Structural FECM: Cointegration in large-scale structural FAVAR models*

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Abstract

Starting from the dynamic factor model for non-stationary data we derive the factor-augmented error correction model (FECM) and, by generalizing the Granger representation theorem, its moving-average representation. The latter is used for the identification of structural shocks and their propagation mechanisms. We show how to implement classical identification schemes based on long-run restrictions in the case of large panels. The importance of the error-correction mechanism for impulse response analysis is analysed by means of both empirical examples and simulation experiments. Our results show that the bias in estimated impulse responses in a FAVAR model is positively related to the strength of the error-correction mechanism and the cross-section dimension of the panel. We observe empirically in a large panel of US and Euro area data that these features have a substantial effect on the responses of several variables to the identified real shock.

Keywords: Dynamic Factor Models, Cointegration, Structural Analysis, Factor-augmented Error Correction Models, FAVAR

JEL-Codes: C32, E17

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1 Introduction

Large dimensional factor models have received considerable attention in the recent econometric literature, starting with the seminal papers by Forni, Hallin, Lippi and Reichlin (2000) and Stock and Watson (2002a, 2002b). While the early applications were mostly reduced form analyses, following the publication of Bernanke, Boivin and Eliasz (2005) more and more attention has been devoted to structural analyses based on Factor Augmented VARs (FAVARs) - see also Stock and Watson (2005).

With few notable exceptions, such as Bai (2004), Bai and Ng (2004) and Barigozzi, Lippi and Luciani (2014) [1], this entire literature does not take account of the possibility of cointegration among the variables under study. Banerjee and Marcellino (2009) suggested including factors extracted from large non-stationary panels in small scale error correction models (ECMs) to proxy for the missing cointegration relations. They labelled the resulting model as the Factor Augmented ECM (FECM). Banerjee, Marcellino and Masten (2014a) showed that FECMs often outperform both FAVARs and standard small scale ECMs in terms of forecasting macroeconomic variables, given the property that FECMs nest both FAVARs and ECMs.

In this paper we focus on the use of FECMs for structural analysis. We start from a dynamic factor model for nonstationary data as in Bai (2004), and show it can be reparameterized to yield a FECM. Bai's asymptotic results can also be applied in our context, when a mixture of I(1) and I(0) factors is allowed, for both the identification of the factor spaces and the estimation of the factors.

We then extend the Granger representation theorem (see, e.g., Johansen,1995) to derive the moving-average representation of the FECM. The latter can be used to identify structural shocks and their propagation mechanism, using similar techniques as those adopted in the structural VAR literature. In particular, our paper provides the first analysis of the long-run scheme for identification of structural shocks in nonstationary panels.[2]

When assessing the properties of the FECM with respect to the FAVAR, we focus on the effects that including the error-correction terms have on the impulse response functions. Using simulation experiments with a design similar to the estimated model in the empirical applications, we consider which features increase the bias in the impulse responses of the FAVAR with respect to those from the FECM. Not surprisingly, the strength of the error-correction mechanism matters. Moreover, as we show in the paper, since the FECM can be approximated to some extent by the FAVAR with a large lag order, over-parameterization and the associated estimation uncertainty also play a role.

[1] In the concluding paragraph of Section 2 of our paper we provide a brief comparison of our work with the results contained in Barigozzi et al. (2014).

[2] Forni et al. (2009) provide an empirical illustration of the stochastic trends analysis of King et al. (1991) in the context of large stationary panels. Eickmeier (2009) works with a nonstationary panel and identification of structural shocks with sign restrictions. The FECM model is also related to the framework used recently to formulate testing for cointegration in panels (see for example Bai, Kao and Ng (2009) and Gengenbach, Urbain and Westerlund (2008).

Finally, we develop two empirical applications where we use our proposed long-run restrictions to identify structural stochastic trends and the effects of their associated shocks on a large set of US and euro area economic variables. Results indicate important effects of omitting the error-correction terms in the FAVAR. Moreover, the FECM impulse responses are broadly in line with economic theory and comparable to the responses to permanent productivity shocks obtained from an estimated DSGE model (Adolfson, Laseen, Linde and Villani, 2007).

The rest of the paper is structured as follows. In Section 2 we discuss the representation of the FECM and its relationship with the FAVAR. In Section 3 we derive the moving-average representation of the FECM and discuss structural identification schemes. In Section 4 we deal with estimation. In Section 5 we present the results of the Monte Carlo experiments. In Section 6 we discuss the two empirical applications. Finally, in Section 7 we summarize the main results and conclude. Appendices A to C present, respectively, an analytical example comparing the FAVAR and FECM responses, results from additional Monte Carlo experiments assessing the finite sample performance of FECM estimators, and a comparison of the empirical FECM and DSGE based responses.

2 The Factor-augmented Error-Correction Model (FECM)

Consider the following dynamic factor model (DFM) for the I(1) scalar process X_{it}:

$$\begin{aligned} X_{it} &= \sum_{j=0}^{p} \lambda_{ij} F_{t-j} + \sum_{l=0}^{m} \phi_{il} c_{t-l} + \varepsilon_{it} \\ &= \lambda_i(L) F_t + \phi_i(L) c_t + \varepsilon_{it}, \end{aligned} \tag{1}$$

where $i = 1, ..., N,\ t = 1, ..., T,\ \ F_t$ is an r_1-dimensional vector of random walks, c_t is an r_2-dimensional vector of I(0) factors, $F_t = c_t = 0$ for $t < 0$, and ε_{it} is a zero-mean idiosyncratic component. Both F_t and c_t are latent, unobserved variables. $\lambda_i(L)$and $\phi_i(L)$ are lag polynomials of finite orders p and m respectively.[3]

The loadings λ_{ij} and ϕ_{ij} are either deterministic or stochastic and satisfy the following restrictions. For $\lambda_i = \lambda_i(1)$ and $\phi_i = \phi_i(1)$ we have $E\left\|\lambda_i\right\|^4 \leq M < \infty$, $E\left\|\phi_i\right\|^4 \leq M < \infty$, and $1/N\sum_{i=0}^{N} \lambda_i \lambda_i'$, $1/N\sum_{i=0}^{N} \phi_i \phi_i'$ converge in probability to positive definite matrices. Furthermore, we assume that $E\left(\lambda_{ij}\varepsilon_{is}\right) = E\left(\phi_{ij}\varepsilon_{is}\right) = 0$ for all i, j and s.

As in Bai (2004), the idiosyncratic components ε_{it} are allowed to be serially and weakly cross correlated:[4]

[3]Our model is capable of accommodating some I(0) X_{it}. In such a case the corresponding λ_{ij}s would be zero, but the main assumptions of the model would still be valid and the theoretical results unaltered. In the simulation experiment in Appendix A we address explicitly the small sample properties of the estimator of model (1) in presence of some X_{it} being I(0).

[4]In Section 4.3 we discuss the strict factor model assumption needed to undertake feasible estimation of the parameters of the FECM equation by equation. The consequences of assuming a strict factor structure are noted in Section 4.4, based on a small scale simulation study presented in Appendix A.

$$\varepsilon_t = \Gamma(L)\varepsilon_{t-1} + v_t,$$

where $\varepsilon_t = [\varepsilon_{1t,...}, \varepsilon_{Nt}]'$, and the vector process $v_t = [v_{1t,...}, v_{Nt}]'$ is white noise.

To derive the FECM and discuss further assumptions upon the model that ensure consistent estimation of the model's components, it is convenient to write first the DFM in static form. To this end, we follow Bai (2004) and define

$$\tilde{\lambda}_{ik} = \lambda_{ik} + \lambda_{ik+1} + ... + \lambda_{ip}, \ \ k = 0, ..., p.$$

Let us in addition define

$$\tilde{\Phi}_i = [\phi_{i0}, ..., \phi_{im}] \, .$$

We can then obtain a static representation of the DFM which isolates the I(1) factors from the I(0) factors:

$$X_{it} = \Lambda_i F_t + \Phi_i G_t + \varepsilon_{it}, \tag{2}$$

where

$$\begin{aligned}
\Lambda_i &= \tilde{\lambda}_{i0}, \\
\Phi_i &= \left[\tilde{\Phi}_i, -\tilde{\lambda}_{i1}, \ldots, -\tilde{\lambda}_{ip}\right], \\
G_t &= \left[c_t', c_{t-1}', ..., c_{t-m}', \Delta F_t', ..., \Delta F_{t-p+1}'\right]'.
\end{aligned}$$

Introducing for convenience the notation $\Psi_i = [\Lambda_i', \Phi_i']'$, the following assumptions are needed for consistent estimation of both the I(1) and I(0) factors: $E\left\|\Psi_i\right\|^4 \leq M < \infty$ and $1/N \sum_{i=0}^{N} \Psi_i \Psi_i'$ converges to a $(r_1(p+1) + r_2(m+1)) \times (r_1(p+1) + r_2(m+1))$ positive-definite matrix.

Grouping across the N variables we have

$$X_t = \Lambda F_t + \Phi G_t + \varepsilon_t \tag{3}$$

where $X_t = [X_{1t}, ..., X_{Nt}]', \Lambda = [\Lambda_1', ..., \Lambda_N']', \Phi = [\Phi_1', ..., \Phi_N']'$ and $\varepsilon_t = [\varepsilon_{1t}, ..., \varepsilon_{Nt}]'$.

As noted above, the idiosyncratic component in (3) is serially correlated. This serial correlation can be eliminated from the error process by premultiplying (2) by

$$I - \Gamma\left(L\right)L$$

where

$$\Gamma\left(L\right) = \begin{bmatrix} \gamma_1\left(L\right) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_N\left(L\right) \end{bmatrix}.$$

Following this transformation, we obtain

$$X_t = (I - \Gamma(L)L)\Lambda F_t + (I - \Gamma(L)L)\Phi G_t + \Gamma(L)X_{t-1} + v_t.$$

Note that $\Gamma(L)$ can be conveniently factorized as

$$\Gamma(L) = \Gamma(1) - \Gamma_1(L)(1-L), \tag{4}$$

which allows us to rewrite the previous expression as

$$\begin{aligned} X_t = {} & \Lambda F_t + \Phi G_t - (\Gamma(1) - \Gamma_1(L)(1-L))(\Lambda F_{t-1} + \Phi G_{t-1}) \\ & + (\Gamma(1) - \Gamma_1(L)(1-L))X_{t-1} + v_t. \end{aligned} \tag{5}$$

With further manipulation we get

$$\begin{aligned} X_t = {} & \Lambda F_t + \Phi G_t - \Gamma(1)\Lambda F_{t-1} + \Gamma_1(L)\Lambda\Delta F_{t-1} - \Gamma(1)\Phi G_{t-1} \\ & + \Gamma_1(L)\Lambda\Phi\Delta G_{t-1} + \Gamma(1)X_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t \end{aligned} \tag{6}$$

or

$$\begin{aligned} \Delta X_t = {} & \Lambda F_t + \Phi G_t - \Gamma(1)\Lambda F_{t-1} + \Gamma_1(L)\Lambda\Delta F_{t-1} - \Gamma(1)\Phi G_{t-1} \\ & + \Gamma_1(L)\Phi\Delta G_{t-1} - (I - \Gamma(1))X_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t \end{aligned} \tag{7}$$

The ECM form of the DFM, i.e., the factor-augmented error-correction model (FECM), then follows directly as

$$\begin{aligned} \Delta X_t = {} & \underbrace{-(I - \Gamma(1))(X_{t-1} - \Lambda F_{t-1})}_{\text{Omitted in the FAVAR}} + \Lambda\Delta F_t + \Gamma_1(L)\Lambda\Delta F_{t-1} \\ & + \Phi G_t - \Gamma(1)\Phi G_{t-1} + \Gamma_1(L)\Phi\Delta G_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t. \end{aligned} \tag{8}$$

Equation (8) is a representation of the DFM in (1) in terms of stationary variables. It contains the error-correction term, $-(I - \Gamma(1))(X_{t-1} - \Lambda F_{t-1})$, which is omitted in the standard FAVAR model that therefore suffers from an omitted variable problem, similar to the case of a VAR in differences in the presence of cointegration.

