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Endogenous Strategic Managerial Incentive Contracts∗

Constantine Manasakis† Evangelos Mitrokostas‡ Emmanuel Petrakis§

Abstract

This paper studies the endogenous structure of incentive contracts that firms' owners offer to their managers, when these contracts are linear combinations either of own profits and own revenues, or of own profits and competitor’s profits or, finally, of own profits and own market share. In equilibrium, each owner has a dominant strategy to reward his manager with a contract combining own profits and competitor’s profits. Contrary to the received literature, the case where there is no ex-ante commitment over any type of contract that each owner offers to his manager is also examined. In equilibrium, each type of contract is an owner’s best response to the competing owner’s choice.

JEL classification: D43; L21

Keywords: Oligopoly; Managerial delegation; Endogenous contracts.

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1 Introduction

Orthodox economic theory treats firms as economic agents whose main objective is to maximize profits. However, already in the 1950s, Baumol (1958) suggested a sales-maximization model of firms’ objective function as a realistic alternative to the profit-maximization one. More recently, Fershtman and Judd (1987) argue that a proper analysis of the firm’s objective function should be undertaken under the prism of the separation of ownership and management.\(^1\) They further argue that such an analysis should incorporate the structure of the incentives that owners offer to managers in order to motivate them.\(^2\)

The purpose of this paper is precisely to study the endogenous emergence of the structure of contracts that firms’ owners offer to their managers so as to motivate them, and how this in turn influences market performance and societal outcomes.

The strategic use of managerial incentive contracts has been introduced in the literature by the path-breaking papers of Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987). In this line of research, each firm’s owner offers an incentive contract to his manager in order to direct him to a more aggressive behavior in the market, so as to force the competing manager to reduce output. In particular, in the above series of papers, in two-staged oligopoly models, in the first stage of the game, profit-maximizing owners choose compensation schemes for their managers that are linear combinations of own profits and own revenues. In the second stage, managers, knowing compensation schemes, compete in quantities. Each firm’s owner, when determining his manager’s incentives, has an opportunity to become a Stackelberg leader, provided that the rival owners do

\(^1\)Managerial theories of the firm and agency theory have emphasized that the aforementioned separation leads to inefficiencies due to asymmetric information and differing objectives of managers and owners (e.g., Williamson, 1964; Jensen and Meckling, 1976; Fama and Jensen, 1983).

\(^2\)The role of non-profit incentives into managerial compensation practices is evident from empirical studies in the agency tradition, such as those of Jensen and Murphy (1990) and Lambert et al. (1991). These studies reveal that executive bonuses and salaries are associated with both firm size and profit level, with the size correlations being the stronger of the two.
not delegate output decisions to managers. In equilibrium, all owners act in the same way at the game’s contract stage and firms end up in a prisoners’ dilemma situation.3

Regarding the structure of managerial incentive contracts, recent empirical evidence has highlighted new perspectives. Gibbons and Murphy (1990), along with Joh (1999) and Aggarwal and Samwick (1999) support that in most cases, firms’ owners, when constructing the incentive contracts for their managers, take into account competitors’ profits along with their own profits. Miller and Pazgal (2001; 2002; 2005) formalize this argument in what they call “Relative Performance” contracts. In a two-staged game, firstly, owners offer to their managers contracts that are linear combinations of own profits and competitors’ profits. In the second stage, managers compete in an oligopolistic industry. Equilibrium analysis suggests that when firms’ owners manipulate their managers’ incentives, they direct the quantity-setting (price-setting) managers to a more (less) aggressive behavior, compared to the strict own-profit maximizing behavior that owners would exercise.

From another perspective, Peck (1988) mentions that market share is highly ranked in managers’ objectives. In a survey for corporate objectives among 1000 American and 1031 Japanese top managers, Peck (1988) documents that increasing market share is ranked third in the American and second in the Japanese sample. Jansen et al. (2007) and Ritz (2005) formalize the case of “Market Share” contracts. In a duopolistic market, in the first stage of a two-staged game, each owner offers to his manager a contract that is a linear combination of own profits and own market share. In the second stage, managers compete either in quantities or in prices. Their main result is that for the case of Cournot (Bertrand) competition, quantities (prices) set from managers compensated with Market Share contracts are higher than those set by strict profit-maximizing owners.

The literature, so far, seems to seek an explanation for the emergence of the various types of managerial incentive contracts.4 The core substance

3 If managers compete in prices, Fershtman and Judd (1987) and Sklivas (1987) find that owners encourage their managers to raise prices and keep sales low.
4 A crucial assumption of the relevant literature is that delegation is observable. Katz (1991) argues that unobservable contracts have no commitment value at all. Fershtman
of the present paper is an attempt to study the endogenous structure of incentive contracts that firms’ owners offer to their managers, when these contracts are linear combinations either of own profits and own revenues (Profits-Revenues contracts), or of own profits and competitor’s profits (Relative Performance contracts) or, finally, of own profits and own market share (Market Share contracts).

Our envisaged product market is a homogenous Cournot duopoly. The examinations are restricted to firms with equally efficient production technologies, reflected in equal (constant) marginal production costs. In this environment, we consider a three-staged game with observable actions: in the first stage, each firm’s owner commits to one type of contract to compensate his manager. In the second stage of the game, given that the types of contracts have become common knowledge and can not be reset, each owner sets the weight (managerial incentive parameter) between own profits and either own revenues, or competitor’s profits, or, finally, own market share. In the third stage of the game, managers compete à la Cournot.

The analysis suggests that each firm’s owner dominant strategy is to reward his manager with a Relative Performance type of contract. Intuitively, although the strategic use of managerial incentive contracts leads to a prisoners’ dilemma situation always, this situation is the least severe in case of the Universal Relative Performance Contracts, implying that this configuration is the least competitive and firms’ profits are the highest. A Relative Performance contract makes a manager’s behavior less susceptible to strategic manipulation by rivals. Less scope for strategic manipulation gives to the rival owner less reason to provide incentives for aggressive behavior to his manager and thus, competition remains softer. The analysis also suggests that competition becomes fiercer the more Profits-Revenue managers are active, as each one of them makes his rival relatively more aggressive. Thus, competition under the Universal Profits-Revenue configuration turns out to

and Judd (1987) support that even if contracts are not observable, they will become common knowledge when the game is being repeated for several periods. More recently, Kockesen and Ok (2004) argue that to the extent that renegotiation is costly and/or limited, in a general class of economic settings, strategic aspects of delegation may play an important role in contract design, even if the contracts are completely unobservable.
be the most fierce.

