constraint

The budget constraint of the consolidated government sector expressed in real terms and aggregate quantities is:

\[ b_t + m_t = R_{t-1} \frac{P_{t-1}}{P_t} b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} + g_t - \tau^c_t c_t - \tau^k_t (\tau^k_t \psi_{t-1} + d_t) - \tau^n_t w_t n_t - \tau^l_t \]

where $b_t$ is the end-of-period total stock of real public debt and $m_t$ is the end-of-period total stock of real money balances. Note that $c_t = \sum_{i=1}^{N} c_{i,t}$, $k_{t-1} = \sum_{i=1}^{N} k_{i,t-1}$, $D_t = \sum_{i=1}^{N} D_{i,t}$, $n_t = \sum_{i=1}^{N} n_{i,t}$, $B_{t-1} = \sum_{i=1}^{N} B_{i,t-1}$ and $T^l_t = \sum_{i=1}^{N} T^l_{i,t}$, and all other variables have been defined above. As above, small letters denote real variables.

In each period, one of the fiscal policy instruments $\tau^c_t$, $\tau^k_t$, $\tau^n_t$, $g_t$, $\tau^l_t$, $b_t$ has to follow residually to satisfy the government budget constraint (see below).
2.4 Decentralized equilibrium (given policy)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) all households maximize utility; (ii) a fraction \((1 - \theta)\) of firms maximize profits by choosing an identical price \(P^*_t\), while the rest, \(\theta\), set their previous period prices; (iii) all constraints, including the government budget constraint, are satisfied; (iv) all markets clear.

To proceed with the solution, we need to define the policy regime. Regarding monetary policy, we assume, as usually, that the nominal interest rate, \(R_t\), is used as a policy instrument, while, regarding fiscal policy, we assume that the residually determined public financing policy instrument is the end-of-period public debt, \(b_t\) (see below for other public financing cases).

Appendix 3 presents the dynamic DE system. It consists of 14 equations in 14 variables \(\{y_t, c_t, n_t, x_t, k_t, m_t, b_t, P_t, P^*_t, \tilde{P}_t, w_t, mc_t, d_t, \tau^k_t\}_{t=0}^\infty\). This is given the independently set policy instruments, \(\{R_t, \tau^k_t, \tau^l_t, g_t, \tau^1_t\}_{t=0}^\infty\), technology \(\{A_t\}_{t=0}^\infty\), and initial conditions for the state variables. All these variables have been defined above, except from \(\tilde{P}_t\) and \(mc_t\), where \(\tilde{P}_t \equiv \left(\sum_{h=1}^{N} [P_t(h)]^{-\delta}\right)^{-\frac{1}{\delta}}\) and \(mc_t\) is the firm’s marginal cost as defined in Appendix 2.

Before we specify the processes of policy instruments and exogenous variables in the next two subsections, and by following the related literature, we transform the above equilibrium conditions. In particular, we express price levels in inflation rates, rewrite the firm’s optimality condition in recursive form and introduce a new equation that helps us to compute household’s expected discounted lifetime utility. Appendix 4 presents details and the resulting transformed DE system consisting of 17 equations in 17 variables.

2.5 Policy rules

Following the related literature, we focus on simple rules meaning that the monetary and fiscal authorities react to a small number of macroeconomic indicators. In particular, we allow the nominal interest rate, \(R_t\), to follow a standard Taylor rule, meaning that it can react to inflation and output as deviations from a policy target, while we allow the distorting fiscal policy instruments, namely, government spending as a share of output, \(s_t^g \equiv \frac{g_t}{y_t}\), and the tax rates on consumption, capital income and labor income, \(\tau_t^c, \tau_t^k, \tau_t^l\), to react to public debt and output, again as deviations from a policy target. The target values are defined below.

In particular, following e.g. Schmitt-Grohé and Uribe (2007), we use policy rules of the functional form:
\[
\log \left( \frac{R_t}{R} \right) = \phi_\pi \log \left( \frac{\Pi_t}{\Pi} \right) + \phi_y \log \left( \frac{yt}{y} \right)
\]
(12)

\[s_i^g - s^g = -\gamma_i^g (l_{t-1} - l) - \gamma_y^g (yt - y)
\]
(13)

\[\tau_i^c - \tau^c = \gamma_i^c (l_{t-1} - l) + \gamma_y^c (yt - y)
\]
(14)

\[\tau_i^k - \tau^k = \gamma_i^k (l_{t-1} - l) + \gamma_y^k (yt - y)
\]
(15)

\[\tau_i^n - \tau^n = \gamma_i^n (l_{t-1} - l) + \gamma_y^n (yt - y)
\]
(16)

where variables without time subscripts denote target values, and \( \phi_\pi, \phi_y, \gamma_i^q, \gamma_y^q \geq 0 \), for \( q \equiv (g, c, k, n) \), are feedback policy coefficients, and where:

\[l_{t-1} \equiv \frac{R_{t-1}b_{t-1}}{y_{t-1}}
\]
(17)

denotes the beginning-of-period public debt burden as share of GDP.

### 2.6 Exogenous stochastic variables

We now define the processes of exogenous stochastic variables. For notational simplicity, we include shocks to TFP only (as we report below, the main results do not change if we add other shocks). In particular, we assume that the TFP follows an AR(1) process:

\[
\log A_t = (1 - \rho^A) \log (A) + \rho^A \log A_{t-1} + \varepsilon^A_t
\]
(18)

where \( 0 \leq \rho^A \leq 1 \) is a persistence parameter and \( \varepsilon^A_t \sim N(0, \sigma^2_A) \).

### 2.7 Final equilibrium system (given feedback policy coefficients)

The full equilibrium system consists of the 17 equations of the transformed DE presented at the end of Appendix 4, and the 5 feedback policy rules as well as the definition of \( l_t \) presented in subsection 2.5. We thus end up with 23 equations in 23 variables \( \{yt, ct, nt, xt, kt, mt, bt, \Pi_t, \Theta_t, \Delta_t, wt, mc_t, dt, r_k^t, z_1^t, z_2^t, V_t, R_t, s_i^g, \tau_i^c, \tau_i^k, \tau_i^n, l_{t=0} \} \). Among them, there are 17 non-predetermined or jump variables, \( \{yt, ct, nt, xt, \Pi_t, \Theta_t, wt, mc_t, dt, r_k^t, z_1^t, z_2^t, V_t, s_i^g, \tau_i^c, \tau_i^k, \tau_i^n \}_{t=0} \), and 6 predetermined or state variables, \( \{R_t, kt, bt, mt, \Delta_t, l_t \}_{t=0} \). This is given
the process of TFP in (18), initial conditions for the state variables and the values of feedback policy coefficients in (12)-(16) which are specified below.

To solve this non-linear difference equation system, we will take a second-order approximation around its steady state solution. We therefore first solve for the steady state in the next section. In turn, we will study the optimal choice of feedback policy coefficients and the resulting transition dynamics.

3 Data, parameterization and steady state

This section solves numerically for the long run of the above economy by using conventional parameter values and data from the euro zone. At steady state, the gross inflation rate is set at one (see below for other cases). Notice that, since policy instruments react to deviations of macroeconomic indicators from their long-run values, feedback policy coefficients do not play any role in the steady state solution.

3.1 Data and parameterization

The fiscal data are from OECD Economic Outlook no. 89. The time unit is meant to be a quarter. Our baseline parameter values are summarized in Table 1.
### Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.33</td>
<td>share of capital</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9926</td>
<td>time preference rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>3.42</td>
<td>parameter related to money demand elasticity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.021</td>
<td>capital depreciation rate (quarterly)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>6</td>
<td>price elasticity of demand</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>inverse of Frisch labour supply elasticity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1</td>
<td>elasticity of public consumption in utility</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$2/3$</td>
<td>price rigidity parameter</td>
</tr>
<tr>
<td>$\chi_m$</td>
<td>0.05</td>
<td>preference parameter related to real money balances</td>
</tr>
<tr>
<td>$\chi_n$</td>
<td>6</td>
<td>preference parameter related to work effort</td>
</tr>
<tr>
<td>$\chi_g$</td>
<td>0.1</td>
<td>preference parameter related to public spending</td>
</tr>
<tr>
<td>$\rho^A$</td>
<td>0.8</td>
<td>serial correlation of TFP shock</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.017</td>
<td>standard deviation of innovation to TFP shock</td>
</tr>
<tr>
<td>$R$</td>
<td>1.0075</td>
<td>long-run nominal interest rate</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>0.19</td>
<td>consumption tax rate</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>0.28</td>
<td>capital tax rate</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.38</td>
<td>labour tax rate</td>
</tr>
<tr>
<td>$s^g$</td>
<td>0.23</td>
<td>government spending as share of output</td>
</tr>
<tr>
<td>$-s^l$</td>
<td>0.2</td>
<td>government transfers as share of output</td>
</tr>
</tbody>
</table>

Using the Euler equation for bonds, the value of the time preference rate, $\beta$, follows so as to be consistent with the average value of the real interest rate in the data, 0.0075 (or 0.03 annually). The real money balances elasticity, $\mu$, is taken from Pappa and Neiss (2005), who estimate this value using UK data; this implies an interest-rate semi elasticity of money demand equal to -0.29 which is a common value in this literature. The elasticity of intertemporal substitution, $\sigma$, the inverse of Frisch labour elasticity, $\eta$, and the price elasticity of demand, $\phi$, are set as in Andrès and Doménech (2006) and Galí (2008) in related studies for the European economy. Regarding the preference parameters in the utility function, $\chi_m$ is chosen so as to obtain a value of real money balances as share of output equal to 1.97 (0.5) quarterly (annually), $\chi_n$ is chosen so as to obtain steady-state labour hours equal to 0.28, while $\chi_g$ is arbitrarily set at 0.1 which is a common valuation of public goods in related utility functions. Other parameters, like $\theta$ measuring Calvo-type nominal fixities, are also set as in related studies of
the euro area (see e.g. Galí et al., 2001). As reported below, our results are robust to changes in these parameter values.