Note that it follows from (3) that

$$X_{t-1} - \Lambda F_{t-1} = \Phi G_{t-1} + \varepsilon_{t-1}, \tag{9}$$

such that it would appear at first sight that the omitted error-correction terms in the FAVAR could be approximated by including additional lags of the I(0) factors. However, by substituting the previous expression into (8) and simplifying we get

$$\Delta X_t = \Lambda \Delta F_t + \Phi \Delta G_t + \Delta \varepsilon_t, \tag{10}$$

which contains a non-invertible MA component. This is problematic from two points of view. Firstly, the structural identification schemes analyzed by Stock and Watson (2005) (see also the survey in Luetkepohl, 2014) rely on inverting the MA process in the idiosyncratic component, and on estimation of v_{it}, the i.i.d. part of the idiosyncratic component. In the presence of a non-invertible MA process, the parameters of the FAVAR model and v_{it} cannot be estimated consistently. Secondly, even if the identification of structural shocks is based only on innovation to the factors and does not require estimation of v_{it}, as in Bernanke et al. (2005), inversion of the MA component is needed to get the endogenous lags in equations for ΔX_{it}. These capture the variable-specific autoregressive dynamics that is unrelated to the common factors, but it affects the impulse responses of ΔX_{ti}.

To elaborate this point further consider the following example. Representation (10) can be alternatively written as

$$\Delta X_t = \Lambda \Delta F_t + \Phi(G_t - G_{t-1}) + \varepsilon_t - \varepsilon_{t-1},$$

which, by using (9) becomes

$$\Delta X_t = -(X_{t-1} - \Lambda F_{t-1}) + \Lambda \Delta F_t + \Phi G_t + \varepsilon_t. \tag{11}$$

At first sight, this is a model that contains an error-correction term, but has a much simpler structure than the FECM in (8). If the identification of structural shocks would be based on innovation to dynamic factors, then such a model would appear to account for the omitted error-correction term in the FAVAR. Note, however, that in order to compute consistent impulse responses to innovations either to F_t or G_t, one still needs to invert the process ε_t so as to get the variable-specific autoregressive dynamics. By doing so, one obtains the FECM representation (8).

In sum, whenever we deal with I(1) data, and many macroeconomic series exhibit this feature, the standard FAVAR model potentially produces biased impulse responses unless we use an infinite number of factors as regressors, or account explicitly for the non-invertible MA structure of the error-process.[5] The analytical example in Appendix A elaborates this point further, and our simulation and empirical analyses below confirm that the omission of the ECM term in the FAVAR may potentially have an important impact on the impulse response functions obtained in typical macroeconomic applications.

To complete the model, we assume that the nonstationary factors follow a vector

[5]For example, our empirical application below is based on the dataset used by Bernanke et al., 2005). They treat 77 out of 120 series as I(1) and use a FAVAR with these variables in differences.

random walk process

$$F_t = F_{t-1} + \varepsilon_t^F, \tag{12}$$

while the stationary factors are represented by

$$c_t = \rho c_{t-1} + \varepsilon_t^c, \tag{13}$$

where ρ is a diagonal matrix with values on the diagonal in absolute term strictly less than one. ε_t^F and ε_t^c are independent of λ_{ij}, ϕ_{ij} and ε_{it} for any i, j, t. As in Bai (2004), it should be noted that the error processes ε_t^F and ε_t^c need not necessarily be *i.i.d.*. They are allowed to be serially and cross correlated and jointly follow a stable vector process:

$$\begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^c \end{bmatrix} = A(L) \begin{bmatrix} \varepsilon_{t-1}^F \\ \varepsilon_{t-1}^c \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{14}$$

where u_t and w_t are zero-mean white-noise innovations to dynamic nonstationary and stationary factors, respectively. Under the stability assumption, we can express the model as

$$\begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^c \end{bmatrix} = [I - A(L)L]^{-1} \begin{bmatrix} u_t \\ w_t \end{bmatrix}. \tag{15}$$

Using (12), (13) and (15) we can write the VAR for the factors as

$$\begin{aligned} \begin{bmatrix} F_t \\ c_t \end{bmatrix} &= \left[\begin{bmatrix} I & 0 \\ 0 & \rho \end{bmatrix} + A(L) \right] \begin{bmatrix} F_{t-1} \\ c_{t-1} \end{bmatrix} - A(L) \begin{bmatrix} I & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} F_{t-2} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix} \\ &= C(L) \begin{bmatrix} F_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \end{aligned} \tag{16}$$

where the parameter restrictions imply that $C(1)$ is a block-diagonal matrix with block sizes corresponding to the partition between F_t and c_t.

The FECM is specified in terms of static factors F and G, which calls for a corresponding VAR specification. Using the definition of G_t and (16) it is straightforward to get the following representation

$$
\begin{bmatrix}
I & 0 & \dots & & & \dots & 0 \\
0 & I & \dots & & & \dots & 0 \\
\vdots & & & & & & \vdots \\
\vdots & & & & & & \vdots \\
0 & \dots & I & 0 & & \dots & 0 \\
-I & \dots & 0 & I & 0 & \dots & 0 \\
0 & \dots & 0 & 0 & I & \dots & 0 \\
\vdots & & & & & & \vdots \\
0 & \dots & & \dots & & \dots & I
\end{bmatrix}
\begin{bmatrix}
F_t \\ c_t \\ c_{t-1} \\ \vdots \\ c_{t-m} \\ \Delta F_t \\ \Delta F_{t-1} \\ \vdots \\ \Delta F_{t-p+1}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
C_{11}(L) & C_{12}(L) & 0 & \dots & & & & \dots & 0 \\
C_{21}(L) & C_{22}(L) & 0 & \dots & & & & \dots & 0 \\
0 & I & 0 & \dots & & & & \dots & 0 \\
\vdots & & \dots & & \dots & & & & \vdots \\
0 & \dots & & \dots & I & 0 & & \dots & 0 \\
-I & \dots & & & \dots & & & \dots & 0 \\
0 & & \dots & & & I & & \dots & 0 \\
\vdots & & & & & & & & \vdots \\
0 & \dots & & & \dots & & I & & 0
\end{bmatrix}
\begin{bmatrix}
F_{t-1} \\ c_{t-1} \\ c_{t-2} \\ \vdots \\ c_{t-m-1} \\ \Delta F_{t-1} \\ \Delta F_{t-2} \\ \vdots \\ \Delta F_{t-p}
\end{bmatrix}
+
\begin{bmatrix}
I & 0 \\ 0 & I \\ 0 & 0 \\ & \\ \vdots & \vdots \\ & \\ \vdots & \vdots \\ & \\ 0 & 0
\end{bmatrix}
\begin{bmatrix}
u_t \\ w_t
\end{bmatrix}
\tag{17}
$$

Using the definition of G_t, the VAR for the static factors, and premultplying the whole expression by the inverse of the initial matrix in (17), the factor VAR can be more compactly written as

$$
\begin{bmatrix} F_t \\ G_t \end{bmatrix} = \begin{bmatrix} M_{11}(L) & M_{12}(L) \\ M_{21}(L) & M_{22}(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ G_{t-1} \end{bmatrix} + Q \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{18}
$$

where the $(r_1(p+1) + r_2(m+1)) \times (r_1 + r_2)$ matrix Q accounts for dynamic singularity of G_t. This is due to the fact that the dimension of the vector process w_t is r_2, which is smaller than or equal to $r_1 p + r_2(m+1)$, the dimension of G_t. Let us assume that the order of the VAR in (18) is n.

To conclude this sub-section, it is convenient to compare our model with that in Barigozzi et al. (2014), who also deal with cointegration in dynamic factor models.

We argue that there are a number of important differences between our framework and the model used by Barigozzi et al. (2014).

First, they work with a static version of the model, with I(1) factors only:

$$
X_{it} = \lambda_{ij} F_t + \varepsilon_{it}, \tag{19}
$$

which is a constrained version of (1) in its specification of the common part of the processes. The assumption of only I(1) factors may simplify substantially the treatment of the model, in particular as far as estimation is concerned, since no attention needs to be paid to separately identifying and estimating the I(1) and I(0) factors, both of which are present in our formulation of the model. However, as we will discuss in more detail in Section 4.1, in order to separately identify and estimate the I(1) and I(0) factors, we need to assume that the idiosyncratic errors are I(0), while this is not necessarily the case with I(1) factors only. Therefore, the restriction involved in considering a simplification of the model to allow only I(1) factors may be offset by a less restrictive assumption on the error processes. It is an empirical issue to determine which is the more restrictive set of assumptions.

As we show in our empirical examples (see Section 4.2), the assumption of I(0) idiosyncratic errors, as well as the presence of I(0) factors, are well supported by the data, making our formulation more relevant to the identification, estimation and structural analysis undertaken in the paper.

Second, they work with the model written as in (10) and focus on how shocks to the common factors propagate to the variables. As mentioned previously, the FAVAR representation in differences with non-invertible errors is not ideal to handle the general structural identification schemes of, e.g., Stock and Watson (2005).

Finally, they assume that the factors F_t follow a VAR model and show that their first differences admit a finite order ECM representation. In order for this representation to be valid they require the existence of cointegration among the I(1) factors.[6] They then combine the latter with (19) to assess how shocks to the factors are transmitted to the variables. Given our focus on modelling the levels of variables, instead of deriving an ECM representation in the unobservable or latent variables, we develop the ECM representation for the variables in (8) and assume, in line with the cointegration literature, that the factors are random walks, possibly with some correlations in the driving errors as in (14).

3 Moving-average representation of the FECM and the Structural FECM

The identification of structural shocks in VAR models usually rests on imposing restrictions upon the parameters of the moving-average representation of the VAR and/or the variance covariance matrix of the VAR errors. An analogous approach is used in the FAVAR model, where the moving-average representation is for both the observable variables and the factors (Stock and Watson, 2005; Lütkepohl, 2014). For vector-error correction models, the derivation of the moving-average representation uses the Granger representation theorem (see, e.g., Johansen, 1995). The FECM is a generalization of error-correction

[6] This is a requirement for which no further clear motivation - beyond theoretical necessity - is presented in their paper, and is in any case unnecessary since it is allowed by the more general Bai (2004) formulation of the model (see page 153) where stationary linear cominations of F_t can always be included as part of G_t.

models to large dynamic panels. For this reason, we first provide a generalization of the Granger representation theorem for nonstationary panels that exhibit cointegration. Then we discuss shock identification.

3.1 The MA representation of the FECM

To start with, we conveniently reparameterize the factor VAR process (18). It contains exactly r_1 unit roots pertaining to F_t.[7] (18) can then be rewritten in differenced form as

$$\begin{bmatrix} \Delta F_t \\ \Delta G_t \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha_M \end{bmatrix} \begin{bmatrix} 0 & I_{r_2} \end{bmatrix} \begin{bmatrix} F_{t-1} \\ G_{t-1} \end{bmatrix} + \begin{bmatrix} M_{11}^*(L) & M_{12}^*(L) \\ M_{21}^*(L) & M_{22}^*(L) \end{bmatrix} \begin{bmatrix} \Delta F_{t-1} \\ \Delta G_{t-1} \end{bmatrix} + Q \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{20}$$

where the coefficient matrices of the matrix polynomials $M_{ij}^*(L)$ are defined from the coefficient matrices in (18) as:

$$M_{ijl}^* = -(M_{ijl+1} + ... + M_{ijn}), \quad l = 1, \ldots, n-1. \tag{21}$$

and $\alpha_M = -\left[I_{r_2} - M_{22}(1)\right]$. With this we can state the following theorem.

Theorem 1 (Granger representation for the FECM) *Given the error-correction representation of the dynamic factor model (8), the moving-average representation of the factor-augmented error-correction model is*

$$\begin{bmatrix} X_t \\ F_t \\ G_t \end{bmatrix} = \begin{bmatrix} \Lambda \\ I_{r_1} \\ 0_{r_2 \times r_1} \end{bmatrix} \omega \sum_{i=1}^{t} u_i + C_1(L) \begin{bmatrix} v_t + [\Lambda, \Phi] Q [u_t', w_t']' \\ Q \begin{bmatrix} u_t \\ w_t \end{bmatrix} \end{bmatrix}. \tag{22}$$

A necessary and sufficient condition for the existence of this representation is $|I_{r_1} - M_{11}^*(1)| \neq 0$.