Then, the analysis is extended towards the case where there is no ex-ante commitment over the type of contract that each owner offers to his manager, implying that each owner can independently shift from one type of contract to another. In this environment, the following two-staged game with observable actions is studied: in the first stage, each owner chooses the type of contract to reward his manager and sets the certain managerial incentive parameter. In the second stage, given that the types of contracts and the parameters that owners have chosen are common knowledge and can not be reset, managers compete in quantities. Interestingly enough, it is found that the magnitude of each individual owner's incentive to deviate from one type of contract to another is zero. Thus, each type of contract, namely, a Profits-Revenues contract, a Relative Performance contract, a Market Share contract is owner $i$’s best response to owner $j$’s choice. The intuitive explanation behind this finding is based on the conditions that must be fulfilled in equilibrium: firstly, since production decisions are taken by managers, in equilibrium, their reaction curves must be intersected. Secondly, the fact that firm $i$’s (firm $j$’s) owner offers an incentive contract to his manager, as a strategic tool in order to become Stackelberg leader against firm $j$ (firm $i$), standard textbook analysis of the Stackelberg model implies that, in equilibrium, there must be tangency between firm $i$’s (firm $j$’s) isoprofit curve and manager $j$’s (manager $i$’s) reaction curve. The two staged game is characterized by multiplicity of equilibria. By employing focal point analysis, it is found that each owner will finally choose to compensate his manager with a Relative Performance contract.

With respect to the societal effects of the alternative managerial incentive contracts, it is found that their use increases total welfare, compared to the benchmark case where it is the owners of the firms that take the production decisions. The strategic use of incentive contracts by owners increases consumers’ surplus and decreases total industry profits. The latter effect is found to be completely compensated by the former. Moreover, the Universal Profits-Revenues Contracts equilibrium leads to the highest total welfare, since it is the most competitive.
Our paper contributes to the literature in two ways. Firstly, by investigating the endogenous structure of incentive contracts that owners offer to their managers, this is, to the best of our knowledge, the first paper that compares market, as well as societal, outcomes of alternative types of managerial incentive contracts. Secondly, this paper is among the first to extend the literature on strategic managerial delegation by considering the case in which there is no ex-ante commitment over any type of contract that each owner offers to his manager.

The rest of the paper is organized as follows: Section 2 presents the model. In Section 3, the different subgames are solved and in Section 4, the subgame Nash perfect equilibrium of managerial contracts is derived. In Section 5, a welfare analysis is carried out and in Section 6, the case where there is no ex-ante commitment over the types of contracts that owners offer to managers is examined. Section 7 offers some concluding remarks.

2 The Model

We consider a homogenous good industry where two firms, denoted by \( i, j = 1, 2, i \neq j \) compete in quantities. The (inverse) demand function for the final good is linear, and is given by:

\[
P(Q) = A - Q
\]

where, \( A > 0 \) and \( Q = q_1 + q_2 \) is the aggregate output. Firms are endowed with constant returns to scale technologies and have constant unit cost \( c_i = c_j = c \). Thus, firm \( i \)'s profits are given by:

\[
\Pi_i = (A - q_i - q_j - c) q_i
\]
Each firm’s owner has the opportunity to compensate his manager by offering to him a “take it or leave it” incentive contract. Each owner can choose one among three alternative types of incentive contracts to compensate his manager. The first is the \textit{Profits-Revenues (PR)} type of contract. Following Fershtman and Judd (1987) and Sklivas (1987), under this type of contract, the incentive structure takes a particular form: the risk-neutral manager $i$ is paid at the margin, in proportion to a linear combination of own profits and own revenues. More formally, firm $i$’s manager will be given incentive to maximize:

$$U_{i}^{PR} = a_{i}^{PR} \Pi_{i} + (1 - a_{i}^{PR}) R_{i}$$

(3)

where $\Pi_{i}$ and $R_{i}$ are firm $i$’s profits and revenues respectively.\footnote{Although in real life the terms of managerial contracts can be determined via owners-managers negotiations, it is a regular assumption in the strategic delegation literature that the market for managers is perfectly competitive and the firms’ owners have all the power during negotiations, i.e., they offer to their managers “take it or leave it” incentive contracts (see Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987; and Miller and Pazgal, 2001; 2002; 2005).}

$a_{i}^{PR}$ is the \textit{managerial incentive parameter} that is chosen optimally by firm $i$’s owner so as to maximize his profits. There is no restriction on $a_{i}^{PR}$, allowing even negative values. If $a_{i}^{PR} < 1$, firm $i$’s manager moves away from strict profit-maximization towards including consideration of sales and thus, he becomes a more aggressive seller in the market. If $a_{i}^{PR} = 1$, manager $i$’s behavior coincides with owner $i$’s objective for strict profit-maximization.

The second type of contract is the \textit{Relative Performance (RP)} one. Following Miller and Pazgal (2001; 2002; 2005), under this type of contract, firm $i$’s owner compensates his manager putting unit weight on own profits and a weight $-a_{i}^{RP}$ on rival’s profits. Thus, manager $i$’s utility function takes the form:\footnote{$U_{i}^{RR}$ will not be the manager’s reward in general. Since the manager’s reward is linear in profits and sales, he is paid $A_{i} + B_{i}U_{i}^{RP}$ for some constants $A_{i}$, $B_{i}$, with $B_{i} > 0$. Since he is risk-neutral, he acts so as to maximize $U_{i}^{RP}$ and the values of $A_{i}$ and $B_{i}$ are irrelevant.}

$$U_{i}^{RP} = (1 - a_{i}^{RP}) \Pi_{i} + a_{i}^{RP} (\Pi_{i} - \Pi_{j})$$

This is equivalent to eq. (4).
When the objective function is written in this manner, it becomes apparent that if $a_{RP}^i > 0$, manager $i$ puts negative weight on the rival firm’s performance. If $a_{RP}^i = 0$, manager $i$’s behavior coincides with owner $i$’s interest for own profit-maximization. Following Miller and Pazgal (2002), in order to avoid the situation where a manager is more concerned with his rival than the performance of his own firm, it is further assumed that $a_{RP}^i \leq 1$.