Concerning the exogenous stochastic variables, we start by setting $\rho^A = 0.8$ and $\sigma_A = 0.017$ for the persistence parameter and the standard deviation respectively of TFP in equation (18) (the value of $\rho^A$ is similar to that in Andrès and Domenéch, 2006, while the value of $\sigma_A$ is close to that in Bi, 2010, and Bi and Kumhof, 2009).

The long-run values of the exogenous policy instruments, $\tau^c_t$, $\tau^k_t$, $\tau^n_t$, $s^q_t$, $s^l_t$, $b_t$, are either set at their data averages, or are calibrated to deliver data-consistent long-run values for the endogenous variables. In particular, $\tau^e$, $\tau^k$, $\tau^n$ are the averages of the effective tax rates in the data. Lump-sum taxes, $s^l$, follow residually in the long run, so as to get a value of 0.43 for total public spending as share of output, $-s^l + s^g$, when at the same time the public debt-to-output ratio is set at 3.4 quarterly (or 0.85 annually) as in the average data over 2008-2011.

3.2 Steady state solution or the "status quo"

Table 2 reports the steady state solution of the model economy when we use the parameter values and the policy instruments in Table 1. The solution makes sense and the resulting great ratios are close to their values in the actual data (recall that, since the time unit is meant to be a quarter, stock variables - like public debt - need to be divided by 4 to give annual values). This is what we call the "status quo". In what follows, we will depart from this status quo solution to study various policy experiments.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Solution</th>
<th>Variables</th>
<th>Solution</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.7381</td>
<td>$d$</td>
<td>0.1230</td>
<td>-</td>
</tr>
<tr>
<td>$c$</td>
<td>0.4594</td>
<td>$r^k$</td>
<td>0.0395</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>0.2784</td>
<td>$z^1$</td>
<td>2.1820</td>
<td>-</td>
</tr>
<tr>
<td>$x$</td>
<td>0.1089</td>
<td>$z^2$</td>
<td>1.8183</td>
<td>-</td>
</tr>
<tr>
<td>$k$</td>
<td>5.1877</td>
<td>$u$</td>
<td>0.8040</td>
<td>-</td>
</tr>
<tr>
<td>$m$</td>
<td>1.4651</td>
<td>$\frac{c}{y}$</td>
<td>0.62</td>
<td>0.57</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>1</td>
<td>$\frac{b}{y}$</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>1</td>
<td>$\frac{x}{y}$</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>1</td>
<td>$\frac{m}{y}$</td>
<td>1.97</td>
<td>-</td>
</tr>
<tr>
<td>$w$</td>
<td>1.4729</td>
<td>$\frac{k}{y}$</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>$mc$</td>
<td>0.8333</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
4 How we model policy

In this section, we explain the policy experiments we focus on (subsection 4.1), the motivation for debt consolidation (subsection 4.2), how we model debt consolidation (subsection 4.3) and how we compute optimized feedback policy rules (subsection 4.4). Recall that, along the transition path, nominal rigidities imply that money is not neutral so that interest rate policy matter to the real economy. Also, recall that, along the transition path, different counter-cyclical fiscal policy rules can have different implications.

4.1 Types of policy action

We will study two environments regarding policy action. In the first, used as a benchmark, the role of policy is only to stabilize the economy against temporary shocks. In particular, we assume that the economy is hit by an adverse temporary TFP shock, as defined in equation (18) above, which produces a contraction in output and a rise in the public debt to output ratio. Then, the policy questions are which policy instrument to use, and how strong the reaction of policy instruments to deviations from targets should be, when these targets are given by the status quo long-run solution. Technically speaking, in this case, we depart from, and end up, at the same steady state, which is the status quo in subsection 3.2 above, while transition dynamics are driven by temporary shocks only.

The second environment is richer. Now the role of policy is twofold: to stabilize the economy against the same TFP shock as above and, at the same time, to improve resource allocation by gradually reducing the public debt ratio over time. The policy questions are as above except that now the policy targets are given by the long-run solution of the reformed economy. Technically speaking, in this case, we depart from the status quo solution, but we end up at a new reformed long-run with lower public debt. Thus, now there are two sources of transition dynamics: temporary shocks and the difference between the initial and the new reformed steady state.

4.2 Is public debt bad?

Before we study the implications of debt consolidation, it is natural to ask "Why is public debt bad?". Although it is widely recognized that we lack a theory of the optimal level of public debt, it is also well documented that a "high" level of public debt hurts the macroeconomy (see e.g. Wren-Lewis, 2010, for a review of the literature). Since the study of such issues is beyond the scope of the current paper, here we just report that, in our DSGE New Keynesian model, a public debt lower than in the recent average data (85% of GDP) is beneficial to the economy.
and this happens through two channels. First, focusing on steady state, a lower public debt creates fiscal space and this can be used, for instance, to cut distorting taxes (see subsection 4.3 below for public financing details). Second, focusing on the transition, our simulations show that, if we arbitrarily assume that the economy is subject to an initial debt shock (see also Cantore et al., 2012), this leads to a fall in output (this happens irrespectively of the fiscal instrument used to react to the public debt gap) and a fall in private investment (this happens when we use income taxes to react to the public debt gap). Results and details are available upon request from the authors.

This can motivate the study of debt consolidation policies below.

4.3 How we model debt consolidation

We assume that the government reduces the share of public debt from 85% (which is its average value in the data over the sample period and is also the status quo solution) to 60%. We choose the target value of 60% simply because it has been the reference rate of the Maastricht Treaty (we report however that our main results are not sensitive to the value of the debt target assumed). Obviously, debt reductions have to be accommodated by adjustments in the tax-spending policy instruments, which, in our model, are the output share of public spending, and the tax rates on capital income, labour income and consumption.

It is widely recognized that the implications of debt consolidation depend heavily on the public financing policy instrument used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see e.g. Leeper et al., 2010, and Leeper, 2010). Therefore, to understand the logic of our results, we will use one fiscal instrument at a time. This means that, along the early costly phase, we allow one of the fiscal policy instruments to react to public debt imbalances, so as to stabilize debt around its new target value of 0.6 and, at the same time, it is the same fiscal policy instrument that adjusts residually in the long-run to close the government budget. Thus, we assume that the same policy instrument bears the short-term cost of, and reaps the medium- and long-term benefit from, debt consolidation. In our experiments, the policy rules for these instruments are as in subsection 2.5 above except that now the targeted values are those of the reformed long-run equilibrium. All other fiscal policy instruments, except the one used for shock stabilization and debt consolidation, remain unchanged and equal to their pre-reform status quo values.

In particular, we work as follows. We first solve and compare the steady state equilibria with and without debt consolidation. In turn, setting, as initial conditions for the state variables, their long-run values from the solution of the economy without debt consolidation (this is the
status quo in subsection 3.2), we compute the equilibrium transition path of each reformed economy under optimized policy rules and in turn compute the associated conditional expected discounted lifetime utility of the household. This is for each method of public financing used. Thus, the feedback policy coefficients along the transition path are chosen optimally. This is further explained in the next subsection.

4.4 How we compute optimized feedback policy rules

Irrespectively of the policy experiments studied, to make the comparison of different policies meaningful, we compute optimized policy rules, so that results do not depend on ad hoc differences in feedback policy coefficients across different policy rules. The welfare criterion is household’s expected discounted lifetime utility, as defined in equation (67) in the Appendix.

We work in two steps. In the first, preliminary, step, we search for the ranges of feedback policy coefficients, as defined in equations (12-16) above, which allow us to get a locally determinate equilibrium (this is what Schmitt-Grohé and Uribe, 2007, call implementable rules). If necessary, these ranges will be further restricted in order to give economically meaningful solutions for the policy instruments; in particular, to give non-negative nominal interest rates, as well as tax rates and public spending ratios between zero and one (see e.g. subsection 5.2 below for numerical results).\footnote{Thus, we implement the zero lower bound (ZLB) for the nominal interest rate simply by restricting the feedback policy coefficients in the Taylor rule, (12). In the same way, since we study optimized fiscal action which may result in large deviations of fiscal instruments from their data averages, we also need to restrict the feedback policy coefficients in the fiscal policy rules, (13)-(16). Keep in mind that these restrictions (on monetary and fiscal feedback policy coefficients) may change when the specification of the model changes; we will thus report ranges in each case studied. The way we work is practically the same as in e.g. Schmitt-Grohé and Uribe (2004), we take a second-order approximation to both the equilibrium conditions and the welfare criterion. As is well known, this is consistent with risk-averse behavior on the part of economic agents and can also help us to avoid possible spurious welfare results that occur when linearization is used.} In this search for determinacy and well-defined policy values, we experiment with one, or more, policy instruments and one, or more, operating targets at a time.