Proof. The FECM (8) can be rewritten as

$$\begin{aligned} \Delta X_t &= \tilde{\alpha}\left(X_{t-1} - \Lambda F_{t-1} - \Phi G_{t-1}\right) + \Lambda \Delta F_t + \Phi \Delta G_t \\ &\quad + \Gamma_1(L)\left(\Lambda \Delta F_{t-1} + \Phi \Delta G_{t-1}\right) - \Gamma_1(L) \Delta X_{t-1} + v_t, \end{aligned} \tag{23}$$

where $\tilde{\alpha} = -(I - \Gamma(1))$. Using (20) we can stack the equations for ΔX_t and the factors

[7] As noted above, cointegration among F_t is ruled out as we can always include the stationary linear combinations of F_t in G_t.

into a single system of equations as

$$\begin{bmatrix} \Delta X_t \\ \Delta F_t \\ \Delta G_t \end{bmatrix} = \alpha\beta' \begin{bmatrix} X_{t-1} \\ F_{t-1} \\ G_{t-1} \end{bmatrix} + \begin{bmatrix} -\Gamma_1(L) & B_1(L) & B_2(L) \\ 0 & M_{11}^*(L) & M_{12}^*(L) \\ 0 & M_{21}^*(L) & M_{22}^*(L) \end{bmatrix} \begin{bmatrix} \Delta X_{t-1} \\ \Delta F_{t-1} \\ \Delta G_{t-1} \end{bmatrix} + \begin{bmatrix} v_t + [\Lambda, \Phi] Q [u_t', w_t']' \\ Q \begin{bmatrix} u_t \\ w_t \end{bmatrix} \end{bmatrix} \tag{24}$$

where $B_1(L) = \Lambda M_{11}^*(L) + \Phi M_{21}^*(L) + \Gamma_1(L)\Lambda$ and $B_2(L) = \Phi M_{22}^*(L) + \Lambda M_{12}^*(L) + \Gamma_1(L)\Phi$ and

$$\underset{N+r_1+r_2 \times N+r_2}{\alpha} = \begin{bmatrix} \tilde{\alpha} & \Phi\alpha_M \\ 0 & 0 \\ 0 & \alpha_M \end{bmatrix} \quad and \quad \underset{N+r_2 \times N+r_1+r_2}{\beta'} = \begin{bmatrix} I & -\Lambda & -\Phi \\ 0 & 0 & I \end{bmatrix}.$$

We can observe that (24) has a structure similar to a standard ECM model with some restrictions imposed. There are $N + r_1 + r_2$ variables driven by r_1 common stochastic trends and therefore there are $N + r_2$ cointegration relationships. The model conforms with the assumptions of the Johansen's version of the Granger representation theorem (Johansen, 1995). In particular

$$\beta_\perp = \left[\Lambda', I_{r_1}, 0_{r_1 \times r_2}\right]', \ \alpha_\perp = \begin{bmatrix} \mathbf{0}_{N \times r_1} \\ \mathbf{I}_{r_1} \\ \mathbf{0}_{r_2 \times r_1} \end{bmatrix}, \ \Xi = I_{N+r_1+r_2} - \begin{bmatrix} -\Gamma_1(1) & B_1(1) & B_2(1) \\ 0 & M_{11}^*(1) & M_{12}^*(1) \\ 0 & M_{21}^*(1) & M_{22}^*(1) \end{bmatrix}$$

and

$$\underset{r_1 \times r_1}{\omega} = \left(\alpha_\perp' \Xi \beta_\perp\right)^{-1} = [(I_{r_1} - M_{11}^*(1))]^{-1}$$

is a full rank matrix by the assumption that the data are at most I(1).[8] Then the generic moving-average representation by the Granger representation theorem can be written as

$$\begin{bmatrix} X_t \\ F_t \\ G_t \end{bmatrix} = C \sum_{i=1}^{t} u_i + C_1(L) \begin{bmatrix} v_t + [\Lambda, \Phi] Q [u_t', w_t']' \\ Q \begin{bmatrix} u_t \\ w_t \end{bmatrix} \end{bmatrix},$$

with

$$C = \beta_\perp \left(\alpha_\perp' \Xi \beta_\perp\right)^{-1},$$

which simplifies to (22). ■

[8]If X_{it} were I(2) processes, ω would be singular. We leave the I(2) case for future research.

3.2 Structural FECM

Our model contains I(1) and I(0) factors with corresponding dynamic factors innovations. From the moving-average representation (22) we can observe that the innovations in the first group have permanent effects on X_t, while the innovations in the second group have only transitory effects. The identification of structural dynamic factor innovations can be performed separately for each group of structural innovations or on both simultaneously. As is standard in SVAR analysis, we assume that structural dynamic factor innovations are linearly related to the reduced-form innovations

$$\varphi_t = \begin{bmatrix} \eta_t \\ \mu_t \end{bmatrix} = H \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{25}$$

where H is a full-rank $(r_1 + r_2) \times (r_1 + r_2)$ matrix. η_t are r_1 permanent structural dynamic factor innovations and μ_t are r_2 transitory structural dynamic factor innovations. It is assumed that $E\varphi_t \varphi_t' = I$ such that $H \Sigma_{u,w} H' = I$.

The moving average representation of the FECM in structural form can be obtained by inserting the two linear transformations above of reduced-form innovations to dynamic factors into the moving-average representation of the FECM given by (22).

3.3 Long-run restrictions

The three most common classes of identification restrictions in the SVAR literature are contemporaneous restrictions, long-run restrictions and sign restrictions.[9] In this paper we focus on long-run restrictions. Specifically, we extend the analysis of structural common stochastic trends of King, Plosser, Stock and Watson (1991) to the case of large nonstationary panels.[10]

The identification of structural innovations with long-run restrictions can be obtained by imposing restrictions on the matrices Λ and ω in the moving-average representation of the FECM (22). By doing this, we replace the long-run effects of reduced-form innovations to factors u_t, i.e.,

$$\Lambda \omega \sum_{i=1}^{t} u_i,$$

[9] The analysis of monetary policy shocks with the FECM using conteporaneous restrictions as in Bernanke et al. (2005) is provided by Banerjee, Marcellino and Masten (2013, 2014b). While there is broad coherence in terms of the basic shape of the impulse responses between the FECM and the FAVAR, the responses may quantitatively differ significantly due to the error-correction terms. The responses of the industrial production, the CPI and wages are very similar. Quite significant differences are observed for money and the yen-dollar exchange rate. The same is true for measures of private consumption.

[10] Barigozzi et al. (2014) also provide an empirical example of a long-run identification scheme in a large-scale modeling framework. However, given that their study does not consider cointegration between the factors and the observable variables, their identification scheme does not therefore consider long-run restrictions on the effects on observable variables.

with the long-run effects of structural innovations denoted η_t, i.e.,

$$\Lambda^* \omega^* \sum_{i=1}^{t} \eta_i,$$

where the matrices Λ^* and ω^* contain restrictions motivated by economic theory.

A common economically motivated identification scheme of permanent shocks, originally proposed by Blanchard and Quah (1989), uses the concept of long-run money neutrality. In this respect, their identification scheme distinguishes real from nominal shocks by imposing zero long-run effects of the nominal shock on real variables.

In a cointegration framework such an identification approach was formalized by King et al. (1991) (see also Warne, 1993). King et al. (1991) analyzed a six-dimensional system of cointegrated real and nominal variables. By imposing a particular cointegration rank, they determined the subset of innovations with permanent effects. Within this subset, they restricted the number of real stochastic trends to one, and identified it by imposing zero restrictions on real variables of all other permanent shocks in the subset. The remaining permanent shocks were allowed to have non-zero effects only on the subset of nominal variables in the cointegrated VAR. We extend the identification approach of King et al. (1991) to large-dimensional panels of non-stationary data using the FECM.

The FECM contains r_1 stochastic trends. Consider the case where $r_1 = 2$. We have two I(1) factors and want to identify one as a real stochastic and the second as a nominal stochastic trend. Accordingly, partition the variables in X_t such that N_1 real variables are ordered first and the remaining $N_2 = N - N_1$ nominal variables are ordered last. The group of real variables contains various measures of economic activity measured in levels, e.g. indexes of industrial production, which are treated as I(1). The identifying restrictions would thus be that the nominal stochastic trend has a zero long-run effect on these variables. Since nominal variables, for example, the levels of different price indexes and nominal wages, are grouped at the bottom of the panel, the restricted loading matrix Λ^* would have the following structure:

$$\Lambda^* = \begin{bmatrix} \Lambda^*_{11} & \mathbf{0} \\ \Lambda^*_{21} & \Lambda^*_{22} \end{bmatrix},$$

where Λ^*_{11} is $N_1 \times 1$ and Λ^*_{21} and Λ^*_{22} are $N_2 \times 1$. More generally, if the objective were to identify only the real stochastic trend with $r_1 > 2$, the dimension of Λ^*_{22} would be $N_2 \times (r_1 - 1)$.

The matrix Λ^* can be identified in the following way. First, the real stochastic trend is allowed to load on all observable variables. This implies that Λ^*_{11} and Λ^*_{21} can be identified as loadings to the first factor - F^r_t - extracted from the whole dataset. Second, we can estimate the residuals from a projection of X_t on F^r_t. Denote these as ε^r_t. Then Λ^*_{22} is identified as the loadings to the $(r_1 - 1)$ factors - denoted F^n_t - extracted from the lower

N_2-dimensional block of ε_t^r.

Note that block diagonality of Λ^* alone does not ensure that nominal shocks do not load to real variables, but we also need (block) diagonality of ω^*. Note also that it is the product $\Lambda^*\omega^*$ that determines the overall long-run effects, implying that zero long-run effect restrictions require $\Lambda^*\omega^*$ to be lower block diagonal, which is achieved by imposing lower (block) diagonality of ω^* in addition to lower (block) diagonality of Λ^*.

The matrix ω^* can be obtained from the estimates of the VAR model (20). Specifically, we can identify ω^* from the long-run covariance matrix

$$\omega E(u_t^F u_t^{F\prime})\omega' = \omega^* E(\eta_t \eta_t')\omega^{*\prime} = \omega^*\omega^{*\prime} \tag{26}$$

where $\eta_t = [\eta_t^{r\prime}, \eta_t^{n\prime}]'$ are the structural innovations and ω^* is lower block diagonal. Empirically, given the definition of ω, it can be replaced by its estimated counterpart, i.e.,

$$\widehat{\omega} = \left[\left(I_{r_1} - \widehat{M}_{11}^*(1)\right)\right]^{-1}.$$

4 Estimation of the FECM

4.1 Order of integration of idiosyncratic errors

For the representation theory, in general the FECM accommodates both I(0) and I(1) idiosyncratic errors. This can be seen from the FECM representation (23). In this form the stationary factors G_t enter the error-correction terms. To estimate the error-correction terms it is thus sufficient to estimate the space spanned by the true factors, which can be achieved under a general specification of the idiosyncratic components.

The idiosyncratic components ε_{it} are allowed to be serially and weakly cross correlated as in Bai (2004). Specifically, along the time series dimension, $\varepsilon_{it} = \gamma_i(L)\varepsilon_{it-1} + v_{it}$. If $\gamma_i(L)$ contains a unit root for some i, for those i, X_{it} and F_t do not cointegrate. Note that the factorization of $\Gamma(L)$ in (4) is also valid in the case it contains unit roots, and the potential presence of I(1) idiosyncratic errors ε_{it} can therefore be accommodated. Hence, the derivation of the FECM does not need the assumption of stationary idiosyncratic components.

The consequence of some of the ε_{it} being I(1) would be the $(I-\Gamma(1))$ matrix containing rows of zeros for all those variables with I(1) idiosyncratic components. The result is expected. A non-stationary idiosyncratic component implies no cointegration between the corresponding X_{it} and F_t. In such a case, there is also no corresponding error-correction mechanism in the equation for ΔX_{it} in (8) for those i whose $\gamma_i(L)$ contain a unit root. In such a case, consistent estimation of the factor space and the corresponding loading matrices can proceed as in Bai and Ng (2004). The remaining parameters of the FECM can be then estimated as discussed below.

For our structural analysis, some other considerations are relevant. Note that under a specification of ε_{it} as given in the previous paragraph, the number of I(1) factors r_1 can in principle be determined by the MQ statistics proposed by Bai and Ng (2004), but the estimation of the factor space that would split it into r_1-dimensional space of I(1) and $(r - r_1)-$dimensional space of I(0) factors is not feasible. Since differentiating between the I(1) and I(0) factors is crucial for the application of our long-run identification scheme of structural shocks, we pay central attention to the alternative assumption that the roots of $\gamma_i(L)$ are inside the unit disc for each i.[11] This assumption implies that X_{it} and F_t cointegrate for all i. It is very important to note that this does not however by any means imply that all bivariate pairs of variables X_{it} and X_{jt}, $j \neq i$, cointegrate mutually.[12]

Moreover, with F_t and G_t identified separately, we have $E\left\|\varepsilon_t^F\right\|^4 \leq M < \infty$, which implies that $1/T^2 \sum_{t=1}^T F_t F_t'$, $1/T \sum_{t=1}^T G_t G_t'$, and the cross-product matrices $1/T^{3/2} \sum_{t=1}^T F_t G_t'$ and $1/T^{3/2} \sum_{t=1}^T G_t' F_t$ converge. The elements of the matrix composed of these four elements jointly converge to form a positive definite matrix, allowing us to apply Bai's (2004) consistency results on factor estimation based on principal components.