The third type of contract is the Market Share ($M$) one. Following Jansen et al. (2007) along with Ritz (2005), under this type of contract, firm $i$’s owner compensates his manager with a contract constituted by a linear combination of own profits and own market share. In this case, manager $i$’s utility function takes the form:

$$U_i^M = \Pi_i + a_i^M \frac{q_i}{q_i + q_j}$$

where the managerial incentive parameter, $a_i^M$, is a non-negative number chosen optimally by owner $i$ in order to maximize his profits.

In order to examine which types of managerial incentive contracts prevail in equilibrium, consider a three-staged game with the following timing: in the first stage, each firm’s owner commits to one of the three types of contracts $D, D = PR, RP, M$ for rewarding his manager. Then, in the second stage of the game, given that the types of contracts have become common knowledge and can not be reset, each owner sets the corresponding managerial incentive parameter $a_i^D$. In the third stage of the game, managers compete
The equilibrium concept employed is the subgame perfect equilibrium.

3 Equilibrium managerial incentive contracts

In this part of the paper, the alternative strategies of firms’ owners, regarding the contracts that they will choose to reward their managers, are investigated. The payoffs of the different subgames are presented in the following Payoff Matrix.

<table>
<thead>
<tr>
<th>OWNER 1</th>
<th>( PR )</th>
<th>( RP )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( RP )</td>
<td>( \Pi_1^{PR}, \Pi_2^{PR} )</td>
<td>( \Pi_1^{(rp-rp)}, \Pi_2^{rp, rp} )</td>
<td>( \Pi_1^{(pr-m)}, \Pi_2^{(pr-m)} )</td>
</tr>
<tr>
<td>( RP )</td>
<td>( \Pi_1^{(rp-pr)}, \Pi_2^{(rp-pr)} )</td>
<td>( \Pi_1^{RP}, \Pi_2^{RP} )</td>
<td>( \Pi_1^{(rp-m)}, \Pi_2^{(rp-m)} )</td>
</tr>
<tr>
<td>( M )</td>
<td>( \Pi_1^{(m-pr)}, \Pi_2^{(m-pr)} )</td>
<td>( \Pi_1^{(m-rp)}, \Pi_2^{(m-rp)} )</td>
<td>( \Pi_1^{M}, \Pi_2^{M} )</td>
</tr>
</tbody>
</table>

3.1 Universal Profits-Revenues Contracts

Under the present candidate equilibrium configuration, in the first stage of the game, firms’ owners simultaneously and non-cooperatively commit to

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8 At this point, it is useful to bear in mind two alternative interpretations of the game. According to the first one, following Fershtman and Judd (1987) and Sklivas (1987), an owner hires a manager and directs him through an appropriate incentive contract. The alternative interpretation is the one presented by Miller and Pazgal (2002). In the latter, the problem faced by the owner of each firm is to choose the best type of manager among those that are available, while each manager is committed to behaving in a certain manner by virtue of his personality type. More specifically, in Miller and Pazgal (2002), potential managers take on a continuum of attitudes toward relative performance which is captured by their type, \( \varphi \). However, the difference between Fershtman and Judd (1987) and Miller and Pazgal (2002) is only semantic, since owners have all the bargaining power (by assumption) when setting the contracts.

9 As a benchmark, the no-delegation case where production decisions are taken by firms’ owners, is considered. In this case, the reaction function is \( q_i^C(q_j^C) = (A - c - q_j^C)/2 \) while equilibrium output, profits and total welfare are \( q_i^{C*} = (A - c)/3 \), \( \Pi_{i}^{C*} = (q_{i}^{C*})^2 \) and \( TW_{i}^{C} = 4(A - c)^2/9 \) respectively. Diagrammatically, the benchmark Cournot case is at point \( E_C \), in Figure 1.
Profits-Revenues contracts to compensate their managers. Thus, in the third stage of the game, manager $i$ chooses $q_i$ in order to maximize his utility given by eq. (3), taking as given the output of his rival, $q_j$, along with the managerial contracts and the managerial incentive parameters set in the previous stages. The first order condition of eq. (3) provides manager $i$’s reaction function:

$$q_{PR}^i(q_{PR}^j) = \frac{A - a_{PR}^i c - q_{PR}^j}{2}$$ (6)

When owner $i$ compensates his manager with a Profits-Revenues contract, the slope of manager $i$’s reaction curve is $\frac{dq_{PR}^i}{dq_{PR}^j} = \frac{dq_{C}^i}{dq_{C}^j} = -\frac{1}{2}$, i.e. it is equal to the reaction curve’s slope in the benchmark case, where it is owner $i$ who decides for the level of production. From eq. (6), manager $i$, when choosing the output level of his firm, considers $a_{PR}^i c$ as the marginal cost of production. Owner $i$, by choosing an incentive parameter $a_{PR}^i < 1$, the aforementioned marginal cost, $a_{PR}^i c$, is lower than that considered under owner $i$’s strict profit-maximizing behavior, $c$. Thus, owner $i$ makes his manager more aggressive, i.e. he directs him to produce output at a level higher than that produced under a strict profit-maximizing behavior. Diagrammatically (Figure 1), as owner $i$ sets $a_{PR}^i \rightarrow 0$ he induces an outward and parallel shift in his manager’s best response curve, from $RC^C_i$ (benchmark Cournot) to $RC_{PR}^i$.

Solving the system of the reaction functions, output levels are:

$$q_{PR}^i(a_{PR}^i, a_{PR}^j) = \frac{A + c(a_{PR}^j - 2a_{PR}^i)}{3}$$ (7)

In the second stage of the game, each owner $i$ chooses $a_{PR}^i$ so as to maximize profits given by:

$$\Pi_{PR}^i(a_{PR}^i, a_{PR}^j) = \frac{[A + c(a_{PR}^j - 2a_{PR}^i)] [A + c(a_{PR}^j + a_{PR}^i - 3)]}{9}$$ (8)

The first order condition of eq. (8) provides owner $i$’s best response function:
\[ a_{PR}^i (q_{PR}^j) = \frac{(A + c) (q_{PR}^j - 6)}{4c} \]  

(9)

Solving the system of the best response functions, the (candidate) equilibrium managerial incentive parameters are:

\[ a_{PR}^i = \frac{-A + 6c}{5c} \]  

(10)

Plugging \( a_{PR}^i \) in eq. (7) and (8), firm \( i \)'s equilibrium output and profits are:

\[ q_{PR}^i = \frac{2(A - c)}{5} \]  

(11)

\[ \Pi_{PR}^i = \frac{2(A - c)^2}{25} \]  

(12)