In the second step, within the ranges found above, we compute the welfare-maximizing values of feedback policy coefficients (this is what Schmitt-Grohé and Uribe, 2005 and 2007, call optimized policy rules). The welfare criterion is to maximize conditional welfare, \(E_0 V_0\), as defined in equation (67), where conditionality refers to the initial conditions chosen; the latter are given by the status quo long-run solution. To this end, following e.g. Schmitt-Grohé and Uribe (2005 and 2007), we take a second-order approximation to both the equilibrium conditions and the welfare criterion. As is well known, this is consistent with risk-averse behavior on the part of economic agents and can also help us to avoid possible spurious welfare results that
may arise when one takes a second-order approximation to the welfare criterion combined with a first-order approximation to the equilibrium conditions (see e.g. Gali, 2008, Malley et al., 2009, and Benigno and Woodford, 2012).

In other words, we first compute a second-order accurate approximation of conditional welfare, and the associated decentralized equilibrium, as functions of feedback policy coefficients by using the perturbation method of Schmitt-Grohé and Uribe (2004) and, in turn, we use a matlab function (fminsearch.m or fminsearchbnd.m) to compute the values of the feedback policy coefficients that maximize the second-order accurate approximation of conditional welfare (our matlab routines are available upon request). In this exercise, as said above, the feedback policy coefficients are restricted to be within some prespecified ranges delivering determinacy as well as meaningful values for policy instruments. We work in this way both without, and with, debt consolidation.

5 Main results

This section presents numerical solutions. We start by presenting the steady state solution of the reformed economy with debt consolidation.

5.1 Steady state utility and output with debt consolidation

The new reformed steady state with debt consolidation is as defined in subsection 4.3 above. In other words, thanks to the fiscal space created by debt reduction, public spending can rise, or a tax rate can be reduced, residually.

Table 3 reports steady-state utility and output under alternative public financing cases. For instance, in the first row of Table 3, the assumption is that it is public spending that takes advantage of debt reduction, in the sense that, once the debt burden has been reduced, public spending can increase relative to its value in the status quo solution. In the other rows, the fiscal space is used to finance cuts in one of the three tax rates. Table 3 reveals that the highest utility and output are achieved when the fiscal space is used to finance a cut in capital tax rates (this is further discussed below).
Table 3: Steady-state utility and output in the reformed economy

<table>
<thead>
<tr>
<th>instrument</th>
<th>steady-state utility</th>
<th>steady-state output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g$</td>
<td>0.799820</td>
<td>0.7418</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.807246</td>
<td>0.7418</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.812269</td>
<td>0.7479</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.810851</td>
<td>0.7460</td>
</tr>
</tbody>
</table>

5.2 Ranges of feedback policy coefficients

Moving to transition, we first check for local determinacy. As is well known, the latter may depend crucially on the values of feedback policy coefficients. We report that (monetary and fiscal) policy guarantees determinacy when the nominal interest rate reacts aggressively to inflation with $\phi_\pi > 1.1$, that is, when the Taylor principle is satisfied, and, at the same time, the fiscal policy instruments, $s^g_t$, $\tau^c_t$, $\tau^k_t$, $\tau^n_t$ react to public liabilities above a critical minimum value, $\gamma^g_t > \gamma^c_t > 0$, where critical minimum values differ across different policy instruments (in particular, $\gamma^g_t = 0.0069$, $\gamma^c_t = 0.0345$, $\gamma^k_t = 0.017$ and $\gamma^n_t = 0.0138$). By contrast, the values of $\phi_y$ and $\gamma^g_y$, measuring respectively the reaction of interest rate policy and fiscal policy to the output gap, are not found to be critical to determinacy.

Nevertheless, as said in subsection 4.4 above, the feedback fiscal policy coefficients on public debt, $\gamma^q_t$, where $q \equiv (g, c, k, n)$, may need to be further restricted in order to get meaningful solutions for the fiscal instruments used, i.e. in order to get $0 \leq s^g_t, \tau^c_t, \tau^k_t, \tau^n_t < 1$. In particular, our computations imply that we need to work within the ranges $\gamma^g_t \in (0.017, 0.15)$ for the capital tax rate and $\gamma^c_t \in (0.0345, 0.2)$ for the consumption tax rate, which are narrower than those required for determinacy only. This makes sense. When debt consolidation is among the policy aims, the fiscal authorities may find it optimal to increase tax rates, and/or reduce public debt.

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8 Actually, we can distinguish two regions of determinacy. In addition to the one discussed above, there is another region in which fiscal policy does not react to public liabilities, i.e. $\gamma^q_t = 0$ for all fiscal instruments, while monetary policy reacts to inflation mildly with $\phi_\pi \leq 1.1$. This region is welfare inferior to the region discussed above. It also contains some sub-areas where determinacy breaks down. Several other papers have distinguished between the same two areas of determinacy (e.g. Leeper, 1991, and Schmitt-Grohé and Uribe, 2007).

9 On the other hand, if we had included a tax on income from government bonds, we would have had to assume a narrower, more restrictive, range of feedback monetary policy reaction to inflation. For instance, if the tax rate is 0.2, monetary policy can guarantee determinacy when the nominal interest rate reacts to inflation with $\phi_\pi > 1.29$ when fiscal policy uses government spending or consumption taxes to react to debt, with $\phi_\pi > 1.43$ when fiscal policy uses capital taxes to react to debt and with $\phi_\pi > 1.57$ when fiscal policy uses labour taxes to react to debt. As the tax rate on income from government bonds increases, the critical minimum value of $\phi_\pi$ required for determinacy also increases (results are available upon request). This is similar qualitatively to the results in Edge and Rudd (2007).
spending, beyond meaningful or historical ranges. Our simulations imply that this applies in particular to the capital tax rate, \( \tau^k_t \), which, if it is left free, it can easily rise above 100% in the short run due to the high value of \( \gamma^k_t \) chosen (this is consistent with the Ramsey-Chamley result that, since capital is inelastic in the very short, the fiscal authorities may find it optimal to confiscate it). To avoid such problems, we restrict ourselves within the above ranges for \( \gamma^k_t \) and \( \gamma^c_t \). Similarly, since in some experiments monetary policy finds it optimal to increase the feedback policy coefficient on inflation, \( \phi^\pi \), to very high values (see also Schmitt-Grohé and Uribe, 2007), we restrict \( \phi^\pi \) within the range \( 1.1 < \phi^\pi \leq 3 \). Thanks to this restriction, the resulting equilibrium nominal interest rate is above the zero lower bound in all solutions reported below and this happens irrespectively of the fiscal policy instrument used jointly with interest rate policy. It is worth reporting that our main results regarding the welfare ranking of policy instruments do not depend on those restrictions (results are available upon request). Also, note that such practice is usual both in the policy literature (see e.g. Cantore et al., 2012), as well as in the theoretical literature on optimal taxation (see e.g. Chamley, 1986).

Thus, the general message is that monetary and fiscal policy need to interact with each other in a specific way for policy to guarantee determinacy or, as Leeper (2010) puts it, there is a "dirty little secret": for monetary policy to control inflation, fiscal policy must behave in a particular manner.

### 5.3 Optimized policy rules and welfare with debt consolidation

Within the above ranges, we can now compute optimized policy rules. Results for the case with debt consolidation are reported in Table 4. The first column lists the pair of policy instruments used (one monetary and one fiscal), the second column reports the optimal reaction of the interest rate to inflation and output, and the third column reports the optimal reaction of each fiscal policy instrument to debt and output. Expected discounted lifetime utility, \( E_0V_0 \), is reported in the last column.

There are three messages from Table 4. First, regarding monetary policy, the interest rate should react aggressively to inflation, while, monetary reaction to the output gap is negligible. Second, regarding fiscal policy, when we use public spending, or consumption taxes, or capital taxes, the fiscal reaction to the output gap is smaller in magnitude than the fiscal reaction to debt. This implies that public spending should fall, while consumption taxes and capital taxes should rise, to address the public debt problem. By contrast, when we use labor taxes, the reaction to output is clearly stronger than the reaction to debt. This implies that the labor tax rate should be reduced at impact to help the real economy. In other words, a particularly
distorting policy instrument, like labor taxes, should be used to address output imbalances rather than to reduce the debt burden. All this is confirmed by impulse response functions shown below. Third, when we rank policy instruments according to expected discounted lifetime utility, $E_0V_0$, the best possible mix is $R_t$ and $s_t^g$. Notice, however, that to the extent that the feedback policy coefficients are chosen optimally, welfare differences, at least in terms of lifetime utility, across different policy mixes are very small.\footnote{When we express welfare differences using consumption equivalents, as in e.g. Lucas (1990), the results are the same. In particular, in Table 4, when the reference regime is $R_t$ and $s_t^g$, the welfare losses expressed in consumption equivalents are $0.0012$ when we use consumption taxes, $0.0013$ when we use capital taxes and $0.0035$ when we use labour taxes.} Keep in mind however that this welfare ranking is in terms of lifetime utility only; shorter time horizons may imply different things (see below).