In sum, there are several reasons, both theoretical and empirical, for working with the hypothesis of I(0) idiosyncratic errors. First, from an economic point of view, integrated errors are unlikely as they would imply that the integrated variables can drift apart in the long run, contrary to general equilibrium arguments. Integrated variables that drift apart are likely to be of marginal importance, and as such they do not contain essential information and can be dropped from the analysis. Second, the basic aim of the paper is to model cointegration in large datasets and to develop long-run identification schemes in this context, see Section 3 above. The proposed scheme requires to estimate the space spanned by the I(1) factors, and this can be consistently done only under stationarity of ε_{it} and application of the principal component estimator to the data in levels (Bai, 2004).

Eventually, whether the idiosyncratic errors ε_{it} are stationary or not is an empirical issue. Below we present two empirical applications. The first one uses a monthly US dataset for the period 1959 - 2003 taken from Bernanke et al. (2005). The authors treat 77 series as I(1). By applying the ADF unit root test to the estimated idiosyncratic components (see details on estimation below), the unit-root null is rejected at 5% significance level for most of the series and at 10% for the remaining few. The second dataset is composed of the Euro area quarterly variables used in Fagan et al. (2001), updated to cover the period 1975 - 2013. It contains 32 I(1) series. The same procedure as in the case of the US dataset results in rejections of the unit-root null at 5% significance level for all 32 idiosyncratic

[11] Different identification schemes that do not rely on distinguishing between I(1) and I(0) factors, such as restrictions on contemporaneous effect of structural shocks, can be analyzed in the FECM without assuming I(0) idiosyncratic errors.

[12] The assertion that stationary idiosyncratic components for all i implies bivariate cointegration between all X_{it}, X_{jt} pairs - limiting the applicability of the Bai (2004) framework - is made by Barigozzi et al. (2014). However their illustration only applies to the case of one integrated factor and does not generalize in the absence of implausible restrictions.

components. The panel unit root test (Bai and Ng, 2004) applied to our datasets also reject the null of no panel cointegration between X_{it} and F_t for both datasets.

Overall, it appears that our assumption of stationary idiosyncratic errors fits the properties of representative macroeconomic datasets. Moreover, it is not restrictive for the derivation of the FECM. As mentioned, the assumption is only required to consistently estimate the I(1) factors (and the corresponding number r_1) as required by the identification scheme of structural shocks discussed in this paper. In other applications of the FECM, including forecasting applications, the assumption of stationary idiosyncratic errors would not be necessary.

4.2 Estimation of the FECM with stationary idiosyncratic components

With stationary idiosyncratic component the FECM model is consistent with the specification of the dynamic factor model analyzed by Bai (2004) that accommodates the presence of I(0) factors along with I(1) factors in the factor model. Our assumptions are consistent with Bai's (2004) and we can therefore rely on Bai's (2004) results on the asymptotic properties of the principal component based factor (and loadings) estimators.

Specifically, the space spanned by the factors can be consistently estimated using principal components. The estimators of F_t are the eigenvectors corresponding to the largest r_1 eigenvalues of XX' normalized such that $\widetilde{F}'\widetilde{F}/T^2 = I$. The stationary factors G_t can be estimated as the eigenvectors corresponding to the next q largest eigenvalues normalized such that $\widetilde{G}'\widetilde{G}/T = I$ (Bai, 2004). Corresponding estimators of the loadings to I(1) factors are then $\widetilde{\Lambda} = X'\widetilde{F}/T^2$, and those to the I(0) factors $\widetilde{\Phi} = X'\widetilde{G}/T$.[13]

Using the estimated factors and loadings, the estimates of the common components are $\widetilde{\Lambda}\widetilde{F}_t$, $\widetilde{\Phi}\widetilde{G}_t$, $\widetilde{\Lambda}\Delta\widetilde{F}_t$ and $\widetilde{\Phi}\Delta\widetilde{G}_t$, while for the cointegration relations it is $X_{t-1} - \widetilde{\Lambda}\widetilde{F}_{t-1}$. Replacing the true factors and their loadings with their estimated counterparts is permitted under the assumptions discussed above and in Bai (2004) (see Bai (2004) Lemmas 2 and 3) so that we do not have a generated regressor problem.[14]

The estimated common components and cointegrating relations can be then used in (8) to estimate the remaining parameters of the FECM by OLS.

Finally, the number of I(1) factors r_1 can be consistently estimated using the criteria developed by Bai (2004) applied to data in levels. The overall number of static factors $r_1(p+1) + r_2(m+1)$ can be estimated using the criteria of Bai and Ng (2002) applied to the data in differences.

[13] In a model similar to ours, Choi (2011) analyzes the generalized principal components estimator that offers some efficiency gains over the classic principal components estimator. Simulation evidence presented below, however, shows that Bai's estimator performs very well even with small sample sizes. For this reason we stick to the standard principal components estimator in this paper.

[14] These assumptions are essentially (1) the common factor structure of the data, (2) heterogeneous loadings with finite fourth moments, (3) mutual orthogonality between u_t, w_t, ε_{it}, λ_{it} and ϕ_{it},, (4) weak dependence of idiosyncratic errors, and (5) N large compared with T for the $I(0)$ factors ($\sqrt{T}/N \to 0$).

4.3 Strict factor model

The FECM specification we have considered so far is heavily parameterized, leading to the curse of dimensionality. To render the estimation empirically feasible, we need to further restrict the idiosyncratic component ε_{it}. In particular, we assume (1) to be a strict factor model: $E(\varepsilon_{it}, \varepsilon_{js}) = 0$ for all i, j, t and s, $i \neq j$. This assumption enables the empirical estimation of the FECM parameters equation by equation by OLS.

The strict factor model assumption is clearly less realistic than stationarity of the idiosyncratic components alone and may be empirically rejected. This implies that we are potentially omitting lags of X_{jt} from the equations for variables X_{it}.[15] In this respect it is important to note that X_{jt} has a factor structure. Consequently, the effects of lags of X_{jt} on X_{it} can be efficiently approximated by including lags of F_t and G_t into the X_{it} equations. This is the approach that we follow in our empirical applicatons. Moreover, the same problem applies also to FAVAR models analyzed, for example, in Stock and Watson (2005) or Lütkepohl (2014). These models, however, relative to the FECM, additionally omit the error-correction terms from the X_{it} equations.

The consequences of assuming a strict dynamic factor model structure are analyzed by means of a simulation experiment, whose main findings we summarize in the next subsection.

4.4 Finite sample properties

Small sample properties of the estimation procedure are analyzed by means of simulation experiments whose details are presented in Appendix B. With these simulation experiments we assess the general small sample properties in presence of I(0) factors and I(0) variables and the effects of an incorrect strict dynamic factor model assumption. In either case, we analyze the small-sample properties of estimated factors and impulse responses generated by the FECM model.

Our simulation experiments indicate that principal component based estimators can recover very well the factor space spanned by a mixture of I(1) and I(0) factors even for N and T less than 50. Moreover, using the estimated factors in the factor VAR replicates accurately the true impulse responses of factors to factor innovations. Finally, inserting the estimated factor responses in the FECM, in combination with the FECM parameters estimated as discussed above, delivers estimated structural impulse responses very close to the true ones.

As for the assumption of strict DFM, we considered as the data-generating process an approximate factor model with cross-correlation structure modelled as in the simulation experiments of Stock and Watson (2002) (see Appendix B for details of the DGP). On generated data we estimated the FECM as discussed above, i.e. by incorrectly assuming a

[15] On a monthly US dataset, similar to the one we use below, Stock and Watson (2005) show that the strict factor model assumption is generally rejected but is of limited quantitative importance.

strict factor model structure. Our simulation results reveal only a marginal deterioration of finite sample properties of the estimated impulse responses relative to the case where the DGP is a strict DFM. Based on this, we conclude that the proposed estimation procedure of the FECM is valid in empirical applications.

5 An evaluation of the effects of the error-correction terms on impulse response analysis

In this section we analyze the effects of omitting the error-correction terms on impulse response analysis by means of simulation experiments, focusing on the role of the strength of error correction and of the sample size, along both the time series and cross section dimensions. In the design of the data-generating process we draw from the empirical analysis of real stochastic trends that is presented in detail in the next section. The estimated responses to a permanent real shock reveal some significant differences between the FECM and the FAVAR. Given that the two models are set up such that the only difference between the two is the presence of the error-correction terms, the simulation evidence presented in this section also facilitates the discussion of the empirically observed differences.

The experiment is designed as follows. We estimate model (23) for the subset of I(1) variables in the US data panel and use the estimated parameters as DGP. The only exceptions are the loading coefficients of the cointegration relations, α. These are drawn from a uniform distribution around mean values as specified below, in order to assess the effects of a different error correction strength. The idiosyncratic components of the data are treated as serially independent and bootstrapped from empirical residuals. The data are driven by factors simulated with the parameters from the estimated factor VAR, combined with bootstrapped factor VAR residuals.

Identification of the real trend requires a division between real and nominal variables in the panel. Our panel contains 55% of real variables and 45% of nominal variables. This relative share is also preserved in the artificially generated data, i.e. out of N generated variables, 55% have parameters that are randomly drawn from the parameters pertaining to real variables. The rest are randomly drawn from the parameters of the subset of nominal variables.

We consider five different parameter configurations. The benchmark sample setup is with $T = 500$ and $N = 100$, which corresponds to the dataset from which the parameters used in the DGP are estimated. The mean value of the error-correction coefficient α is set to -0.50.

We consider three deviations from this parameter setup. The first is the variation in the strength of error correction, with mean α set to -0.25. The remaining two modifications alter the sample size. First, we halve the time series dimension to 250, and second we halve the cross-section dimension to 50.

For each parameter set we take 100 random draws of the parameter set and factor process. Within each of these random draws, the confidence intervals of the impulse responses are estimated through 100 bootstrap replications. The confidence intervals are used to measure the differences between the estimated impulse responses computed with the FAVAR model and those with the FECM. The results of the Monte Carlo experiment are presented in Table 1.

Table 1: Importance of the error-correction term - results of the Monte Carlo experiment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	-0.50		-0.25		-0.50		-0.50	
T	500		500		250		500	
N	100		100		100		50	
	% of FAVAR responses outside the FECM conf. intervals							
	Confidence interval coverage (%)							
Horizon	67	90	67	90	67	90	67	90
3	57.44	39.22	51.28	31.34	48.29	28.40	60.12	40.32
6	53.25	34.47	53.03	33.82	43.31	24.13	53.56	35.24
12	42.98	24.03	45.38	27.09	31.42	15.21	39.38	20.76
18	43.14	25.47	42.86	24.10	29.22	13.45	38.24	20.34
24	39.69	22.04	40.31	22.07	28.22	12.85	34.68	17.60
36	35.21	17.20	37.60	20.04	28.43	12.87	30.98	14.52
48	35.26	16.99	37.74	19.53	29.29	14.03	31.14	14.96
60	35.99	18.27	40.44	21.55	28.63	13.58	34.52	16.72
any	89.12	67.74	86.37	62.84	80.44	54.59	88.80	65.70

The simulation results show that in the basic specification of our empirically-motivated data generating process the effects of omitting the error-correction term are sizeable. At short horizons over 50% of FAVAR responses lie outside the FECM 67% confidence intervals. The share decreases with the horizon, but stabilises above 35%. The respective figures for 90% confidence interval are 30% and 17%. The bottom line of the table tells us that almost 90% of FAVAR impulse responses lie outside 67% FECM confidence interval for at least one period at any horizon. For the 90% confidence interval the share is roughly two thirds. Across the modifications that we consider to the basic data-generating process the shares vary, but they still remain of similar magnitude.[16]

The effect of the strength of the error-correction can be evaluated by comparing the benchmark parameter specification in columns 1 and 2 to columns 3 and 4 that report the simulation results with weaker degree of error correction. The occurrence of significant differences in the estimated impulse responses is smaller at shorter horizons and somehow higher at longer horizons, but the differences are more pronounced at shorter horizons. The share of the FAVAR impulse responses that are different for at least one period at any horizon decreases with weaker error-correction. These results suggest that the effect of omitted error-correction mechanism on impulse response analysis is positively related to the strength of the error-correction on average, but the fact that this is not uniform

[16] The FECM in the simulation experiment contains 3 endogenous lags (uniform across equations), while the factors enter contemporaneously and with one lag. We repeated the same experiment also with one and three of both endogenous lags and lags of factors. The results, available upon request, are robust and fully in line with those presented in Table 1.

across the horizon suggests that the effect remains important also with relatively weak error-correction mechanism.

The effect of a smaller time series dimension of the panel is not uniform across the time elapsed after the shocks. Within the first 12 periods, the differences are less frequent. At longer horizons, however, the frequency increases.