3.2 Universal Relative Performance Contracts

Under the present candidate equilibrium configuration, in the first stage of the game, firms’ owners simultaneously and non-cooperatively commit to Relative Performance contracts. Thus, in the third stage of the game, manager \( i \) chooses \( q_i \) in order to maximize his utility given by eq. (4), taking as given the output of his rival, \( q_j \), along with the managerial contracts and the managerial incentive parameters set in the previous stages. The first order condition of eq. (4) provides manager \( i \)'s reaction function:

\[ q_{RP}^i (q_{RP}^j) = \frac{A - c - (1 + a_{RP}^i) q_{RP}^j}{2} \]  

(13)

When owner \( i \) compensates his manager with a Relative Performance contract, the slope of manager \( i \)'s reaction curve is \( \frac{da_{RP}^i}{dq_{RP}^j} = -\frac{(1 + a_{RP}^i)}{2} < \frac{da^C}{dq_j} = \frac{da_{RP}^i}{dq_{RP}^j} \), since \( a_{RP}^i > 0 \). From eq. (13), manager \( i \), when choosing the output level of his firm, considers \(- (1 + a_{RP}^i)\) as the competing manager’s best response to the level of output that he sets. By choosing an incentive parameter \( a_{RP}^i > 0 \), the competitor’s best response considered by the manager \( i \), \(- (1 + a_{RP}^i)\), is lower than that considered under a strict own profit-maximizing behavior, \(-q_j^C\). Thus, owner \( i \) makes his manager more
aggressive, i.e. he directs him to produce a level of output, higher than that produced under a strict own profit-maximizing behavior. Diagrammatically (Figure 1), as owner $i$ sets $a_i^{RP} \rightarrow 1$, he changes the slope of his manager’s reaction curve, compared with $q_i^C(q_j^C)$, from $RC_i^C$ to $RC_i^{RP}$.

Solving the system of the reaction functions, output levels are:

$$q_i^{RP}(a_i^{RP}, a_j^{RP}) = \frac{(A - c)(a_i^{RP} + 1)}{3 + a_i^{RP} + a_j^{RP} - a_i^{RP}a_j^{RP}} \quad (14)$$

In the second stage of the game, each owner $i$ chooses $a_i^{RP}$ so as to maximize profits given by:

$$\Pi_i^{RP}(a_i^{RP}, a_j^{RP}) = \frac{(A - c)^2(a_i^{RP} + 1)(1 - a_i^{RP}a_j^{RP})}{(3 + a_i^{RP} + a_j^{RP} - a_i^{RP}a_j^{RP})^2} \quad (15)$$

The first order condition of eq. (15) provides owner $i$’s best response function:

$$a_i^{RP}(a_j^{RP}) = \frac{1 - a_j^{RP}}{1 + 3a_j^{RP}} \quad (16)$$

Now, exploiting the problem’s symmetry, it is assumed that $a_i^{RP} = a_j^{RP} = a^{RP}$. Hence, the symmetric equilibrium managerial incentive parameter is:

$$a_i^{RP^*} = \frac{1}{3} \quad (17)$$

Plugging $a_i^{RP^*}$ in eq. (14) and (15), firm $i$’s equilibrium output and profits are:

$$q_i^{RP^*} = \frac{3(A - c)}{8} \quad (18)$$

$$\Pi_i^{RP^*} = \frac{3(A - c)^2}{32} \quad (19)$$

### 3.3 Universal Market Share Contracts

Under the present candidate equilibrium configuration, in the first stage of the game, firms’ owners simultaneously and non-cooperatively commit to Market Share contracts. Thus, in the third stage of the game, manager
Then, since the Jacobian determinant:

\[
\det J = \begin{vmatrix} \frac{\partial R_i}{\partial q_1} & \frac{\partial R_i}{\partial q_2} \\ \frac{\partial R_j}{\partial q_1} & \frac{\partial R_j}{\partial q_2} \end{vmatrix} = (q_1 + q_2)^3 (a_1 + a_2) (q_1 + q_2)^2 > 0
\]

This is a system of analytically non-solvable equations. Jansen et al. (2007) prove that for any “feasible” pair \((a_1^M, a_2^M)\) of weights, the reaction curves are smooth and concave, intersecting once, say at \([q_{1M}^M(a_1^M, a_2^M), q_{2M}^M(a_1^M, a_2^M)]\).

Then, since the Jacobian determinant:

\[
J_F = \begin{bmatrix} \frac{\partial R_i}{\partial q_1} & \frac{\partial R_i}{\partial q_2} \\ \frac{\partial R_j}{\partial q_1} & \frac{\partial R_j}{\partial q_2} \end{bmatrix} = (q_1 + q_2)^3 (a_1 + a_2) (q_1 + q_2)^2 > 0
\]

the Implicit Function Theorem implies that the functions \(q_{1M}^M(a_1^M, a_2^M)\) and \(q_{2M}^M(a_1^M, a_2^M)\) have continuous partial derivatives with respect to the weights \(a_1^M\) and \(a_2^M\).\(^{10}\) Plugging \(q_{1M}^M\) and \(q_{2M}^M\) in eq. (2), firm \(i\)'s profits in the second stage of the game are given by:

\[
\Pi_i^M(a_1^M, a_2^M) = [A - a_i M(a_1^M, a_2^M) - q_j M(a_1^M, a_2^M) - c] q_i M(a_1^M, a_2^M) \quad (21)
\]

Then, the first-order conditions for firms 1 and 2, for an equilibrium in the second stage of the game, are formulated:

\[
\begin{align*}
\frac{\partial \Pi_1^M(a_1^M, a_2^M)}{\partial a_1} = 0 & \iff \frac{\partial q_1}{\partial a_1} \left( A - c - 2q_1^M - q_2^M \right) - q_1^M \frac{\partial q_1}{\partial a_1} = 0 \\
\frac{\partial \Pi_2^M(a_1^M, a_2^M)}{\partial a_2} = 0 & \iff \frac{\partial q_2}{\partial a_2} \left( A - c - q_1^M - 2q_2^M \right) - q_2^M \frac{\partial q_2}{\partial a_2} = 0 \quad (22)
\end{align*}
\]