### Table 4: Optimal monetary reaction to inflation and output and optimal fiscal reaction to debt and output

<table>
<thead>
<tr>
<th>Policy instruments</th>
<th>Optimal interest-rate reaction to inflation and output</th>
<th>Optimal fiscal reaction to debt and output</th>
<th>Lifetime utility $E_0V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t$ $s_t^g$</td>
<td>$\phi_\pi = 1.1$</td>
<td>$\gamma_l^g = 0.1927$</td>
<td>109.2534</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0011$</td>
<td>$\gamma_y^g = 0.1134$</td>
<td></td>
</tr>
<tr>
<td>$R_t$ $\tau_t^c$</td>
<td>$\phi_\pi = 2.7983$</td>
<td>$\gamma_l^c = 0.1943$</td>
<td>109.0918</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0098$</td>
<td>$\gamma_y^c = 0.0036$</td>
<td></td>
</tr>
<tr>
<td>$R_t$ $\tau_t^k$</td>
<td>$\phi_\pi = 2.9323$</td>
<td>$\gamma_l^k = 0.1499$</td>
<td>109.0754</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0037$</td>
<td>$\gamma_y^k = 0.0007$</td>
<td></td>
</tr>
<tr>
<td>$R_t$ $\tau_t^n$</td>
<td>$\phi_\pi = 2.983$</td>
<td>$\gamma_l^n = 0.0129$</td>
<td>108.7748</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0728$</td>
<td>$\gamma_y^n = 0.1138$</td>
<td></td>
</tr>
</tbody>
</table>

The optimized policy rules shape the motion of public debt over time. Figure 1 shows the resulting path of the public debt-to-GDP ratio. The duration of the debt consolidation phase, or equivalently the speed of debt reduction, depend heavily on the fiscal policy instrument used. In particular, more than 95% of debt consolidation should be achieved within 20 time periods (5 years), if we use the public spending ratio, $s_t^g$; within 20 time periods (5 years), if we use the consumption tax rate, $\tau_t^c$; and within 50 time periods (12.5 years), if we use the capital tax rate, $\tau_t^k$. On the other hand, if we use the labor tax rate, $\tau_t^n$, the debt-to-output ratio...
should converge very slowly to its 60% target looking like a unit-root process. The general idea is that the more distorting the policy instrument is, the slower the debt adjustment should be.

Figure 1: The path of public debt as share of output

5.4 Welfare over various time horizons with and without debt consolidation

We now study what happens to welfare over various time horizons. This is important because, for several (political-economy) reasons, economic agents’ behavior can be short sighted. Setting the feedback policy coefficients as in Table 4 above, the expected discounted utility at various time horizons is reported in Table 5. In the same Table, we also report results without debt consolidation other things equal (these are the numbers in parentheses). As said, without debt consolidation, we again compute optimized policy rules but now the economy starts from, and returns to, the status quo solution.

There are two messages from Table 5. First, when we focus on lifetime utility only, welfare differences between debt consolidation and no debt consolidation look to be small. But, a more careful inspection of the results in Table 5 reveals that this happens only because short-term effects work in opposite direction from medium- and long-term effects, so that the net, or lifetime, effects are small. In particular, the comparison of outcomes with consolidation to outcomes without consolidation implies that, in most cases, consolidation is costly in the short

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11When expressed in consumption equivalents, the welfare gains of debt consolidation are 0.0036 when we use government spending, 0.0025 when we use consumption taxes, 0.0024 when we use capital taxes and 0.00024 when we use labor taxes. In each case, the reference regime is the case without debt consolidation.
run and that these costs are not trivial. By contrast, after the first 60 periods or 15 years, debt consolidation becomes superior across all cases and this more than offsets its short-term costs, so that eventually lifetime, or net, utility becomes higher with debt consolidation. Second, without debt consolidation, and to the extent that feedback policy coefficients are optimally chosen, the choice of the fiscal policy instrument used is trivial. Welfare differences appear after the second decimal point across all time horizons (these are the numbers in parentheses). On the other hand, with debt consolidation, the choice of the policy instrument matters more (these are the numbers without parentheses). Now, except from the case in which we care only about the short run, the best policy mix is \( R_t - \sigma_t^g \). In the short run, by contrast, the best mix is \( R_t - \tau_t^k \) for the reasons explained above.

### Table 5: Welfare over different time horizons with, and without, debt consolidation

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>60 periods</th>
<th>Lifetime utility ( E_0V_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_t ) ( s_t^g )</td>
<td>1.5655</td>
<td>3.1990</td>
<td>7.9406</td>
<td>39.7726</td>
<td>109.2534</td>
</tr>
<tr>
<td>(1.6029)</td>
<td>(3.1822)</td>
<td>(7.7822)</td>
<td>(39.0939)</td>
<td>(108.7662)</td>
<td></td>
</tr>
<tr>
<td>( R_t ) ( \tau_t^k )</td>
<td>1.4188</td>
<td>2.8775</td>
<td>7.3441</td>
<td>39.1210</td>
<td>109.0918</td>
</tr>
<tr>
<td>(1.6024)</td>
<td>(3.1818)</td>
<td>(7.7809)</td>
<td>(39.0883)</td>
<td>(108.749)</td>
<td></td>
</tr>
<tr>
<td>( R_t ) ( \tau_t^n )</td>
<td>1.6667</td>
<td>3.2929</td>
<td>7.9569</td>
<td>39.8577</td>
<td>109.0754</td>
</tr>
<tr>
<td>(1.6030)</td>
<td>(3.1825)</td>
<td>(7.7827)</td>
<td>(39.0870)</td>
<td>(108.7452)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Results without debt consolidation in parentheses. Periods denote quarters.

6 Adding labor market rigidities

The previous analysis has assumed away any rigidities in the labor market. This is questionable since it is widely believed that labor market rigidities are a key feature of the European economy (see e.g. Blanchard, 2004). In this subsection, we extend the model to allow for such rigidities.

To avoid further complicating the model, but to also help it replicate the stylized facts in
Europe regarding inertia in wage adjustment, we follow the setup employed in Blanchard and Galí (2007), Malley et al. (2009) and many others. In particular, we assume that the nominal wage rate at time $t$ is a weighted average of the nominal wage in the previous period, $t - 1$, and the nominal wage that would arise in case the labor market worked perfectly. Expressing variables in real term, this implies that the real wage at time $t$ follows:

$$w_t \equiv \left( w_{t-1} \frac{P_{t-1}}{P_t} \right)^\gamma (MRS_t)^{1-\gamma}$$  \hfill (19)

where $0 \leq \gamma \leq 1$ measures the degree of wage sluggishness and $MRS_t \equiv \frac{\chi_n (1+\tau_t) n_t^q}{(1-\tau_t) c_{t,t}}$ (see equation (30) in Appendix 1). The idea behind this partial adjustment model is that real wages respond only sluggishly to current conditions in the labor market. As pointed out by Blanchard and Galí (2007), "this is a parsimonious way of modeling the slow adjustment of wages to labor market conditions, as found in a variety of models of real wage rigidities, without taking a stand on what is the "right" model". In other words, although ad hoc, this specification can be consistent with a number of possible sources of rigidity in European labor markets, e.g. institutional, legal and socio-political rigidities and safety nets, etc. Finally, notice that this modeling has the following advantages: (i) if $\gamma = 0$, the standard neoclassical model obtains; (ii) in the steady-state, i.e. when $w_t = w_{t-1} = w$, it follows that again $w = MRS$. If $\gamma = 1$, we have full persistence in wage setting. In our numerical solutions below, we set $\gamma = 0.9$, which is close to the value used by Malley et al. (2009) for a number of European economies.

### 6.1 Results with labor market rigidities

The model is resolved using the new specification in the labor market. Regarding ranges of feedback policy coefficients guaranteeing determinacy and meaningful solutions for the policy instruments, we report that these ranges practically remain as in subsection 5.2 above, except that now, when we use capital and labor taxes on the side of fiscal policy, the minimum boundaries of interest rate reaction to inflation are related to the minimum boundaries of fiscal policy reaction to public debt (as we allow the fiscal reaction to public debt to rise, the monetary reaction to inflation needs also to rise to guarantee determinacy).

Within these ranges, the new results are reported in Tables 6 and 7, which are like Tables 4 and 5 respectively. There are three messages. First, optimal fiscal reaction to the output gap is now much bigger than in the case without labor market rigidities. That is, now the fiscal authorities find it optimal to also react to the recession so that debt reduction is not their only concern. Actually, in the case of public spending, capital taxes and labor taxes, the
coefficient on the output gap is bigger than the coefficient on the debt gap; this means that now public spending should rise, and capital and labor taxes should fall, in order to stimulate the economy, and only in turn be used to address the public debt problem.