The effect of the cross-section dimension is opposite to what we observe for the effect of the time series dimension. With fewer series in the panel, obtaining statistically different impulse responses between the FAVAR and the FECM becomes slightly more probable at short horizons (below 6) and less probable at longer horizons. This is again an indication that the error-correction mechanism might be empirically important for impulse response analysis even at moderate sample sizes.

Overall, this simulation experiment confirms the relevance of the inclusion of error correction terms in FAVAR models, suggesting that their omission can have sizeable effects, also in rather small panels and with error-correction mechanisms of moderate strength. Additional simulation experiments reported in Appendix B instead provide support for a good finite sample performance of the FECM based estimated impulse responses.

6 Empirical applications

In this section we illustrate the identification of permanent productivity shocks and their effects in the context of two empirical applications. In both we focus on the empirical importance of the error-correction mechanism for the analysis of structural shocks.

The first application uses the dataset of Bernanke et al. (2005). It contains 120 variables for the US, spanning the period 1959 - 2003 at monthly frequency. 77 variables are treated by the authors as I(1) (see data description in Appendix 1 of Bernanke et al., 2005).

The second application is to the Euro area (EA) and is based on quarterly data for the period 1975 - 2013, an updated version of the dataset used by Fagan et al. (2001). It contains 38 macroeconomic series, of which 32 are I(1).[17] Data are seasonally adjusted at source. The only exception is the consumer price index, which we seasonally adjust using the X-11 procedure.

The Bai (2004) IPC2 information criterion indicates $r_1 = 2$ for both the US and EA datasets. The choice of the total number of estimated factors r is based on Bai and Ng (2004). Their PC3 criterion indicates 4 factors in total for the EA dataset. For the US dataset none of the Bai and Ng (2004) criteria gives inconclusive evidence. For comparability with the EA dataset and our previous analysis with US data in Banerjee, Marcellino and Masten (2014a), we set also the total number of factors for the US dataset to 4.

[17]The data and the corresponding list of variables can be downloaded from the Euro area business cycle network webpage (www.eabcn.org/area-wide-model).

Both datasets therefore contain both I(1) and I(0) variables, which we model in the following way. Denote by X_{it}^1 the I(1) variables and by X_{it}^2 the I(0) variables. Naturally, the issue of cointegration applies only to X_{it}^1. As a consequence, the I(1) factors load only to X_{it}^1 and not to X_{it}^2. In other words, the fact that X_{it}^2 are assumed to be I(0) implies $\Lambda_i^2 = 0$, which is a restriction that we take into account in model estimation.

Our empirical FECM is then

$$\Delta X_{it}^1 = \alpha_i (X_{it-1}^1 - \Lambda_i F_{t-1}) + \Lambda_i^1(L)\Delta F_t + \Phi_i^1(L) G_t + \Gamma^1(L)\Delta X_{it-1}^1 + v_{it}^1 \quad (27)$$

$$X_{it}^2 = \Phi_i^2(L) G_t + \Gamma^2(L)\Delta X_{it-1}^2 + v_{it}^2 \quad (28)$$

The model for the I(1) variables in (27) is the FECM, while the model for the I(0) variables in (28) is a standard FAVAR with the restrictions that I(1) factors do not load onto I(0) variables.

The FAVAR model is as follows:

$$\Delta X_{it}^1 = \Lambda_i^1(L)\Delta F_t + \Phi_i^1(L) G_t + v_{it}^1 \quad (29)$$

$$X_{it}^2 = \Lambda_i^2(L)\Delta F_t + \Phi_i^2(L) G_t + v_{it}^2 \quad (30)$$

(29) differs from (27) in that it does not include the error-correction term. (30) differs from (28) by not taking into account the restriction $\Lambda_i^2 = 0$.

The lag structure of the models is the following. Both the FAVAR model and the FECM contain three endogenous lags, while the factors enter contemporaneously and with one additional lag. This additional lag of factors serves to proxy for potentially omitted lags of X_j variables in equations for X_i, $i \neq j$. Robustness of the results has been checked by varying the number of endogenous lags from 1 to 6, and lags of factors from 0 to 3. Results turn out to be robust and are available upon request.

The lag structure of the FECM equations is common for the US and EA datsets. The specification differs for the factor VAR. For the US data we follow Bernanke et al. (2005) and Stock and Watson (2005) and set the number of lags to 13. For the EA data, which is on quarterly frequency, we set the number of lags to 6.

To provide *prima facie* evidence of the importance of the error-correction terms in (27) we tested their significance with a standard t-test equation by equation. In the US dataset 63 out 77 equations have a statistically significant α_i at the 5% significance level. The average partial R^2 of these terms is 2.8%, while the maximum reaches 23.4%. In the EA dataset 27 out of 32 I(1) variables have a statistically significant α_i at the 5% significance level. The average partial R^2 is 1.6%, while the maximum reaches 8.2%. These figures confirm the importance of including the error-correction term in modelling variables that are originally I(1), but are modelled in differences in FAVAR applications. The average size of the partial R^2 implies a limited partial contribution of the error-correction term to the goodness of fit of the estimated equations. However, even in such circumstances omitting the error-correction terms could lead to significant distortions in estimated impulse responses.

The space spanned by F_t and G_t is estimated by the principal components on the data in levels (Bai, 2004). Our simulations reported in Appendix B give us confidence that this space is estimated consistently. Our assumption of cointegration between X_{it} and F_t is valid if the ε_{it} series is stationary. The panel unit root test (Bai and Ng, 2004) applied to our datasets rejects the null of no panel cointegration between X_{it} and F_t (see also results reported in penultimate paragraph of Section 4.1).

6.1 Results for the US

We first present the analyisis of structural permanent productivity shocks based on US data. The impulse responses to an identified permanent real shock are presented in Figure 1. The top left panel contains the responses of the real permanent trend (factor), the remaining variables are those for which Bernanke et al.(2005) report results.[18] Each subplot contains the impulse responses obtained with the FECM (solid line) and the FAVAR (dashed line) together with 90% bootstrapped confidence intervals of the FECM impulse responses.[19]

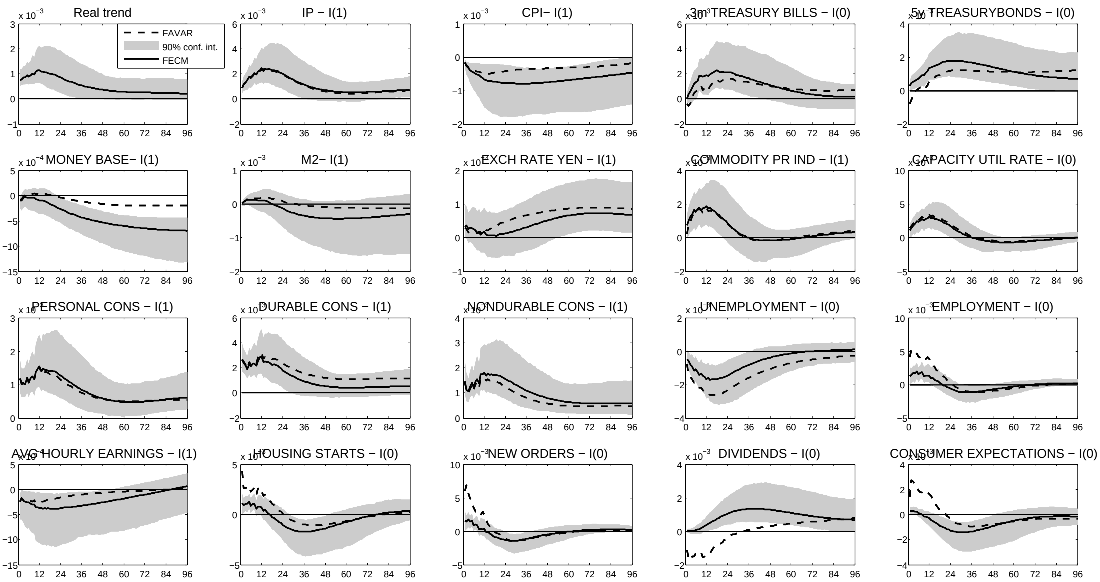
The impulse responses are broadly in line with economic theory. Along the adjustment path the real factor exhibits a hump-shaped response and after three years it levels off at the new higher steady state. Similar in shape are the positive responses of industrial production and measures of real private consumption. Prices decrease, but only temporarily. This effect is considerably larger in the FECM than in the FAVAR. The feature is exhibited also for other prices in the panel, but the corresponding impulse responses are not presented in Figure 1. Interest rates gradually increase, reflecting an increase in real rates associate with an increase in productivity. Moreover, while the short rate returns to equilibrium, the effect on the 5-year rate is positive, which implies a slightly steeper yield curve. The responses of money related variables are negative and again considerably more so for the FECM. Consistently with higher interest rates, the dollar appreciates. Employment temporarily increases along the adjustment path, while unemployment decreases. The initial response of the average wage rate is negative and turns positive only gradually and with a significant lag. Such a response is an indication of a skill-biased technological change. Consistent with theory are also the responses of housing starts, orders and dividends.

Figure 1 does not give a full account of the effects of the error-correction term on impulse responses. The differences in the responses between the FAVAR and the FECM for

[18]Impulse responses for the remaining 101 variables of the panel are available upon request.

[19]Concerning the estimation of the FAVAR, it is worth mentioning that in the present application, which serves to illustrate the method, we do not consider the potential dynamic singularity in the variance-covariance matrix of stationary factors G_t. A more general treatment is at present beyond the scope of this paper.

Figure 1: Impulse responses to real stochastic trend in the US- FAVAR Vs FECM



I(0) variables in the figure are not due to the error-correction term, but due to restrictions the FECM contains (see (28) and (30)). Full account of the empirical effect of the error-correction term is given in Table 2. It reports the percentage of variables (out of 77 I(1) variables in the panel) for which the impulse response obtained with the FAVAR model lie outside the confidence interval of the FECM impulse responses at different horizons. In addition to the results for all 77 variables, we also group the variables according to their economic meaning.

Taking into account all 77 I(1) variables, we observe that within the first 6 months after the shock only a limited number of impulse responses differ significantly. At the 12-month horizon roughly a third of impulse responses differ at 67% confidence level, and 11% at 90%. For the three-year horizon, these shares increase to 44% and 30% respectively and remain stable at longer horizons.

Looking across categories of variables, we can group the variables in three groups according to the size of the effect of the error-correction terms. The strongest effect is observed for money aggregates, prices and wages. For theses categories the share of different impulse responses can exceed 50% according to the 90% confidence interval of the FECM responses. In the second group we have private consumption and orders, for which the shares of different responses exceed 50% if we consider narrower, 67% confidence intervals. For output, exchange rates and stock prices we observe that neglecting cointegration between variables and factors has only a limited effect on the impulse responses analysis.

Finally, when in Section 4.3 we discussed the implications of the strict factor model

Table 2: Percentage of FAVAR responses outside the FECM confidence intervals

Variables		Horizon									
	CI coverage	3	6	12	24	36	48	60	72	84	96
All	67	0.0	14.3	31.2	41.6	44.2	44.2	41.6	44.2	45.5	45.5
	90	0.0	0.0	11.7	27.3	29.9	28.6	31.2	31.2	24.7	27.3
Output	67	0.0	0.0	11.1	16.7	16.7	16.7	16.7	27.8	27.8	27.8
	90	0.0	0.0	0.0	11.1	11.1	11.1	11.1	11.1	16.7	27.8
Employment	67	0.0	0.0	5.9	17.6	17.6	17.6	11.8	17.6	17.6	11.8
	90	0.0	0.0	0.0	11.8	11.8	5.9	5.9	5.9	5.9	11.8
Consumption	67	0.0	0.0	40.0	60.0	60.0	60.0	60.0	60.0	60.0	80.0
	90	0.0	0.0	0.0	20.0	20.0	0.0	20.0	20.0	20.0	20.0
Orders	67	0.0	0.0	50.0	50.0	50.0	50.0	100.0	100.0	100.0	100.0
	90	0.0	0.0	0.0	50.0	0.0	0.0	0.0	0.0	0.0	0.0
Exchange Rates	67	0.0	0.0	25.0	25.0	25.0	25.0	25.0	25.0	50.0	50.0
	90	0.0	0.0	0.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
Stock Prices	67	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Money	67	0.0	11.1	22.2	44.4	66.7	66.7	66.7	55.6	55.6	66.7
	90	0.0	0.0	0.0	22.2	33.3	44.4	55.6	55.6	55.6	55.6
Wages	67	0.0	50.0	100.0	100.0	100.0	100.0	50.0	50.0	50.0	50.0
	90	0.0	0.0	50.0	50.0	50.0	50.0	50.0	50.0	0.0	0.0
Prices	67	0.0	60.0	86.7	100.0	100.0	100.0	93.3	93.3	93.3	86.7
	90	0.0	0.0	53.3	73.3	86.7	86.7	86.7	86.7	53.3	46.7

assumption for estimation, we noted that the omission of cross-equation terms can be proxied by the inclusion of lagged factors. Hence, to assess the robustness of our results, we have considered alternative specifications of the lag structure of the FECM and the FAVAR, with combinations of zero and three lags of factors. Specifications with more than three lags of factors were not considered in order to avoid overfitting.[20] Results obtained with the alternative lag structures (available upon request) show a great degree of similarity to the results presented in Table 2 and Figure 1.