Moreover,\(^{10}\) Jansen et al. (2007) prove that the second-order conditions for the second stage of the game do hold.
\[
\frac{\partial q_i^M}{\partial a_i} = -\frac{1}{J_F} \det \begin{bmatrix}
\frac{\partial R_i}{\partial a_i} & \frac{\partial R_i}{\partial q_i^M} \\
\frac{\partial R_i}{\partial a_i} & \frac{\partial R_i}{\partial q_i^M}
\end{bmatrix}
\]
\[
\frac{\partial q_i^M}{\partial a_i} = -\frac{1}{J_F} \det \begin{bmatrix}
\frac{\partial R_i}{\partial a_i} & \frac{\partial R_i}{\partial q_i^M} \\
\frac{\partial R_i}{\partial a_i} & \frac{\partial R_i}{\partial q_i^M}
\end{bmatrix}
\]

Plugging \(\frac{\partial q_i^M}{\partial a_i}\), \(\frac{\partial q_i^M}{\partial a_2}\), \(\frac{\partial q_i^M}{\partial a_2}\) and \(\frac{\partial q_i^M}{\partial a_2}\) in eq. (22) and solving the system of eq. (20) and (22), manager \(i\)'s equilibrium managerial incentive parameter and firm \(i\)'s equilibrium output are: \(^{11}\)

\[
q_i^{M^*} = \left( 2 - 10\sqrt{2} \right) (A - c)^2
\]
\[
q_i^{M^*} = \left( 4 + \sqrt{2} \right) (A - c)
\]

Plugging \(q_i^{M^*}\) in eq. (2), firm \(i\)'s equilibrium profits are:

\[
\Pi_i^{M^*} = \frac{(A - c)^2}{10 + \sqrt{2}}
\]

### 3.4 Coexistence of Profits-Revenues and Relative Performance Contracts

Consider the asymmetric candidate equilibrium configuration \((pr - rp)\) where firm 1's owner compensates his manager with a Profits-Revenues contract while firm 2's owner rewards his manager with a Relative Performance one. \(^{12}\)

Under the present candidate equilibrium, in the third stage of the game, manager 1 (2) chooses \(q_1\) \((q_2)\) in order to maximize his utility given by eq. (3) [(4)]. The corresponding reaction curves for managers 1 and 2 are given by eq. (6) and eq. (13) respectively. Solving the system of the reaction functions, output levels are:

\(^{11}\)The formulas for \(\frac{\partial q_i^M}{\partial a_i}\), \(\frac{\partial q_i^M}{\partial a_2}\), \(\frac{\partial q_i^M}{\partial a_2}\) and \(\frac{\partial q_i^M}{\partial a_2}\) are given in Appendix A1.

\(^{12}\)Of course, due to the symmetric industry structure, the reverse configuration where owner 1 compensates his manager under a Relative Performance contract while owner 2 rewards his manager under a Profits-Revenues one, is as well (implicitly) proposed as a candidate configuration.
In the second stage of the game, owners set the incentive parameters so as to maximize profits given by:

\[ \Pi_{pr1} (a_{pr1}, a_{rp2}) = (A + c) (1 - 2a_{pr1}) \]

\[ \Pi_{pr2} (a_{pr1}, a_{rp2}) = (A + c) (a_{pr1} - a_{pr1} a_{rp2} - 2) \]

The corresponding best response functions for owners 1 and 2 are:

\[ a_{pr1} (a_{rp2}) = A (a_{rp2} - 1) + c (3a_{rp2} + 5) \]

\[ a_{pr2} (a_{pr1}) = A + c (a_{pr1} - 2) \]

Solving the system of the best response functions, (candidate) equilibrium managerial incentive parameters are:

\[ a_{1}^{(pr-rp)*} = a_{2}^{(pr-rp)*} = 1 \]

Note that \( a_{1}^{(pr-rp)*} = a_{2}^{(pr-rp)*} = 1 \). This implies that when owner \( i \) compensates his manager with a Profits-Revenues contract, while owner \( j \) compensates his manager with a Relative Performance contract, owner \( j \)'s best response to owner \( i \) is a manager who is concerned with firm \( i \)'s profits as much as he is concerned with his firm's profits. This implies that owner \( j \)'s best response to owner \( i \) is a manager whose behavior reaches its maximum level of aggressiveness, i.e. \( a_{2}^{(pr-rp)*} = 1 \). At the same time, owner \( i \)'s best response to owner \( j \) is a manager with a strict profit-maximizing behavior.
Plugging \( a_1^{(pr-rp)^*} \) and \( a_2^{(pr-rp)^*} \) in eq. (26), (27), (28) and (29), equilibrium outputs and profits are:

\[
q_1^{(pr-rp)^*} = \frac{(A - c)}{4} \quad q_2^{(pr-rp)^*} = \frac{(A - c)}{2} \tag{33}
\]

\[
\Pi_1^{(pr-rp)^*} = \frac{(A - c)^2}{16} \quad \Pi_2^{(pr-rp)^*} = \frac{(A - c)^2}{8} \tag{34}
\]

Note that, \( q_2^{(pr-rp)^*} > q_1^{(pr-rp)^*} \) and \( \Pi_2^{(pr-rp)^*} > \Pi_1^{(pr-rp)^*} \). Intuitively, when owner \( i \) compensates his manager with a Profits-Revenues contract, the slope of his manager’s reaction curve is given by \( \left| \frac{dq^{PR}_i}{dq_j} \right| = \frac{1}{2} \). When owner \( i \) compensates his manager with a Relative Performance contract, the corresponding slope is \( \left| \frac{dq^{RP}_i}{dq_j} \right| = \frac{1+a^{RP}_i}{2} \). Since \( a^{RP}_i > 0 \) always holds, it is straightforward that \( \left| \frac{dq^{RP}_i}{dq_j} \right| > \left| \frac{dq^{PR}_i}{dq_j} \right| \), i.e. the reaction curve of the manager who is compensated under the Relative Performance contract is steeper than that corresponding to the Profits-Revenues contract. In the present configuration of contracts, the point where manager 1’s reaction curve intersects manager 2’s reaction curve implies that the manager 2 sets output at a level higher than the competing manager does, i.e. \( q_2^{(pr-rp)^*} > q_1^{(pr-rp)^*} \) which results in \( \Pi_2^{(pr-rp)^*} > \Pi_1^{(pr-rp)^*} \).