Table 6: Optimal monetary reaction to inflation and output and optimal fiscal reaction to debt and output

<table>
<thead>
<tr>
<th>Policy instruments</th>
<th>Optimal interest-rate reaction to inflation and output</th>
<th>Optimal fiscal reaction to debt and output</th>
<th>Lifetime utility $E_0V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t \ s^g_t$</td>
<td>$\phi_\pi = 2.9988$</td>
<td>$\gamma^g_t = 0.1368$</td>
<td>109.067</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0001$</td>
<td>$\gamma^g_y = 0.3907$</td>
<td></td>
</tr>
<tr>
<td>$R_t \ \bar{\tau}^i_t$</td>
<td>$\phi_\pi = 2.9988$</td>
<td>$\gamma^c_t = 0.2598$</td>
<td>109.0807</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0186$</td>
<td>$\gamma^c_y = 0.1514$</td>
<td></td>
</tr>
<tr>
<td>$R_t \ \bar{\tau}^k_t$</td>
<td>$\phi_\pi = 1.1$</td>
<td>$\gamma^k_t = 0.0119$</td>
<td>109.174</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0306$</td>
<td>$\gamma^k_y = 0.424$</td>
<td></td>
</tr>
<tr>
<td>$R_t \ \bar{\tau}^n_t$</td>
<td>$\phi_\pi = 1.102$</td>
<td>$\gamma^n_t = 0.0143$</td>
<td>109.1488</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0456$</td>
<td>$\gamma^n_y = 0.8083$</td>
<td></td>
</tr>
</tbody>
</table>

All this is confirmed by impulse response functions shown below. Second, when there are rigidities in the labor market, it is better to use income tax rates along with the interest rate. That is, under debt consolidation, the mixes $R_t - \bar{\tau}^k_t$ and $R_t - \bar{\tau}^n_t$ score better than $R_t - s^g_t$ and this is the case both in the very short run and in the long run. Intuitively, this follows naturally from the first result above: since the emphasis should be now given to the real economy, it is better to use fiscal instruments, like income taxes, which can more more effectively stimulate the economy and which are relatively close to the heart of the labor market imperfection. Third, monetary reaction to the output gap remains small as in Table 4.
Table 7: Welfare at different time horizons with, and without, debt consolidation

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>60 periods</th>
<th>Lifetime utility $E_0V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t \ s_t^g$</td>
<td>1.5659</td>
<td>3.1442</td>
<td>7.8545</td>
<td>39.6939</td>
<td>109.067</td>
</tr>
<tr>
<td></td>
<td>(1.6018)</td>
<td>(3.1808)</td>
<td>(7.7771)</td>
<td>(39.0880)</td>
<td>(108.8049)</td>
</tr>
<tr>
<td>$R_t \ t_i^c$</td>
<td>1.3337</td>
<td>2.7513</td>
<td>7.2392</td>
<td>39.1385</td>
<td>109.0807</td>
</tr>
<tr>
<td></td>
<td>(1.6005)</td>
<td>(3.1783)</td>
<td>(7.7786)</td>
<td>(39.0736)</td>
<td>(108.759)</td>
</tr>
<tr>
<td>$R_t \ t_i^k$</td>
<td>1.5963</td>
<td>3.1645</td>
<td>7.7320</td>
<td>39.1589</td>
<td>109.174</td>
</tr>
<tr>
<td></td>
<td>(1.6011)</td>
<td>(3.1795)</td>
<td>(7.7739)</td>
<td>(39.0732)</td>
<td>(108.756)</td>
</tr>
<tr>
<td>$R_t \ t_i^n$</td>
<td>1.6090</td>
<td>3.1730</td>
<td>7.7272</td>
<td>39.0825</td>
<td>109.1488</td>
</tr>
<tr>
<td></td>
<td>(1.6012)</td>
<td>(3.1793)</td>
<td>(7.7792)</td>
<td>(39.0863)</td>
<td>(108.7728)</td>
</tr>
</tbody>
</table>

Notes: Results without debt consolidation in parentheses. Periods denote quarters.

The resulting public debt dynamics are shown in Figure 2. The general message is that now it is optimal to reduce public debt more gradually than before. In particular, when we use public spending or consumption taxes, debt adjustment should take place at a slower pace during the first 5-10 periods or quarters. But the difference from Figure 1 becomes more striking when we use capital and labour income taxes. Now, it is clearly optimal to let the debt-to-output ratio further rise in the very short run, so as to help the real economy recover first, and only then decrease the debt gradually by following an almost unit root process to its 60% target.
6.2 Impulse response functions of optimized fiscal instruments with and without labor rigidities

To make our results clearer, we also provide the impulse response functions of the optimized fiscal policy instruments studied above. This is in Figure 3. Impulse response functions are shown as log-linear deviations from the status-quo solution. Solid lines correspond to the model without wage rigidities. Broken lines correspond to the model with wage rigidities. Recall that there are two driving forces of dynamics in our model: an adverse shock to TFP causing a recession and the debt consolidation reform.

As can be seen in Figure 3, public spending should fall, and consumption taxes should rise, with and without wage rigidities. Thus, the concern for public debt dominates the concern for the output gap, when we make use of a relatively non-distorting fiscal instrument, like public spending and consumption taxes. On the other hand, the degree of wage rigidities plays an important role if we use capital taxes. If there are no labor market rigidities, capital taxes should rise to bring public debt down. But, if there are labor market rigidities, the change in capital taxes should be very mild (actually, as shown in Figure 3, the capital tax rate should be cut initially) to help the real economy recover first. The emphasis on real activity becomes even more obvious when we use labor taxes which, as we have seen, are particularly distorting. Now, labor taxes should be reduced so as to counter the recession first and only later on should be raised to address the public debt problem. Naturally, this gets more obvious in the presence of labor market rigidities.
7 Robustness

In this section, we examine robustness to alternative parameterizations, to adding new shocks and to several generalizations of the model. The bottom line will be that the main results remain unaffected.

7.1 Alternative parameterizations of the model

We first report that our main results are robust to changes in the magnitude of key parameter values. Among the latter, we have, in particular, experimented with changes in the values of the Calvo parameter in the firm’s problem, $\theta$, and the preference parameter for public goods, $\chi_g$, whose values are relatively unknown. Our main results do not change within the ranges $0.33 \leq \theta \leq 0.75$ and $0 \leq \chi_g \leq 0.09$.

Results are available upon request.

7.2 Adding new shocks

We also report that our main results are robust to assuming a more volatile economy. This can be captured by a higher standard deviation of the existing TFP shock, or by adding new shocks, like policy shocks in the feedback rules (12)-(16) as well as shocks to the time preference
It is worth pointing out here that, when extrinsic volatility rises (meaning a higher standard deviation and/or more sources of stochasticity), we need, in most cases, to further restrict the range of feedback policy coefficients in order to guarantee determinacy and well-defined values for the policy instruments employed for cyclical stabilization and debt consolidation.

Results are available upon request.

7.3 More general economic environments

We next consider robustness to richer economic environments. We first allow policymakers to react to the so-called natural level of output; second, we generalize the model by allowing for trend inflation.

First, we study the case in which, in each period, policymakers react to the current natural level of output. The latter is defined as the level of output that would arise in the absence of nominal fixities. Technically, this means that the policy target for output in the feedback policy rules, (12)-(16), is now time-varying and its value follows from the fictional case in which, other things equal, the Calvo parameter is set at zero. Thus, we first solve for this fictional case (always computing welfare-maximizing feedback policy coefficients and the associated equilibrium values of the endogenous variables including output) and, in turn, we use the resulting time path of natural output as a time-varying target in the policy rules (12)-(16). The new results, when policymakers react to the natural level of output, are reported in Table A.1 in Appendix 5, which is like Table 4 in the main text (note that, since nominal fixities play no role in the long run, the steady state solution remains the same as in Table 2). Comparison of these results implies that the main results do not change. For instance, the welfare ranking remains as in Table 4. On the other hand, there is a quantitative difference: optimal monetary policy reaction to the output gap is now much stronger than it was in Table 4. This makes sense: since the output target is more ambitious, the monetary authorities find it optimal to react also to the real economy. Thus, although the feedback policy coefficient on inflation is still higher than the coefficient on output, response to output is welfare superior relative to the simple rule in Table 4 where the interest rate responded solely to inflation.

Second, we generalize the model by allowing for trend inflation. This means that the (gross) steady state rate of inflation is not 1 but it is 1.00375 which is the average value in the data.

\[13\] We model shocks to the time preference rate by using an AR(1) process like that in equation (18) used for the TFP shock. Our main results (e.g. the welfare ranking of various fiscal policy instruments, the way monetary and fiscal policy instruments should react to economic indicators, and the comparison between debt consolidation and non-debt consolidation) do not change.
This requires the recalibration of the model. The new results are reported in Tables A.2 and A.3 in Appendix 6, which are respectively like Tables 2 and 4 in the main text. As is known (see e.g. Ascari, 2004), trend inflation can play a non-trivial role. For instance, steady state output falls with inflation (see page 650 in Ascari, 2004, and page 215 in Wickens, 2008, for interpretation of the output loss as trend inflation rises). Here we get similar results; steady state output and welfare are lower in Table A.2 with positive trend inflation than in Table 2 with zero trend inflation. Nevertheless, the welfare ranking of policy instruments, as well as the properties of the optimized policy rules, reported in Table A.3, remain as in Table 4.

8 Concluding remarks and possible extensions

This paper studied the optimal mix of monetary and fiscal policy actions in a New Keynesian model of a closed economy. The aim was to welfare rank different fiscal (tax and spending) policy instruments when the central bank followed a Taylor rule for the nominal interest rate. We did so when the policy task was to stabilize the economy against shocks and to reduce public debt over time.

Since the results have been listed in the Introduction, we close with some extensions. A natural extension is to see what changes in an open economy setup. Actually, in two companion papers (Philippopoulos et al., 2013 and 2014), we have done so by examining respectively the case of a semi-small open economy facing sovereign risk premia when it borrows from abroad and the case of a multi-country model of a currency union with debtor and creditor country-members. We find that openness can change results.