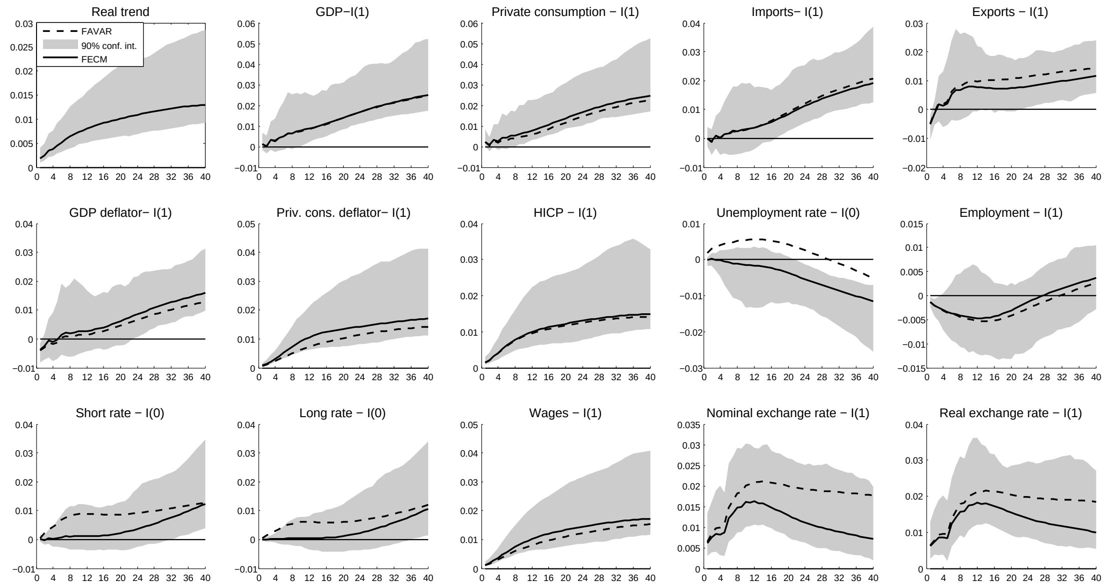
6.2 Results for the euro area

The stochastic trend analysis for the euro area variables is summarized in Figure 2. In the top left corner we see that a shock to the real stochastic trend leads to its permanent increase and levels off at the new equilibrium level after 7 years. Unlike the US, its response is not hump-shaped. Key measures of output and private economic activity - GDP, private consumption, imports and exports - respond positively and are significantly different from zero at new equilibrium levels. The impulse responses obtained for output and consumption variables with the FAVAR are quite similar, which is in line with responses for the US. Prices also respond positively. The unemployment rate gradually declines. Employment initially declines and improves only with a significant lag. The responses of interest rates are positive, but with a significant lag, which is not present in the FAVAR responses. Both the nominal and the real exchange rate appreciate.

The differences in responses between the FECM and the FAVAR model are less pronounced than in the case of the US model. Such a result is fully expected given the outcome of our simulation experiments in Section 5. The Euro area dataset is considerably smaller

[20]Because the model contains four factors, including up to three lags in addition to contemporaneous terms implies sixteen terms with factors in each equation.

Figure 2: Impulse responses to real stochastic trend in the Euro area - FAVAR Vs FECM



than the US dataset, especially in the cross sectional dimension. The simulation results show that in such a case the differences between the two models become more difficult to detect.

To facilitate a structural interpretation of the identified real stochastic trend, we compare our impulse responses with the impulse responses reported by Adolfson et al. (2007) for an estimated DSGE model of the Euro area. The model of Adolfson et al. (2007) contains a stochastic productivity trend, which allows them to estimate the model on raw, non detrended data. Their impulse responses to a positive and permanent productivity shock, reproduced in Appendix C, share a great degree of similarity with our impulse responses. The signs of responses are matched for most of the variables we report in Figure 2. Measures of economic activity respond positively, as do prices and interest rates, wages increase and, finally, the real exchange rate appreciates. The only notable difference between our and their application is the response of the labor market. While a negative response of unemployment is fully consistent with theory, the initial decline and only gradual recovery of employment is at odds with the theoretical model. We observe also some differences in terms of the shape of the responses, which are more delayed and persistent in our case. In other words, our impulse responses level off at new equilibrium levels more gradually than in the DSGE model of Adolfson et al. (2007).

The stochastic trend response for the US case is different in its basic shape, namely, hump-shaped, but conditional on this feature, the adjusting dynamics of other variables are very comparable.The only notable difference in the US case is a temporary negative response of prices and wages. For the remaining variables, real output, private consumption, interest rates and the real exhange rate, the responses are consistent with the DSGE evidence. Such direct comparability of basic shapes of the responses allows us to interpret the stochastic real trend identified with our approach as the structural stochastic productivity trend.

7 Conclusions

In this paper we analyse the implications of cointegration for structural FAVAR models. Starting from a dynamic factor model for non-stationary data, we derive the factor-augmented error-correction model (FECM), its moving-average representation, and discuss estimation of the model parameters and of the impulse response functions, relying on the asymptotic theory developed in Bai (2004).

Our simulation experiments indicate that principal component based estimators (with a mixture of I(1) and I(0) factors) can recover very well the factor space. Moreover, using the estimated factors in the factor VAR replicates accurately the true factor responses. Finally, inserting the estimated factor responses in the FECM, in combination with the estimated FECM parameters delivers estimated structural impulse responses very close to the true ones.

Structural analysis in the FECM can be conducted as in structural VARs. We provide the first analysis of long-run restrictions to identify a permanent productivity shock in the context of large cointegrated panels. Accounting for cointegration has important effects on the impulse responses to this shock as it reveals significant differences between the FECM and the FAVAR. Moreover, the FECM generates responses broadly in line with the theoretical DSGE analysis of, e.g., Adolfson et al. (2007).

The relevance of the error correction terms to avoid biases in FAVAR responses to shocks are also confirmed by means of simulations experiments. Simulation results show that the differences between the impulse response functions obtained by the FECM and the FAVAR are on average more pronounced the higher is the strength of the error-correction and the higher are the cross-section and the time series dimensions of the panel. Moreover, the differences in impulse responses are frequent also in samples of moderate size and with moderate strength of the error-correction mechanism.

Overall, these results suggest that the FECM that exploits the information in the levels of nonstationary variables to explicitly model cointegration provides an empirically important extension of classical FAVAR models for structural modelling. Other identification schemes such as sign restrictions could be also adopted in a FECM context. A detailed analysis of these is beyond the scope of this paper but provides an interesting topic for further research.

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Appendix A: Impulse response analysis in the FECM and FAVAR - an analytical illustration

We illustrate analytically the computation of structural responses using the FECM rather than the FAVAR with a simple but comprehensive example. The example may easily be seen to be a special case of the general specification introduced in the main text, obtained by restricting the dimension of the factor space and of the variables of interest studied.

We suppose that the large information set available can be summarized by one $I(1)$ common factor, f, and that the econometrician is particularly interested in the response of one of the many variables, x_1, and that she can choose any of the three following models. First, a FECM, where the explanatory variables of the FAVAR are augmented with a term representing the (lagged) deviation from the long run equilibrium of x_1 and f. Second, a FAVAR model where the change in x_1 (Δx_1) is explained by an infinite number of its own lags and by lags of the change in f. And, third, the same model but with a finite number of lags. We want to compare the differences in IRFs resulting from the three models.

To start with, let us consider a system consisting of the two variables x_1 and x_2 and of one factor f. The factor follows a random walk process,

$$f_t = f_{t-1} + \varepsilon_t, \tag{31}$$

where ε_t is a structural shock and we are interested in the dynamic response to this shock. The factor loads directly on x_2,

$$x_{2t} = f_t + u_t, \tag{32}$$

while the process for x_1 is given in ECM form as

$$\Delta x_{1t} = \alpha \left(x_{1t-1} - \beta f_{t-1} \right) + \gamma \Delta f_{t-1} + v_t, \quad \alpha < 0. \tag{33}$$

or

$$\Delta x_{1t} = \alpha \left(x_{1t-1} - \beta f_{t-1} \right) + \gamma \varepsilon_{t-1} + v_t. \quad \alpha < 0 \tag{34}$$

Here the processes ε_t and v_t are assumed $i.i.d.(0, I_N)$, while u_t is allowed to have a moving average structure, i.e. $u_t = u_t^* / \left(1 - \eta L\right)$, $|\eta| < 1$ and u_t^* is $i.i.d.(0, \sigma_{u^*}^2)$ Hence, the DGP is a FECM.

Note that the moving-average representation of x_{1t} can be written as

$$\begin{aligned}
x_{1t} &= (1+\alpha)^h x_{1t-h} \\
&+ (1+\alpha)^{h-1}(-\alpha\beta(\varepsilon_{t-h}+\varepsilon_{t-h-1}+...+\varepsilon_{-h}) + +\gamma\varepsilon_{t-h} + v_{t-h+1}) \\
&+ (1+\alpha)^{h-2}(-\alpha\beta(\varepsilon_{t-h+1}+\varepsilon_{t-h}+...+\varepsilon_{-h+1}) + \gamma\varepsilon_{t-h+1} + v_{t-h+2}) \\
&\vdots \\
&- (\alpha\beta(\varepsilon_{t-1}+\varepsilon_{t-2}+...+\varepsilon_{1}) + \gamma\varepsilon_{t-1} + v_t.
\end{aligned}$$

Based on this, the impulse response function takes the following form:

$$\frac{\partial\Delta x_{1t+h}}{\partial\varepsilon_t} = \frac{\partial x_{1t+h}}{\partial\varepsilon_t} - \frac{\partial x_{1t+h-1}}{\partial\varepsilon_t} = -(1+\alpha)^{h-1}\alpha\beta + \alpha(1+\alpha)^{h-2}\gamma.$$

The FECM representation of x_1 can also be written as a FAVAR. In fact, since the error-correction term $x_{1t} - \beta f_t$ evolves as

$$\begin{aligned}
x_{1t} - \beta f_t &= (\alpha+1)(x_{1t-1} - \beta f_{t-1}) + \gamma\Delta f_{t-1} + v_t - \beta\varepsilon_t \\
&= \frac{\gamma\Delta f_{t-1}}{1-(\alpha+1)L} + \frac{v_t - \beta\varepsilon_t}{1-(\alpha+1)L},
\end{aligned}$$

we can re-write equation (33) as

$$\Delta x_{1t} = \gamma\Delta f_{t-1} + \frac{\alpha\gamma\Delta f_{t-2}}{1-(\alpha+1)L} + v_t + \frac{\alpha(v_{t-1} - \beta\varepsilon_{t-1})}{1-(\alpha+1)L}, \tag{35}$$

which is a FAVAR of infinite order. The corresponding moving-average representation then follows directly as

$$\Delta x_{1t} = \gamma\varepsilon_{t-1} + \frac{\alpha\gamma\varepsilon_{t-2}}{1-(\alpha+1)L} + v_t + \frac{\alpha(v_{t-1} - \beta\varepsilon_{t-1})}{1-(\alpha+1)L}. \tag{36}$$

This implies that the impulse responses of the infinite-order FAVAR model would be

$$\frac{\partial\Delta x_{1t+h}}{\partial\varepsilon_t} = -(1+\alpha)^{h-1}\alpha\beta + \alpha(1+\alpha)^{h-2}\gamma.$$

We therefore see that only using a FAVAR with an infinite number of lags allows us to recover the same IRFs as in the FECM. However, in practice, a short lag length is used in the FAVAR, so that the resulting responses will be different from those from the FECM, the more so the poorer the finite lag approximation is to the infinite order FAVAR.

A simulation experiment whose design is based on a frequently-used panel of US macroeconomic data, presented in Table 1 in Section 5, reveals that the differences in the impulse responses obtained by the FECM and the (finite order) FAVAR can be substantial.

8 Appendix B: Finite sample properties of the FECM estimators

With the simulation experiments reported here we address three questions related to the finite sample properties of the FECM estimators. First, we investigate whether the principal component based estimator efficiently estimates the space spanned by both the I(1) and I(0) factors. The second issue is concerned with retrieving the impulse responses to innovations to dynamic factors conditional on sample size. The third issue is related to estimation of the FECM under the strict DFM assumption when the data generating process is an approximate DFM.