### 3.5 Coexistence of Profits-Revenues and Market Share Contracts

Consider the asymmetric candidate equilibrium configuration \((pr - m)\) where firm 1’s owner compensates his manager with a Profits-Revenues contract while firm 2’s owner rewards his manager with a Market Share one. Under the present candidate equilibrium, in the third stage of the game, manager 1 (2) chooses \( q_1 \) (\( q_2 \)) in order to maximize his utility given by eq. (3) [(5)]. The equilibrium quantities of the third stage of the game satisfy the following system of equations:
\[
A - d_{1p}^r c - 2q_{1p}^m - q_{2m}^m = RC_1^{(pr-m)}(a_{1p}^r, a_{2p}^m, q_{1p}^r, q_{2m}^m) = 0 \\
A - c - 2q_{2m}^m - q_{1p}^r + a_{2m}^m \frac{q_{1p}^r}{q_{1p}^r + q_{2m}^m} = RC_2^{(pr-m)}(a_{1p}^r, a_{2m}^m, q_{1p}^r, q_{2m}^m) = 0 
\]

(35)

This is a system of analytically non-solvable equations. Since the Jacobian determinant:
\[
J_F = \begin{bmatrix}
\frac{\partial R_1}{\partial q_{1p}^r} & \frac{\partial R_1}{\partial q_{2m}^m} \\
\frac{\partial R_2}{\partial q_{1p}^r} & \frac{\partial R_2}{\partial q_{2m}^m}
\end{bmatrix} = 3 + \left(\frac{q_{1p}^r + 3q_{2m}^m}{q_{1p}^r + q_{2m}^m}\right)^2 > 0
\]

the Implicit Function Theorem implies that the functions \( q_1^{(pr-m)} = q_1^{(pr-m)}(a_{1p}^r, a_{2m}^m) \) and \( q_2^{(pr-m)} = q_2^{(pr-m)}(a_{1p}^r, a_{2m}^m) \) have continuous partial derivatives with respect to the weights \( a_{1p}^r \) and \( a_{2m}^m \). Plugging \( q_1^{(pr-m)} \) and \( q_2^{(pr-m)} \) in eq. (2), the first-order conditions, for an equilibrium of the second stage of the game, are formulated. Then, following the procedure applied in the case of Universal Market Share Contracts, equilibrium managerial incentive parameters and outputs are:\(^{13}\)

\[
a_{1p}^{*(pr-m)} = \frac{4A - 3c - \sqrt{17}(A - c)}{c} \quad a_{2p}^{*(pr-m)} = \frac{(10 - \sqrt{17})(A - c)^2}{8} \\
q_1^{*(pr-m)} = \frac{(3\sqrt{17} - 11)(A - c)}{4} \quad q_2^{*(pr-m)} = \frac{(5 - \sqrt{17})(A - c)}{2} 
\]

(36)

(37)

Plugging \( q_1^{*(pr-m)} \) and \( q_2^{*(pr-m)} \) in eq. (2), equilibrium profits for firms 1 and 2 are:

\[
\Pi_1^{*(pr-m)} = \frac{8(A - c)^2}{53 + 13\sqrt{17}} \quad \Pi_2^{*(pr-m)} = \frac{(\sqrt{17} - 5)^2(A - c)^2}{8} 
\]

(38)

\(^{13}\)The formulas for \( \frac{\partial q_{1p}^r}{\partial a_{1p}^r}, \frac{\partial q_{2m}^m}{\partial a_{1p}^r}, \frac{\partial q_{1p}^r}{\partial a_{2m}^m} \) and \( \frac{\partial q_{2m}^m}{\partial a_{2m}^m} \) are given in Appendix A2.
3.6 Coexistence of Market Share and Relative Performance Contracts

Consider the asymmetric candidate equilibrium configuration $(m - rp)$ where firm 1’s owner compensates his manager with a Market Share contract while firm 2’s owner rewards his manager with a Relative Performance one. Under the present candidate equilibrium, in the third stage of the game, manager 1 (2) chooses $q_1$ ($q_2$) in order to maximize his utility given by eq. (5) [(4)]. The equilibrium quantities of the third stage of the game satisfy the following system of equations:

\[
\begin{align*}
A - c - 2q_1^m - q_2^{rp} + a_2^{rp} \frac{q_1^m}{(q_1^m + q_2^{rp})} &= RC_1^{(m-rp)}(a_1^m, a_2^{rp}, q_1^m, q_2^{rp}) = 0 \\
A - c + (a_2^{rp} - 1)q_1^m - 2q_2^{rp} &= RC_2^{(m-rp)}(a_1^m, a_2^{rp}, q_1^m, q_2^{rp}) = 0
\end{align*}
\]  

(39)

This is a system of analytically non-solvable equations. Since the Jacobian determinant:

\[
J_F = \begin{vmatrix}
\frac{\partial R_1}{\partial q_1^m} & \frac{\partial R_1}{\partial q_2^{rp}} \\
\frac{\partial R_2}{\partial q_1^m} & \frac{\partial R_2}{\partial q_2^{rp}}
\end{vmatrix} = 3 + a_2^{rp} + \frac{a_1^m[(1-a_2^{rp})q_1^m + q_2^{rp}(3+a_2^{rp})]}{(q_1^m + q_2^{rp})^3} > 0
\]

the Implicit Function Theorem implies that the functions $q_1^{(m-rp)} = q_1^{(m-rp)}(a_1^m, a_2^{rp})$ and $q_2^{(m-rp)} = q_2^{(m-rp)}(a_1^m, a_2^{rp})$ have continuous partial derivatives with respect to the weights $a_1^{rp}$ and $a_2^{rp}$. Plugging $q_1^{(m-rp)}$ and $q_2^{(m-rp)}$ in eq. (2), the first-order conditions, for an equilibrium of the second stage of the game, are formulated. Then, following the procedure applied in the case of Universal Market Share Contracts, equilibrium managerial incentive parameters and outputs are:

\[
a_1^{(m-rp)*} = 0 \quad a_2^{(m-rp)*} = 1
\]

(40)

\[
q_1^{(m-rp)*} = \frac{(A - c)}{4} \quad q_2^{(m-rp)*} = \frac{(A - c)}{2}
\]

(41)

Plugging $q_1^{(rp-m)*}$ and $q_2^{(rp-m)*}$ in eq. (2), equilibrium profits for firms 1

\[\text{The formulas for } \frac{\partial q_1^m}{\partial a_1^{rp}}, \frac{\partial q_2^{rp}}{\partial a_1^{rp}}, \frac{\partial q_1^m}{\partial a_2^{rp}} \text{ and } \frac{\partial q_2^{rp}}{\partial a_2^{rp}} \text{ are given in Appendix A3.} \]
and 2 are:

\[
\Pi_1^{(m-\text{rp})^*} = \frac{(A - c)^2}{16} \quad \Pi_2^{(m-\text{rp})^*} = \frac{(A - c)^2}{8}
\]  