This type of work can be further extended in several ways. For instance, it would be interesting to study the implications of less conventional monetary policy instruments, like the case in which the central bank acts as a lender of last resort. Similarly, it would be interesting to use a more detailed decomposition of public spending (like spending on infrastructure, education services, pensions, etc) and reexamine the attractiveness of each one of those spending categories as a tool for macroeconomic stabilization (but this would require a much richer model where each public spending category functions a well-specified economic role). Finally, one could allow for different policy reaction to economic fundamentals during booms and during recessions (although, as far as we know, policy regime switching is a challenging task computationally; see e.g. Foerster et al., 2013). We leave these extensions for future work.
References


9 Appendices

9.1 Appendix 1: Households

This Appendix provides details and the solution of household’s problem. There are \( i = 1, 2, ..., N \) households. Each household \( i \) acts competitively to maximize expected lifetime utility.

9.1.1 Household’s problem

Household \( i \)'s expected lifetime utility is:

\[
E_0 \sum_{t=0}^{\infty} \beta^t U (c_{i,t}, n_{i,t}, m_{i,t}, g_t) \tag{20}
\]

where \( c_{i,t} \) is \( i \)'s consumption bundle (defined below), \( n_{i,t} \) is \( i \)'s hours of work, \( m_{i,t} \equiv \frac{M_{i,t}}{P_t} \) is \( i \)'s real money balances, \( g_t \) is per capita public spending, \( 0 < \beta < 1 \) is the time discount rate, and \( E_0 \) is the rational expectations operator conditional on the current period information set.

In our numerical solutions, we use the period utility function (see also e.g. Gali, 2008):

\[
u_{i,t} (c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{c_{1-\sigma}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{1+\eta}}{1+\eta} + \chi_m \frac{m_{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} \tag{21}\]

where \( \chi_n, \chi_m, \chi_g, \sigma, \eta, \mu, \zeta \) are preference parameters.

The period budget constraint of each household \( i \) is in nominal terms:

\[
(1 + \tau^e_t) P_t c_{i,t} + P_t x_{i,t} + B_{i,t} + M_{i,t} =
(1 - \tau^k_t) (r^F_t P_t k_{i,t-1} + D_{i,t}) + (1 - \tau^n_t) W_t n_{i,t} + R_{t-1} B_{i,t-1} + M_{i,t-1} - T^l_{i,t} \tag{22}\]

where \( P_t \) is the general price index, \( x_{i,t} \) is \( i \)'s real investment, \( B_{i,t} \) is \( i \)'s end-of-period nominal government bonds, \( M_{i,t} \) is \( i \)'s end-of-period nominal money holdings, \( r^F_t \) is the real return to inherited capital, \( k_{i,t-1} \), \( D_{i,t} \) is \( i \)'s nominal dividends paid by firms, \( W_t \) is the nominal wage rate, \( R_{t-1} \) is the gross nominal return to government bonds between \( t - 1 \) and \( t \), \( T^l_{i,t} \) is nominal lump-sum taxes/transfers to each \( i \) from the government, and \( \tau^e_t, \tau^k_t, \tau^n_t \) are respectively tax rates on private consumption, capital income and labour income.
Dividing by $P_t$, the budget constraint of each $i$ in real terms is:

\[
(1 + \tau_t^i) c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} = (1 - \tau_t^k) (r_{t}^k k_{i,t-1} + d_{i,t}) + \\
+ (1 - \tau_t^n) w_t n_{i,t} + R_{t-1} \frac{P_{t-1}}{P_t} b_{i,t-1} + \frac{P_{t-1}}{P_t} m_{i,t-1} - \tau_{i,t}^l
\]

(23)

where small letters denote real variables, i.e. $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$, $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$, $w_t \equiv \frac{W_t}{P_t}$, $d_{i,t} \equiv \frac{D_{i,t}}{P_t}$, $\tau_{i,t}^l \equiv \frac{T_{i,t}^l}{P_t}$, at individual level.

The motion of physical capital for each household $i$ is:

\[
k_{i,t} = (1 - \delta) k_{i,t-1} + x_{i,t}
\]

(24)

where $0 < \delta < 1$ is the depreciation rate of capital.

Household $i$’s consumption bundle at $t$, $c_{i,t}$, is a composite of $h = 1, 2, ..., N$ varieties of goods, denoted as $c_{i,t}(h)$, where each variety $h$ is produced monopolistically by one firm $h$. Using a Dixit-Stiglitz aggregator, we define:

\[
c_{i,t} = \left[ \sum_{h=1}^{N} \lambda[c_{i,t}(h)]^{\phi-1} \right]^{1/\phi}
\]

(25)

where $\phi > 0$ is the elasticity of substitution across goods produced and $\sum_{h=1}^{N} \lambda = 1$ are weights (to avoid scale effects, we assume $\lambda = 1/N$).

Household $i$’s total consumption expenditure is:

\[
P_t c_{i,t} = \sum_{h=1}^{N} \lambda P_t(h) c_{i,t}(h)
\]

(26)

where $P_t(h)$ is the price of variety $h$.

**9.1.2 Household’s optimality conditions**

Each household $i$ acts competitively taking prices and policy as given. Following the literature, we first suppose that the household chooses its desired consumption of the composite good, $c_{i,t}$, and, in turn, chooses how to distribute its purchases of individual varieties, $c_{i,t}(h)$. Details are available upon request.

The first-order conditions include the budget constraint above and:

\[
\frac{c_{i,t}^\sigma}{(1 + \tau_t^i)} = \beta E_t \frac{c_{i,t+1}^\sigma}{(1 + \tau_{t+1}^i)} \left[ (1 - \tau_{t+1}^k) r_{t+1}^k + (1 - \delta) \right]
\]

(27)
\[
\frac{c_{i,t}^{-\sigma}}{(1 + \tau_t^c)} = \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} R_t \frac{P_t}{P_{t+1}}
\]  
(28)

\[
\chi_m m_{i,t}^{-\mu} - \frac{c_{i,t}^{-\sigma}}{(1 + \tau_t^c)} + \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{P_t}{P_{t+1}} = 0
\]  
(29)

\[
\chi_n n_{i,t}^{-\gamma} = \frac{(1 - \tau_t^\mu)}{(1 + \tau_t^\mu)} w_t
\]  
(30)

\[
c_{i,t}(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_{i,t}
\]  
(31)

Equations (27) and (28) are respectively the Euler equations for capital and bonds, (29) is the optimality condition for money balances, (30) is the optimality condition for work hours and (31) shows the optimal demand for each variety of goods.

9.1.3 Implications for price bundles

Equations (26) and (31) imply that the general price index is:

\[
P_t = \left[ \sum_{h=1}^N \lambda[P_t(h)]^{1-\phi} \right]^{\frac{1}{1-\phi}}
\]  
(32)

9.2 Appendix 2: Firms

This Appendix provides details and the solution of firm’s problem. There are \( h = 1, 2, ..., N \) firms. Each firm \( h \) produces a differentiated good of variety \( h \) under monopolistic competition facing Calvo-type nominal fixities. firm \( i \) acts competitively to maximize expected lifetime utility.

9.2.1 Demand for firm’s product

Each firm \( h \) faces demand for its product, \( y_t(h) \), coming from households’ consumption and investment, \( c_t(h) \) and \( x_t(h) \), where \( c_t(h) \equiv \sum_{i=1}^N c_{i,t}(h) \) and \( x_t(h) \equiv \sum_{i=1}^N x_{i,t}(h) \), and from the government, \( g_t(h) \). Thus, the demand for each firm’s product is:

\[
y_t(h) = c_t(h) + x_t(h) + g_t(h)
\]  
(33)

where from above:

\[
c_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_t
\]  
(34)
and similarly:

\[
x_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} x_t \tag{35}
\]

\[
g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} g_t \tag{36}
\]

where \( c_t = \sum_{i=1}^{N} c_{i,t}, \ x_t = \sum_{i=1}^{N} x_{i,t} \) and \( g_t \) is public spending.

Since, at the economy level:

\[
y_t = c_t + x_t + g_t \tag{37}
\]

the above equations imply that the demand for each firm’s product is:

\[
y_t(h) = c_t(h) + x_t(h) + g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t \tag{38}
\]

### 9.2.2 Firm’s problem

Each firm \( h \) nominal profits in period \( t \), \( D_t(h) \), defined as:

\[
D_t(h) = P_t(h)y_t(h) - P_t^{\kappa}k_{t-1}(h) - W_t n_t(h) \tag{39}
\]

All firms use the same technology represented by the production function:

\[
y_t(h) = A_t[k_{t-1}(h)]^\alpha [n_t(h)]^{1-\alpha} \tag{40}
\]

where \( A_t \) is an exogenous stochastic TFP process whose motion is defined below.

Under imperfect competition, profit maximization is subject to the demand function derived above, namely:

\[
y_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t \tag{41}
\]

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm \( h \) faces an exogenous probability \( \theta \) of not being able to reset its price. A firm \( h \), which is able to reset its price \( P_t^\text{res}(h) \) to maximize the sum of discounted expected nominal profits for the next \( k \) periods in which it may have to keep its price fixed.
9.2.3 Firm’s optimality conditions

Following the related literature, we follow a two-step procedure. We first solve a cost minimization problem, where each firm $h$ minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price. Details are available upon request.