The exact theoretical structure of (17) is rather specific. Given that the factors estimated by principal components are only a rotation of the true factors, fitting a VAR to them will not retrieve the theoretical structure given by (17) directly. This is however unnecessary, and with the simulation experiment we address two questions which enable us to attack the issue of consistency indirectly but completely. The first is how precisely PCA retrieves the space spanned by the factors in finite samples. Bai (2004) provides simulation evidence for the case with I(1) factors only and shows that the method works well also for relatively small panels. Our setting explicitly allows for both I(1) and I(0) factors and verifies the Bai simulation results in this more general scenario. Second, we test whether the impulse responses obtained from the VAR based on the estimated factors correspond to the true impulse responses obtained with the true model (17) and (8).

The design of the Monte Carlo experiment is the following. The factors are generated by a VAR such as (16) with one I(1) and one I(0) factor and two lags of each factor. The sum of the autoregressive coefficients for the I(0) factors is set to 0.7. The two factors are independent, i.e. the VAR coefficients matrices are diagonal and u_t and w_t are independent $N(0,1)$ processes. F_t and c_t enter (1) contemporaneously and with one lag, i.e. $p = m = 1$. The loadings λ_{ij}, ϕ_{ij}, $j = 0, 1$, are drawn from a standard normal distribution. Finally, the idiosyncratic component is serially correlated. This is modelled by setting the order of $\gamma_i(L)$ to two and drawing the values of γ_{i1} and γ_{i2} from $N(0.4, 0.01)$ and $N(0.2, 0.01)$ respectively.[21]

The factors are estimated from the generated Xs in levels by principal components, imposing the true number of factors. It follows from the representation of the FECM that there is one I(1) factor - F_t, and three I(0) factors - ΔF_t, c_t and c_{t-1}.

To check whether the principal components retrieve the space spanned by the factors we follow Bai (2004) and estimate the following projection

$$\begin{bmatrix} F_t^0 \\ c_t^0 \end{bmatrix} = \delta \begin{bmatrix} \hat{F}_t \\ \hat{c}_t \end{bmatrix} + v_t$$

[21]We conducted also robustness checks by varying the persistence in the idiosyncratic components. Results, available from the authors upon request, exhibit high degree of robustness.

where F_t^0, c_t^0 denote true factors and $\hat{F}_t, \hat{c}_t$ the estimated factors. We then rotate the estimated factors towards the true factors by

$$\begin{bmatrix} \tilde{F}_t \\ \tilde{c}_t \end{bmatrix} = \hat{\delta} \begin{bmatrix} \hat{F}_t \\ \hat{c}_t \end{bmatrix}.$$

The correlation between $\tilde{F}_t$ and F_t^0, and $\tilde{c}_t$ and c_t^0 indicates how precisely PCA estimates the space spanned by the factors.

Using $\tilde{F}_t$ and $\tilde{c}_t$ we then fit a VAR of order two and estimate the parameters of the FECM given by (8). The estimated VAR is then used to obtain the impulse responses of rotated factors to unit shocks to $\tilde{F}_t$. The resulting responses, combined with the estimated parameters of the FECM, yield the impulse responses of the Xs.

Table 3: Correlation between true and estimated factors

(1)	(2)	(3)	(4)	(5)	(6)
		Correlation between			
		$\tilde{F}_t$ and $\hat{F}_t$	$\tilde{c}_t$ and $\hat{c}_t$	$\tilde{F}_t$ and $\hat{F}_t$	$\tilde{c}_t$ and $\hat{c}_t$
T	N	I(1) variables		I(1) and I(0) variables	
30	50	0.989	0.964	0.995	0.986
50	50	0.995	0.975	0.993	0.989
50	100	0.998	0.989	0.995	0.990
50	250	0.999	0.997	0.997	0.997
50	500	0.999	0.998	0.999	0.998
100	250	0.999	0.998	0.999	0.997
100	500	1.000	0.998	1.000	0.999
100	1000	1.000	0.999	1.000	0.999
250	500	1.000	0.999	1.000	0.999
250	1000	1.000	0.999	1.000	0.999
500	100	1.000	0.993	0.999	0.994
500	250	1.000	0.997	1.000	0.998
500	500	1.000	0.999	1.000	0.999

Notes: Panel with only I(1) data in columns 3 and 4. Panel with I(1) and I(0) data in columns 5 and 6.

The impulse responses are computed for 100 periods. The VAR for the factors is estimated with the unit root imposed in the equation for $\tilde{F}_t$.[22] In order to mimic the practice in the empirical example, we do not impose the mutual independence of the (dynamic) factors.

The experiment consists of 1000 replications. Within each iteration we generate a new set of parameters and iterate 100 times on random draws of the error processes u_t, w_t and v_{it} to get the distribution of impulse responses. The confidence intervals of the impulse responses are averaged over the 1000 replications and compared to the true impulse responses.

Table 3 reports the correlation coefficients between the true and the estimated and rotated factors for different combinations of T and N. As we can see, principal components capture the space spanned by the factors quite successfully, even at moderate sample sizes. The correlations increase with both T and N.

[22]The key results are unaltered if the unit root is not imposed in estimation. The only difference is to be found in lower efficiency (as reflected in the width of the confidence intervals). Results available from the authors upon request.

Table 4 reports measures of coherence between true and estimated impulse responses for the two factors. In particular, columns (3) and (4) contain the share of periods the true impulse responses of both factors, either to a shock to the I(1) factors (upper panel) or a shock to the I(0) factors (lower panel), are outside the bootstrapped 95% confidence intervals. The results show that virtually no true impulse response is outside the confidence interval of the responses to the shock to I(1) factors. The shares of responses outside the confidence interval to a shock to the I(0) factor do not exceed the theoretical 5% level. Columns (5) - (8) contain the differences between true impulse responses and the responses averaged across the Monte Carlo replications, which gives a measure of the bias in finite samples.[23] Similar observations apply both to responses to a shock to the I(1) factor (upper panel), and to a shock to the I(0) factor (lower panel). We can observe that the impulse responses converge to the true responses quite fast with both T and N. As expected, also the width of the confidence intervals generally decreases with both N and T (while holding the other constant).

Table 4: Impulse responses of factors - strict DFM

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		% of true IRs outside 95% CI		True IR - mean IR at horizon				Conf. int. width at horizon			
T	N	F_t	c_t	3	12	24	100	3	12	24	100
				Responses of F_t to shock to F_t							
30	50	0.0	0.0	-0.10	-0.10	-0.10	-0.10	1.42	1.47	1.47	1.47
50	50	0.0	0.0	-0.04	-0.03	-0.03	-0.03	1.03	1.03	1.04	1.04
50	100	0.0	0.0	-0.05	-0.04	-0.04	-0.04	1.06	1.08	1.08	1.08
50	250	0.0	0.0	-0.03	-0.03	-0.03	-0.03	1.03	1.06	1.06	1.06
50	500	0.0	0.0	-0.03	-0.02	-0.02	-0.02	1.03	1.04	1.04	1.04
100	250	0.0	0.0	-0.02	-0.02	-0.02	-0.02	0.93	0.94	0.94	0.94
100	500	0.0	0.0	-0.01	-0.01	-0.01	-0.01	0.92	0.93	0.93	0.93
100	1000	0.0	0.0	-0.02	-0.02	-0.02	-0.02	0.92	0.92	0.92	0.92
250	500	0.0	0.0	-0.01	-0.01	-0.01	-0.01	0.86	0.86	0.86	0.86
250	1000	0.0	0.0	-0.01	-0.01	-0.01	-0.01	0.86	0.85	0.86	0.86
500	100	0.0	0.0	-0.03	-0.04	-0.04	-0.04	0.90	0.90	0.90	0.90
500	250	0.0	0.0	-0.01	-0.01	-0.01	-0.01	0.85	0.84	0.84	0.84
500	500	0.0	0.0	-0.01	-0.01	-0.01	-0.01	0.84	0.83	0.83	0.83
				Responses of c_t to shock to c_t							
30	50	0.0	1	-0.04	-0.01	0.00	0.00	0.74	0.16	0.03	0.00
50	50	0.0	1	-0.09	-0.01	0.00	0.00	0.68	0.14	0.02	0.00
50	100	0.0	1	-0.09	-0.01	0.00	0.00	0.67	0.13	0.02	0.00
50	250	0.0	1	-0.07	-0.01	0.00	0.00	0.66	0.12	0.01	0.00
50	500	0.0	1	-0.08	-0.01	0.00	0.00	0.65	0.12	0.01	0.00
100	250	0.0	1	-0.10	-0.01	0.00	0.00	0.62	0.10	0.01	0.00
100	500	0.0	1	-0.10	-0.01	0.00	0.00	0.62	0.10	0.01	0.00
100	1000	0.0	1	-0.10	-0.01	0.00	0.00	0.62	0.10	0.01	0.00
250	500	0.0	3	-0.11	-0.01	0.00	0.00	0.58	0.07	0.00	0.00
250	1000	0.0	3	-0.11	-0.01	0.00	0.00	0.58	0.07	0.00	0.00
500	100	0.0	5	-0.14	-0.02	0.00	0.00	0.59	0.07	0.00	0.00
500	250	0.0	3	-0.12	-0.01	0.00	0.00	0.57	0.06	0.00	0.00
500	500	0.0	5	-0.12	-0.01	0.00	0.00	0.57	0.06	0.00	0.00

Notes: 1000 Monte Carlo replications

Corresponding to Table 4 for the factors, Table 5 reports equivalent results for impulse responses of Xs. To facilitate presentation all statistics are averaged over N variables.

[23]Note that the generated factors are independent, but independence is not imposed when working with estimated factors. Because of this, the cross-equation responses of factors are not zero, but still quantitatively limited. For this reason and in order to save space, Table 4 reports only the responses of factors to own shocks. Detailed results are available upon request.

For the impulse responses of Xs we also observe that only a negligible share of impulse responses deviates from the 95% confidence intervals. The largest shares reported in column 3 are below 0.5%. These results suggest that the estimation method successfully retrieves the impulse responses to shocks. Similar observations to those of factors about the convergence of the impulse responses and their distribution apply also to the impulse responses of Xs (see columns 4 - 11 in Table 5).

Table 5: Estimation of impulse responses of observable variables - average across Xs - strict DFM

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		% of true IRs outside	True IR - mean IR at horizon				Conf. int. width at horizon			
T	N	95% CI	3	12	24	100	3	12	24	100
				Shock to F_t						
30	50	0.00	0.10	0.14	0.15	0.14	5.25	4.05	4.03	4.03
50	50	0.00	0.07	0.08	0.09	0.09	3.01	2.02	2.01	2.00
50	100	0.00	0.07	0.05	0.06	0.06	2.81	1.99	1.98	1.97
50	250	0.00	0.08	0.04	0.05	0.05	2.75	1.88	1.88	1.87
50	500	0.00	0.08	0.03	0.04	0.04	2.71	1.83	1.82	1.83
100	250	0.00	0.10	0.02	0.03	0.03	1.36	0.76	0.76	0.76
100	500	0.00	0.10	0.01	0.02	0.02	1.30	0.70	0.69	0.69
100	1000	0.00	0.09	0.02	0.03	0.03	1.19	0.69	0.69	0.69
250	500	0.02	0.11	0.01	0.01	0.01	0.61	0.27	0.26	0.26
250	1000	0.03	0.10	0.01	0.01	0.01	0.64	0.27	0.27	0.27
500	100	0.13	0.09	0.04	0.05	0.04	0.34	0.19	0.19	0.19
500	250	0.34	0.11	0.01	0.01	0.01	0.44	0.17	0.17	0.17
500	500	0.39	0.11	0.01	0.01	0.01	0.42	0.16	0.16	0.16
				Shock to c_t						
30	50	0.00	0.40	0.01	0.01	0.01	11.03	3.13	3.20	3.23
50	50	0.02	0.40	0.01	0.01	0.00	5.00	1.83	1.84	1.85
50	100	0.02	0.36	0.02	0.02	0.02	4.20	1.52	1.59	1.60
50	250	0.02	0.38	0.02	0.01	0.01	4.47	1.46	1.49	1.50
50	500	0.03	0.38	0.01	0.01	0.01	4.16	1.56	1.63	1.64
100	250	0.10	0.34	0.01	0.01	0.01	1.96	0.96	0.99	0.99
100	500	0.11	0.33	0.01	0.01	0.01	1.77	0.89	0.89	0.89
100	1000	0.11	0.33	0.02	0.01	0.01	1.94	0.95	0.97	0.97
250	500	0.49	0.34	0.01	0.00	0.00	0.86	0.54	0.54	0.54
250	1000	0.46	0.31	0.01	0.00	0.00	0.86	0.58	0.59	0.59
500	100	1.12	0.34	0.02	0.01	0.01	0.58	0.49	0.50	0.50
500	250	0.98	0.33	0.01	0.00	0.00	0.55	0.39	0.38	0.37
500	500	0.94	0.30	0.01	0.00	0.00	0.52	0.39	0.39	0.39

Notes: 1000 Monte Carlo replications. Results in the table refer to mean impulse responses across N variables. Absolute deviations between true and estimated impulse responses.