(42)

The fact that \( a_1^{(m-\text{rp})^*} = 0 \) while \( a_2^{(m-\text{rp})^*} = 1 \), implies that when owner \( j \) compensates his manager with a Relative Performance contract, while owner \( i \) compensates his manager with a Market Share contract, owner \( j \)'s best response to owner \( i \) is a manager who is concerned with firm \( i \)'s profits as much as he is concerned with his firm's profits. Subsequently, owner \( j \)'s best response to owner \( i \) is a manager whose behavior reaches its maximum level of aggressiveness, i.e. \( a_2^{(m-\text{rp})^*} = 1 \). At the same time, owner \( i \)'s best response to owner \( j \) is a manager with a strict profit-maximizing behavior. This is the reason why, \( q_2^{(m-\text{rp})^*} > q_1^{(m-\text{rp})^*} \) which results in \( \Pi_2^{(m-\text{rp})^*} > \Pi_1^{(m-\text{rp})^*} \).

4 Equilibrium managerial incentive contracts

In this part of the paper, firms owners’ strategies in the first stage of the game are investigated. In order to answer the question with respect of the optimal structure of contracts that owners choose to compensate their managers, firms’ profits under the different subgames are compared. The results are summarized in the following Proposition:

Proposition 1 Under Cournot competition and ex-ante commitment over the type of contract that each firm’s owner chooses, each owner’s dominant strategy is to reward his manager with a Relative Performance type of contract.

This Proposition establishes a strategic rationale for committing to Relative Performance contracts. The rest of this section is devoted for giving the intuition behind this result in detail.

Comparing equilibrium output and profits under the three symmetric subgames, i.e. the symmetric configurations of managerial contracts, with those obtained in the benchmark case where production decisions are taken by owners, the following Corollary summarizes the findings:
Corollary 1 (i) \( q_i^{PR^*} > q_i^{C^*} \) and \( \Pi_i^{PR^*} < \Pi_i^{C^*} \).

(ii) \( q_i^{RP^*} > q_i^{C^*} \) and \( \Pi_i^{RP^*} < \Pi_i^{C^*} \).

(iii) \( q_i^{M^*} > q_i^{C^*} \) and \( \Pi_i^{M^*} < \Pi_i^{C^*} \).

The reason why the use of a managerial incentive contract increases output is the following: owner \( i \), by using an incentive contract strategically, directs his manager to a more aggressive behavior in order to force the competing manager to reduce output. Because each owner acts in the same way at the game’s contract stage, firms end up in a prisoners’ dilemma situation. Naturally, the increased market supply, in comparison to the no-delegation case, leads to lower profits. These findings are in the spirit of the well-known result in the industrial organization literature that firms competing in quantities have no incentive to engage in Stackelberg warfare.

Then, equilibrium output and profits under the three symmetric configurations of managerial contracts are ranked. The following Corollary summarizes the findings:

Corollary 2 (i) \( q_i^{RP^*} < q_i^{M^*} < q_i^{PR^*} \).

(ii) \( \Pi_i^{RR^*} > \Pi_i^{M^*} > \Pi_i^{PR^*} \).

Intuitively, the strategic use of managerial incentive contracts leads to a prisoners’ dilemma situation always. This situation is the least severe in case of the Universal Relative Performance Contracts candidate equilibrium. Moreover, it turns out that the Universal Market Share Contracts candidate equilibrium is more (less) competitive than the Universal Relative Performance (Profits-Revenues) Contracts one. This implies that the former equilibrium, when compared with the later, is characterized by higher output and lower profits.

Considering the asymmetric equilibria as well, the results are the following:

Corollary 3 \( q_2^{(pr-rp)^*} = q_2^{(m-mp)^*} > q_2^{(pr-m)^*} > q_1^{(pr-m)^*} > q_1^{(pr-rp)^*} = q_1^{(m-mp)^*} > q_i^{C^*} \).
With respect to \( q_2^{(pr−rp)} = q_2^{(m−rp)} \) and \( q_1^{(pr−rp)} = q_1^{(m−rp)} \), recall that
when owner \( i \) compensates his manager either with a Profits-Revenues contract or with a Market Share contract and owner \( j \) compensates his manager with a Relative Performance contract, owner \( j \)'s best response to owner \( i \) is a manager whose behavior reaches its maximum level of aggressiveness. At the same time, owner \( i \)'s best response to owner \( j \) is a manager with a strict profit-maximizing behavior. This is the reason why the subgames \( (pr−rp) \) and \( (m−rp) \) lead to identical firm-level quantities and profits. Note also that \( q_i^{S^*} \in \left[ q_1^{(pr−m)}, q_2^{(pr−m)} \right] \), \( S = PR, M, RP \), implying that in the symmetric configurations of managerial incentive contracts, each manager sets output at a level lower (higher) than the level set by the manager compensated with the contract that intensifies the competition relatively less (more), since there is no asymmetry that an owner can exploit.

Corollary 3 leads to the following ranking regarding the equilibrium profits of the different subgames:

**Corollary 4**

\[
\Pi_i^{C*} > \Pi_2^{(pr−rp)*} = \Pi_2^{(m−rp)*} > \Pi_2^{(pr−m)*} > \Pi_1^{(pr−m)*} > \Pi_1^{(pr−rp)*} = \Pi_1^{(m−rp)*}.
\]

As for the equilibrium profits, \( \Pi_i^{S^*} \in \left[ \Pi_1^{(pr−m)*}, \Pi_2^{(pr−m)*} \right] \), \( S = PR, M, RP \).

Regarding the industry-level equilibrium output \( Q = \sum_{i=1}^{2} q_i \) and total profits \( T\Pi = \sum_{i=1}^{2} \Pi_i \), the results are the following:

**Corollary 5**

(i) \( Q^{PR*} > Q^{(pr−m)*} > Q^{M*} > Q^{RP*} > Q^{(pr−rp)*} = Q^{(m−rp)*} > \)

\( Q^{C*} \).

(ii) \( T\Pi^{C*} > T\Pi^{RP*} > T\Pi^{(pr−rp)*} = T\Pi^{(m−rp)*} > T\Pi^{M*} > T\Pi^{(pr−m)*} > T\Pi^{PR*}. \)

The results stated in Corollary 5 reveal the competitiveness of the alternative configurations of managerial incentive contracts. Recall that although the strategic use of managerial incentive contracts leads to a prisoners’ dilemma situation always, this situation is the least severe in case of the
Universal Relative Performance Contracts, implying that this configuration is the least competitive and firms’ profits are the highest. The intuition behind this result goes as follows: a Relative Performance contract makes a manager’s behavior less susceptible to strategic manipulation by rivals. Less scope for strategic manipulation gives to the rival owner less reason to provide incentives for aggressive behavior to his manager and thus, competition remains softer.