The solution to the cost minimization problem gives the input demand functions:

$$w_t = mc_t(1 - a)A_t[k_{t-1}(h)]^\alpha[n_t(h)]^{-\alpha}$$ (42)

$$r_t^k = mc_t a A_t[k_{t-1}(h)]^{\alpha-1}[n_t(h)]^{1-\alpha}$$ (43)

where $mc_t = \Psi'_t(.)$ is the marginal nominal cost with $\Psi_t(.)$ denoting the associated minimum nominal cost function for producing $y_t(h)$ at $t$.

Then, the firm chooses its price, $P_t^#(h)$, to maximize nominal profits written as:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^#(h) y_{t+k}(h) - \Psi_{t+k}(y_{t+k}(h)) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm and where $y_{t+k}(h) = \left[ \frac{P_t^#(h)}{P_{t+k}} \right]^{-\phi} y_{t+k}$.

The first-order condition gives:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^#(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ P_t^#(h) - \frac{\phi}{\phi - 1} \Psi'_{t+k} \right\} = 0$$ (44)

Dividing by the aggregate price index, $P_t$, we have:

$$E_t \sum_{k=0}^{\infty} \theta^k [\Xi_{t,t+k}] \left[ \frac{P_t^#(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ P_t^#(h) - \frac{\phi}{\phi - 1} mc_{t+k} \frac{P_{t+k}}{P_t} \right\} = 0$$ (45)

Therefore, the behaviour of firm $h$, which can reset its price, is summarized by the above three conditions (42), (43) and (45).

Each firm $h$ which can reset its price in period $t$ solves an identical problem, so $P_t^#(h) = P_t^#$ is independent of $h$, and each firm $h$ which cannot reset its price just sets its previous period price $P_t(h) = P_{t-1}(h)$. Then, it can be shown that the evolution of the aggregate price level
is given by:

\[(P_t)^{1-\phi} = \theta (P_{t-1})^{1-\phi} + (1 - \theta) \left( P^#_{t} \right)^{1-\phi} \] (46)

### 9.3 Appendix 3: Decentralized equilibrium (given policy instruments)

We now combine the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. In this DE, (i) all households maximize utility (ii) a fraction \((1 - \theta)\) of firms maximize profits by choosing the identical price \(P^#_{t}\), while the rest, \(\theta\), set their previous period prices (iii) all constraints are satisfied and (iv) all markets clear (details are available upon request).

The DE can be summarized by the following equilibrium conditions (quantities are in per capita terms):

\[
\frac{c^*_{t}\sigma}{(1 + \tau^c_t)} = \beta E_t \frac{c_{t+1}\sigma}{(1 + \tau^c_{t+1})} \left[ (1 - \tau^k_{t+1}) r^k_{t+1} + (1 - \delta) \right] \] (47)

\[
\frac{c^*_{t}\sigma}{(1 + \tau^c_t)} = \beta E_t \frac{c_{t+1}\sigma}{(1 + \tau^c_{t+1})} \frac{P_t}{P_{t+1}} \] (48)

\[
\chi m m^\mu_{t} - \frac{c^*_{t}\sigma}{(1 + \tau^c_t)} + \beta E_t \frac{c_{t+1}\sigma}{(1 + \tau^c_{t+1})} \frac{P_t}{P_{t+1}} = 0 \] (49)

\[
\chi n n^\mu_{t} = \frac{(1 - \tau^\mu_t)}{(1 + \tau^c_t)} w_t \] (50)

\[
k_t = (1 - \delta) k_{t-1} + \Delta x_t \] (51)

\[
E_t \sum_{k=0}^{\infty} \theta^k \left\{ z_{t,t+k} \left[ \frac{P^#_{t+k}}{P_{t+k}} \right]^{-\phi} y_{t+k} \left( \frac{P^#_{t+k}}{P_{t+k}} - \frac{\phi}{\phi - 1} mc_{t+k} \frac{P_{t+k}}{P_{t}} \right) \right\} = 0 \] (52)

\[
w_t = mc_t(1 - a) \frac{y_t}{n_t} \] (53)

\[
r^k_t = mc_t \frac{y_t}{k_t} \] (54)

\[
d_t = y_t - w_t n_t - r^k_t k_{t-1} \] (55)

\[
y_t = \frac{1}{(\bar{P}_t/P_t)^{\sigma}} \Lambda_t k^\mu_{t-1} n_t^{1-\sigma} \] (56)
\[ b_t + m_t = R_{t-1} b_{t-1} \frac{P_{t-1}}{P_t} + m_{t-1} \frac{P_{t-1}}{P_t} + g_t - \tau_t c_t - \tau_t n_t - \tau_t \left( \tau_t k_{t-1} + d_t \right) - \tau_t \]

\[ y_t = c_t + x_t + g_t \]

\[ (P_t)^{1-\phi} = \theta (P_{t-1})^{1-\phi} + (1 - \theta) \left( P_t^# \right)^{1-\phi} \]

\[ (\tilde{P}_t)^{-\phi} = \theta (\tilde{P}_{t-1})^{-\phi} + (1 - \theta) \left( P_t^# \right)^{-\phi} \]

where \[ \Xi_{t,t+k} \equiv \frac{\beta^k c_{t+k}^{\infty}}{c_t^{\infty}} \frac{P_{t+k}}{P_t} \tau_t^{r_k} \] and \[ \tilde{P}_t \equiv \left( \sum_{h=1}^{N} [P_t (h)]^{-\phi} \right)^{-\frac{1}{\phi}} \]. Thus, \[ (\tilde{P}_t)^{-\phi} \] is a measure of price dispersion.

We thus have 14 equilibrium conditions for the DE. To solve the model, we need to specify the policy regime and thus classify policy instruments into endogenous and exogenous. Regarding the conduct of monetary policy, we assume that the nominal interest rate, \( R_t \), is used as a policy instrument, while, regarding fiscal policy, we assume that the residually determined public financing policy instrument is the end-of-period public debt, \( b_t \). Then, the 14 endogenous variables are \( \{ y_t, c_t, n_t, k_t, m_t, b_t, P_t, P_t^#, \tilde{P}_t, w_t, m_c t, d_t, \tau_t k_t \} \). This is given the independently set policy instruments, \( \{ R_t, g_t, \tau_t c_t, \tau_t n_t, \tau_t \} \), technology, \( \{ A_t \} \), and initial conditions for the state variables.

### 9.4 Appendix 4: Decentralized equilibrium transformed (given policy instruments)

We now rewrite the above equilibrium conditions, first, by using inflation rates rather than price levels, second, by writing the firm’s optimality condition (35) in recursive form and, third, by introducing a new equation that helps us to compute expected discounted lifetime utility. Details for each step are available upon request.

#### 9.4.1 Variables expressed in ratios

We define three new endogenous variables, which are the gross inflation rate \( \Pi_t \equiv \frac{P_t}{P_{t-1}} \), the auxiliary variable \( \Theta_t \equiv \frac{P_t^#}{P_t} \), and the price dispersion index \( \Delta_t \equiv \left( \frac{P_t}{\Pi_t} \right)^{-\phi} \). We also find it convenient to express the two exogenous fiscal spending policy instruments as ratios of GDP, \( s_t^g \equiv \frac{g_t}{y_t} \) and \( s_t^l \equiv \frac{\tau_t}{y_t} \).

Thus, from now on, we use \( \Pi_t, \Theta_t, \Delta_t, s_t^g, s_t^l \) instead of \( P_t, P_t^#, \tilde{P}_t, g_t, \tau_t \) respectively.
9.4.2 Equation (52) expressed in recursive form

Following Schmitt-Grohé and Uribe (2007), we look for a recursive representation of (52):

\[
E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^#}{P_{t+k}^#} \right]^{-\phi} y_{t+k} \left\{ \frac{P_t^#}{P_t} - \phi (\phi - 1)^{mc_{t+k} P_{t+k}} P_t \right\} = 0 \tag{61}
\]

We define two auxiliary endogenous variables:

\[
z_1^t \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^#}{P_{t+k}^#} \right]^{-\phi} y_{t+k} \frac{P_t^#}{P_t} \tag{62}
\]

\[
z_2^t \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^#}{P_{t+k}^#} \right]^{-\phi} y_{t+k} mc_{t+k} \frac{P_t^#}{P_t} \tag{63}
\]

Using these two auxiliary variables, \(z_1^t\) and \(z_2^t\), we come up with two new equations which enter the dynamic system and allow a recursive representation of (61). Thus, we replace equation (52) above with:

\[
z_1^t = \frac{\phi}{(\phi - 1)} z_2^t \tag{64}
\]

where:

\[
z_1^t = \Theta_t^{-\phi-1} y_t + \beta \theta E_t \frac{c_{t+1}^{1-\sigma}}{c_t^{1-\sigma}} \left[ \frac{1 + \tau_t^e}{\tau_{t+1}^e} \right] \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi-1} \left( \frac{1}{\Pi_{t+1}} \right)^{-\phi} z_1^{t+1} \tag{65}
\]

\[
z_2^t = \Theta_t^{-\phi} y_t mc_t + \beta \theta E_t \frac{c_{t+1}^{1-\sigma}}{c_t^{1-\sigma}} \left[ \frac{1 + \tau_t^e}{\tau_{t+1}^e} \right] \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi} \left( \frac{1}{\Pi_{t+1}} \right)^{1-\phi} z_2^{t+1} \tag{66}
\]

Thus, from now on, instead of (52), we use (64), (65) and (83) and add two new endogenous variables, \(z_1^t\) and \(z_2^t\).