Motivated by the empirical applications in the paper, we next consider one modification to the data generating process. Both datasets we use contain both I(1) and I(0) variables and we want to investigate how the presence of I(0) variables affects the finite sample properties of the estimated factors. The setting of the experiment can be easily adapted by restricting some of the loadings of F_t to zero.

We focus on the US dataset, because of its larger dimensions. The dataset used in contains 120 variables, 43 of which are treated as I(0). To replicate this feature we restrict roughly 36% of the loadings of F_t to zero in each sample setup. The factors are extracted from generated data using PCA without imposing the zero restrictions on the loadings.

Simulation results, presented in columns 5 and 6 of Table 3, reveal that the presence of I(0) variables in the panel does not have a detrimental effect on estimation since even at moderate sample sizes PCA successfully retrieves the space spanned by dynamic factors.

Overall, our simulation experiments indicate that principal component based estimators (with a mixture of I(1) and I(0) factors) can recover very well the factor space. Moreover, using the estimated factors in the factor VAR replicates accurately the true factor responses. Finally, inserting the estimated factor responses in the FECM, in combination with the estimated FECM parameters, delivers estimated structural impulse responses very close to the true ones.

Table 6: Impulse responses of factors - approximate DFM

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		% of true IRs outside 95% CI		True IR - mean IR at horizon				Conf. int. width at horizon			
T	N	F_t	c_t	3	12	24	100	3	12	24	100
				Responses of F_t to shock to F_t							
30	50	0	0	-0.04	-0.03	-0.03	-0.03	1.12	1.16	1.16	1.18
50	50	0	0	-0.03	-0.02	-0.02	-0.02	0.99	1.02	1.02	1.02
50	100	0	0	-0.02	-0.02	-0.02	-0.02	1.00	1.00	1.00	1.00
50	250	0	0	-0.02	-0.01	-0.01	-0.01	0.97	0.99	0.99	1.00
50	500	0	0	-0.02	-0.01	-0.01	-0.01	1.00	1.02	1.02	1.02
100	250	0	0	-0.01	-0.01	-0.01	-0.01	0.92	0.91	0.91	0.91
100	500	0	0	-0.01	-0.01	-0.01	-0.01	0.91	0.90	0.90	0.90
100	1000	0	0	-0.01	-0.01	-0.01	-0.01	0.91	0.91	0.92	0.92
250	500	0	0	0.00	0.00	0.00	0.00	0.86	0.86	0.86	0.86
250	1000	0	0	0.00	0.00	0.00	0.00	0.85	0.84	0.84	0.84
500	100	0	0	0.00	0.00	0.00	0.00	0.83	0.82	0.82	0.82
500	250	0	0	0.00	0.00	0.00	0.00	0.83	0.83	0.83	0.83
500	500	0	0	0.00	0.00	0.00	0.00	0.83	0.82	0.82	0.82
				Responses of c_t to shock to c_t							
30	50	0	1	-0.01	-0.01	0.00	0.00	0.71	0.15	0.02	0.00
50	50	0	1	-0.05	-0.01	0.00	0.00	0.65	0.10	0.01	0.00
50	100	0	1	-0.05	-0.01	0.00	0.00	0.65	0.11	0.01	0.00
50	250	0	1	-0.05	-0.01	0.00	0.00	0.66	0.13	0.02	0.00
50	500	0	1	-0.06	-0.01	0.00	0.00	0.66	0.12	0.02	0.00
100	250	0	1	-0.09	-0.01	0.00	0.00	0.61	0.09	0.01	0.00
100	500	0	1	-0.09	-0.01	0.00	0.00	0.61	0.10	0.01	0.00
100	1000	0	1	-0.09	-0.01	0.00	0.00	0.62	0.09	0.01	0.00
250	500	0	3	-0.11	-0.01	0.00	0.00	0.58	0.07	0.00	0.00
250	1000	0	3	-0.11	-0.01	0.00	0.00	0.58	0.07	0.01	0.00
500	100	0	3	-0.11	-0.01	0.00	0.00	0.56	0.06	0.00	0.00
500	250	0	3	-0.11	-0.01	0.00	0.00	0.56	0.05	0.00	0.00
500	500	0	5	-0.11	-0.01	0.00	0.00	0.56	0.05	0.00	0.00

Notes: 1000 Monte Carlo replications

The final issue analyzed with the simulation experiment is the estimation of the FECM under the strcit DFM assumption, while the data-generating process is an approximate DFM. To this end we modofy the data-generating process from above to include cross-correlated idiosyncratic errors. In particular, we follow the structure of the Monte Carlo experiment in Stock and Watson (2002) and set

$$(1-\gamma_i(L))\,\varepsilon_{it} = \left(1+b^2\right)v_{it} + bv_{i-1,t} + bv_{i+1,t},$$

where the parameters of $\gamma_i(L)$ are set as above and $b = 1$. The data are thus generated by an approximate dynamic factor model and on these data we estimate the FECM as proposed in the paper, i.e. by omitting the (lags of) X_{jt} from the equations of X_{it} according to the (incorrect) strict DFM assumption.

The results, equivalent to those in Tables 4 and 5, are presented in Tables 6 and 7.

Principal components yield consistent estimates of the factor space even in presence of cross-correlated idosyncratic errors, which is why we put our attention to the properties of the impulse responses of observable variables X_{it}. What can be observed by comparing Tables 5 and 7 is a great degree of similarity of results. The only notable difference are the impulse responses at short horizons (horizon 3 in the tables) for which we see that the presence of cross-correlation of the idiosyncratic component leads to slighlty larges bias than observed in the case of a strict DFM GDP. The bias decreases both with the impulse response horizon and both dimensions of the data panel. Similar observations apply to estimated confidence intervals. These results suggest that estimating the FECM under the strict DFM assumption does not lead to a significant bias in estimated impulse responses and/or inefficiency of inference based on bootstrapped confidence intervals.

Table 7: Estimation of impulse responses of observable variables - average across Xs - approximate DFM

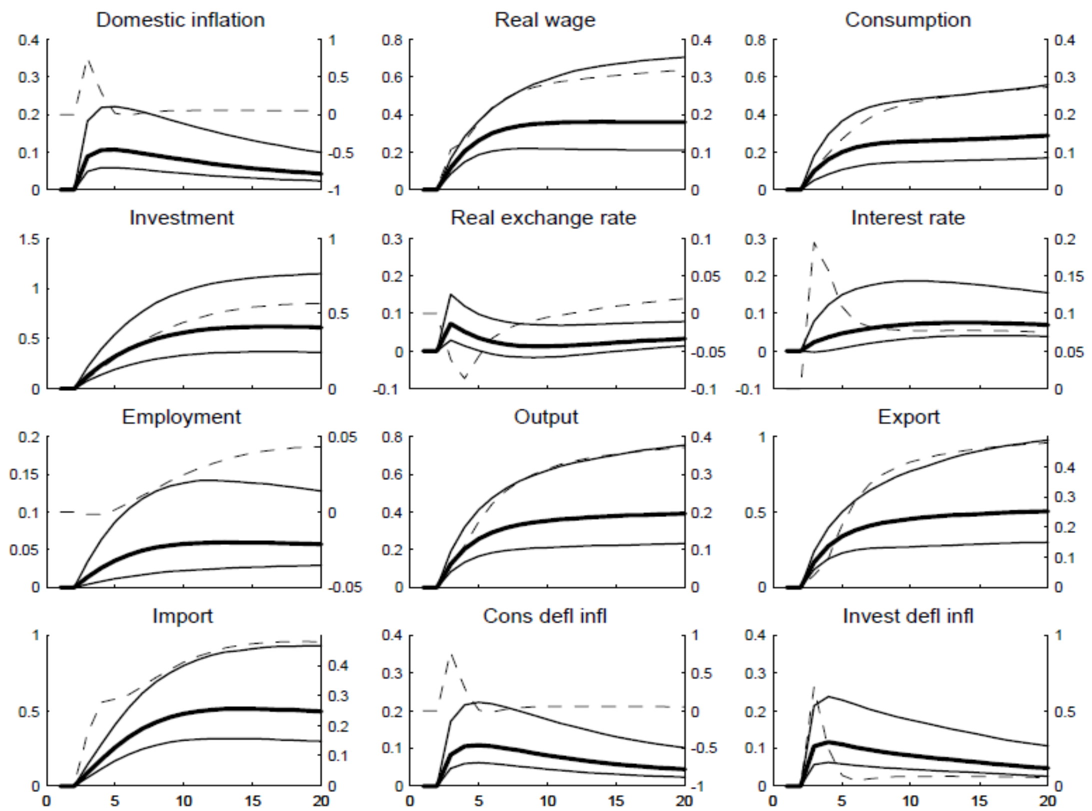
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		% of true IRs outside	True IR - mean IR at horizon				Conf. int. width at horizon			
T	N	95% CI	3	12	24	100	3	12	24	100
Shock to F_t										
30	50	0.02	0.16	0.05	0.05	0.05	4.22	3.18	3.18	3.15
50	50	0.18	0.17	0.03	0.03	0.03	2.30	1.43	1.42	1.41
50	100	0.12	0.19	0.05	0.05	0.05	2.57	1.71	1.73	1.74
50	250	0.21	0.17	0.05	0.05	0.05	2.52	1.70	1.70	1.70
50	500	0.14	0.15	0.03	0.03	0.03	2.59	1.54	1.53	1.53
100	250	0.26	0.15	0.02	0.02	0.02	1.22	0.55	0.55	0.55
100	500	0.28	0.14	0.02	0.02	0.02	1.14	0.52	0.52	0.52
100	1000	0.32	0.15	0.02	0.02	0.02	1.22	0.54	0.54	0.53
250	500	0.78	0.12	0.01	0.01	0.01	0.56	0.18	0.18	0.18
250	1000	0.73	0.13	0.00	0.00	0.00	0.61	0.19	0.19	0.19
500	100	1.13	0.14	0.00	0.00	0.00	0.42	0.11	0.11	0.11
500	250	0.86	0.13	0.00	0.00	0.00	0.43	0.13	0.13	0.13
500	500	0.98	0.13	0.00	0.00	0.00	0.48	0.12	0.12	0.12
Shock to c_t										
30	50	0.06	0.39	0.03	0.03	0.03	6.66	2.13	2.15	2.17
50	50	0.12	0.40	0.02	0.02	0.02	3.68	1.28	1.34	1.34
50	100	0.07	0.33	0.02	0.01	0.01	4.12	1.43	1.44	1.45
50	250	0.12	0.41	0.01	0.01	0.01	3.87	1.34	1.36	1.36
50	500	0.14	0.40	0.02	0.02	0.02	3.72	1.33	1.35	1.35
100	250	0.38	0.36	0.01	0.00	0.00	1.81	0.92	0.94	0.94
100	500	0.38	0.35	0.01	0.00	0.00	1.93	0.85	0.86	0.86
100	1000	0.39	0.36	0.01	0.01	0.01	1.79	0.88	0.91	0.91
250	500	1.43	0.35	0.01	0.00	0.00	0.71	0.52	0.53	0.53
250	1000	1.45	0.34	0.01	0.00	0.00	0.75	0.54	0.54	0.54
500	100	1.99	0.34	0.01	0.00	0.00	0.47	0.45	0.44	0.44
500	250	2.24	0.34	0.01	0.00	0.00	0.44	0.38	0.38	0.39
500	500	2.20	0.33	0.01	0.00	0.00	0.43	0.40	0.40	0.40

Notes: 1000 Monte Carlo replications. Results in the table refer to mean impulse responses across N variables. Absolute deviations between true and estimated impulse responses.

Appendix C: DSGE evidence on impulse responses to innovations to a stochastic productivity trend

In this Appendix we report the responses to a positive and permanent productivity shock obtained with the Adolfson et al. (2007) estimated DSGE model of the Euro area, for comparison with the FECM results presented in Section 6.2.

Figure 3: Impulse responses to a permanent technology shock for the EA from Adolfson et al. (2007)



Note: Benchmark impulse responses under price rigidity and imperfect exchange rate (pass-through solid, left axis) and flexible prices and wages (dashed, right axis).