9.4.3 Lifetime utility written as a first-order dynamic equation

To compute social welfare, we follow Schmitt-Grohé and Uribe (2007) by defining a new endogenous variable, \(V_t\), whose motion is:

\[
V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\phi}}{1+\phi} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} + \chi_g (s_t^g y_t)^{1-\zeta} + \beta E_t V_{t+1} \tag{67}
\]

where \(V_t\) is the expected discounted lifetime utility of the household at any \(t\).

Thus, from now on, we add equation (67) and the new variable \(V_t\) to the equilibrium system.
9.4.4 Equations of transformed DE

Using the above, the final non-linear stochastic system is:

\[
\frac{c_t^{-\sigma}}{(1 + \tau_t^c)} = \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \left[ (1 - \tau_t^k) r_{t+1}^k + (1 - \delta) \right] \tag{68}
\]

\[
\frac{c_t^{-\sigma}}{R_t} \frac{1}{(1 + \tau_t^c)} = \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{1}{\Pi_{t+1}} \tag{69}
\]

\[
\chi_m m_t^{-\mu} - \frac{c_t^{-\sigma}}{(1 + \tau_t^c)} + \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{1}{\Pi_{t+1}} = 0 \tag{70}
\]

\[
\frac{n_t^m}{c_t^{-\sigma}} = \frac{(1 - \tau_t^m)}{(1 + \tau_t^c)} w_t \tag{71}
\]

\[
k_t = (1 - \delta) k_{t-1} + x_t \tag{72}
\]

\[
z_t^1 = \frac{\phi - 1}{\phi} z_t^2 \tag{73}
\]

\[
w_t = mc_t(1 - a) \frac{y_t}{n_t} \tag{74}
\]

\[
r_t^k = mc_t a \frac{y_t}{k_{t-1}} \tag{75}
\]

\[
d_t = y_t - w_t n_t - r_t^k k_{t-1} \tag{76}
\]

\[
y_t = \frac{1}{\Delta_t} A_t k_{t-1}^a n_t^{1-a} \tag{77}
\]

\[
b_t + m_t = R_t b_{t-1} - \frac{1}{\Pi_t} + m_{t-1} \frac{1}{\Pi_t} + s_t^q y_t - \tau_t^c c_t - \tau_t^n w_t n_t - \tau_t^k \left[ r_t^k k_{t-1} + d_t \right] - s_t^l y_t \tag{78}
\]

\[
y_t = c_t + x_t + s_t^q y_t \tag{79}
\]

\[
\Pi_t^{1-\phi} = \theta + (1 - \theta) [\Theta_t \Pi_t]^{1-\phi} \tag{80}
\]

\[
\Delta_t = (1 - \theta) \Theta_t^{-\phi} + \theta \Pi_t^{\phi} \Delta_{t-1} \tag{81}
\]

\[
z_t^1 = \Theta_t^{-\phi} y_t + \beta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi-1} \left( \frac{1}{\Pi_{t+1}} \right)^{-\phi} z_{t+1}^1 \tag{82}
\]
\[ z_t^2 = \Theta_t^{-\phi} y_t mc_t + \beta \theta E_t \frac{c_{t+1}^{\sigma}}{c_t^{\sigma}} \frac{1 + \tau_t^y}{1 + \tau_t^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi} \left( \frac{1}{\Pi_{t+1}} \right) \left( 1 - \zeta \right) \]  

(83)

\[ V_t = \frac{c_t^{1-\sigma}}{1 - \sigma} + \chi_m \frac{m_t^{1-\mu}}{1 - \mu} - \chi_n \frac{n_t^{1+\phi}}{1 + \phi} + \chi_g \frac{(s_t^g y_t)^{1-\zeta}}{1 - \zeta} + \beta E_t V_{t+1} \]  

(84)

There are 17 equations in 17 endogenous variables, \( \{y_t, c_t, n_t, x_t, k_t, m_t, b_t, \Pi_t, \Theta_t, \Delta_t, w_t, mc_t, d_t, r_t^k, \tau_t^l, z_t^l, z_t^2, V_t\}_{t=0}^{\infty} \). This is given the independently set policy instruments, \( \{R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n, s_t^l\}_{t=0}^{\infty} \), technology, \( \{A_t\}_{t=0}^{\infty} \), and initial conditions for the state variables. Recall that \( \{R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n\}_{t=0}^{\infty} \) follow the feedback rules specified above, while \( \{s_t^l\}_{t=0}^{\infty} \) remains constant and equal to its average value in the data.

### 9.5 Appendix 5: Using the natural level of output as a policy target

In this Appendix, the policy target for output, in each period, is the level without nominal fixities. Then, working as explained in the text, Table 4 changes to:

<table>
<thead>
<tr>
<th>Policy instruments</th>
<th>Optimal interest-rate reaction to inflation and output</th>
<th>Optimal fiscal reaction to debt and output</th>
<th>Lifetime utility</th>
<th>( E_0 V_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_t ) ( s_t^g )</td>
<td>( \phi_x = 1.1 ) ( \gamma_t^g = 0.1922 )</td>
<td>( \phi_y = 0.0337 ) ( \gamma_t^g = 0.0992 )</td>
<td>109.2589</td>
<td></td>
</tr>
<tr>
<td>( R_t ) ( \tau_t^c )</td>
<td>( \phi_x = 3 ) ( \gamma_t^c = 0.2 )</td>
<td>( \phi_y = 0.1966 ) ( \gamma_t^c = 0.0499 )</td>
<td>109.0948</td>
<td></td>
</tr>
<tr>
<td>( R_t ) ( \tau_t^k )</td>
<td>( \phi_x = 3 ) ( \gamma_t^k = 0.15 )</td>
<td>( \phi_y = 0.3759 ) ( \gamma_t^k = 0.1744 )</td>
<td>109.0752</td>
<td></td>
</tr>
<tr>
<td>( R_t ) ( \tau_t^n )</td>
<td>( \phi_x = 3 ) ( \gamma_t^n = 0.0092 )</td>
<td>( \phi_y = 0.3783 ) ( \gamma_t^n = 0.0958 )</td>
<td>108.7812</td>
<td></td>
</tr>
</tbody>
</table>

### 9.6 Appendix 6: Adding steady-state inflation

In this section, we allow for steady state, or trend, inflation. Then, working as explained in the text, Table 2 in the main text changes to:
Table A.2: Steady state solution or "the status quo"

<table>
<thead>
<tr>
<th>Variables</th>
<th>Long-run solution</th>
<th>Variables</th>
<th>Long-run solution</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0.7378</td>
<td>$d$</td>
<td>0.12</td>
<td>-</td>
</tr>
<tr>
<td>$c$</td>
<td>0.459</td>
<td>$r^k$</td>
<td>0.039</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>0.2784</td>
<td>$z^1$</td>
<td>2.16</td>
<td>-</td>
</tr>
<tr>
<td>$x$</td>
<td>0.11</td>
<td>$z^2$</td>
<td>1.80</td>
<td>-</td>
</tr>
<tr>
<td>$k$</td>
<td>5.19</td>
<td>$V$</td>
<td>108.238</td>
<td>-</td>
</tr>
<tr>
<td>$m$</td>
<td>1.3</td>
<td>$u$</td>
<td>0.8009</td>
<td>-</td>
</tr>
<tr>
<td>$b$</td>
<td>2.51</td>
<td>$l$</td>
<td>3.438</td>
<td>-</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>1.00375</td>
<td>$\frac{c}{y}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Theta$</td>
<td>1.00773</td>
<td>$\frac{b}{y}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$</td>
<td>1.00027</td>
<td>$\frac{z}{y}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>1.47246</td>
<td>$\frac{m}{y}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$mc$</td>
<td>0.83</td>
<td>$\frac{k}{y}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

while Table 4 in the main text changes to:

Table A.3: Optimal monetary reaction to inflation and output
and optimal fiscal reaction to debt and output

<table>
<thead>
<tr>
<th>Policy instruments</th>
<th>Optimal interest-rate reaction to inflation and output</th>
<th>Optimal fiscal reaction to debt and output</th>
<th>Lifetime utility $E_0V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t \ s^g_t$</td>
<td>$\phi_\pi = 1.1$                        $\gamma^g_\pi = 0.2942$</td>
<td>$\gamma^g_\pi = 0.0292$</td>
<td>108.9356</td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.0005$                       $\gamma^g_y = 0.23$</td>
<td>$\gamma^g_y = 0.5274$</td>
<td>108.6972</td>
</tr>
<tr>
<td>$R_t \ t^c_t$</td>
<td>$\phi_\pi = 2.7610$                       $\gamma^c_\pi = 0.15$</td>
<td>$\gamma^c_\pi = 0.0037$</td>
<td>108.686</td>
</tr>
<tr>
<td>$R_t \ t^k_t$</td>
<td>$\phi_\pi = 1.1123$                       $\gamma^n_\pi = 0.0135$</td>
<td>$\gamma^n_\pi = 0.0437$</td>
<td>108.348</td>
</tr>
</tbody>
</table>