Note also that competition becomes fiercer the more Profits-Revenue managers are active, as each one of them makes his rival relatively more aggressive. Thus, competition under the Universal Profits-Revenue configuration turns out to be the most fierce.

5 Welfare analysis

In this Section we perform a welfare analysis. More specifically, total welfare under the alternative configurations of contracts is compared with total welfare in the benchmark case where production decisions are taken by firms’ owners.

Let total welfare $TW$ be defined as the sum of firms’ profits and consumers’ surplus.

$$TW^D = T\Pi^D + CS^D,$$  

$D = PR, RP, M, (pr-rp), (pr-m), (m-rp)$ (43)

with $T\Pi^D$ and $CS^D = \frac{1}{2} (Q^D)^2$ being the overall industry profits and consumers’ surplus respectively. Using equilibrium results, total welfare for each configuration of contracts is given in Appendix B. The results are summarized in the following Proposition:

**Proposition 2** Under Cournot competition:

(i) The use of managerial incentive contracts increases total welfare, i.e., $TW^D > TW^C$.

(ii) The Universal Profits-Revenues Contracts (Universal Relative Performance Contracts) equilibrium leads to the highest (lowest) total welfare,
i.e., $SW^{PR} > SW^{(pr-m)} > SW^M > Q^{RP} > SW^{(pr-rp)} = SW^{(m-rp)}$.

As far as the first part of the Proposition is concerned, it has already been clear that, compared with the no-delegation case, the strategic use of an incentive contract by an owner, in order to become Stackelberg leader, distorts his manager’s incentives away from profit-maximization, increasing output and consumers’ surplus. This tends to increase total welfare. Because each owner acts in the same way, in equilibrium, the increased market supply leads to lower industry profits than those obtained under the benchmark no-delegation case. The reduced total industry profits tend to decrease total welfare, but this effect is completely compensated by the higher consumers’ surplus. It is found that the positive effect of increased consumers’ surplus on total welfare dominates the negative effect of decreased profits which results in $TW^D > TW^C$.

Regarding the second part of the Proposition, it has already been mentioned that competition becomes fiercer the more Profits-Revenue managers are active, as each one of them makes his rival relatively more aggressive. The fiercer the competition, the higher the consumers’ surplus and the lower the firms’ profits. Note also that since it is the “consumers’ surplus” effect that dominates on total welfare, the ranking of total welfare is identical with the ranking of the industry-level equilibrium total output, stated in Corollary 5 (i).

## 6 Equilibrium managerial incentive contracts under no commitment

So far analysis was grounded on the assumption that each firm’s owner commits to a particular type of contract in the first stage of the game. In this part of the paper, the case where there is no ex-ante commitment over the types of contracts that owners offer to managers is investigated. In order to examine which types of managerial incentive contracts prevail in equilibrium, we consider a two-staged game with the following timing: in the first stage, each firm’s owner chooses one type of contract to reward his manager and
sets the corresponding managerial incentive parameter $a_i^D$, $D = PR, RP, M$. The crucial, yet (due to the symmetric industry) reasonable assumption here is that the precise contract (the type of contract and the managerial incentive parameter) that owner $i$ sets is not observable by the rival owner, before contract-setting is everywhere completed. Thus, each owner can independently shift from one type of contract to another. In the second stage of the game, managers compete a la Cournot. To investigate the conditions under which, a candidate equilibrium configuration of strategies, in the types of contracts (one for each owner) at the first stage, is proposed. Subsequently, it is checked whether or not it survives all possible deviations. If yes, the proposed equilibrium is a Subgame Perfect Nash Equilibrium.

Consider the symmetric configuration where both firms’ owners choose to compensate their managers with Profits-Revenues contracts. Figure 1 offers the visualization of this candidate equilibrium (point $E^{PR}$). Note that in equilibrium, two conditions must be fulfilled: firstly, since production decisions are taken by managers, their reaction curves $RC_1^{PR}$ and $RC_2^{PR}$ must be intersected. Secondly, the fact that firm 1’s (firm 2’s) owner offers an incentive contract to his manager, as a strategic tool in order to become Stackelberg leader against firm 2 (firm 1), implies that, in equilibrium, there must be tangency between firm 1’s (firm 2’s) isoprofit curve $\Pi^1$ ($\Pi^2$) and manager 2’s (manager 1’s) reaction curve $RC_2^{PR}$ ($RC_1^{PR}$).

Universal Profits-Revenues Contracts is an equilibrium configuration only if no owner has an incentive to unilaterally deviate by offering to his manager either a Relative Performance contract or a Market Share one. Therefore, two possible deviations have to be checked.

Regarding the first deviation, suppose that owner 2 sticks to the Profits-Revenues contract, believing that owner 1 will offer to his manager the same type of contract. In the first stage of the game, owner 2 sets the parameter $a_2^{PR^*} = \frac{1}{3}$ that corresponds to the Universal Profits-Revenues Contracts equilibrium. Owner 1 deviates towards compensating his manager with a Relative Performance contract. Thus, owner 1 uses his reaction function $a_1^{(rp-pr)}(a_2)$ with $a_2 = a_2^{PR^*} = \frac{1}{3}$ to optimally adjust the parameter $a_1$ for his manager’s contract. Thus, owner 1 sets:
\[ a_{1d}^{PR} = \frac{1}{2} \]

The deviant owner 1's profits will be \( \Pi_{1d}^{PR} = \Pi_1^{(r_p-p_r)}(a_1, a_2) \), where \( a_1 = a_{1d}^{PR} \) and \( a_2 = a_{2d}^{PR} = \frac{1}{3} \).

\[ \Pi_{1d}^{PR} = \frac{2(A - c)^2}{25} \]

Interestingly enough \( \Pi_{1d}^{PR} = \Pi_1^{PR} \), implying that the magnitude of owner 1's incentive to deviate from the Universal Profits-Revenues Contracts equilibrium towards compensating his manager with a Relative Performance contract, is zero.
Diagrammatically, when owner 1 deviates from the Universal Profits-Revenues Contracts candidate equilibrium towards compensating his manager with a Relative Performance contract, owner 1 directs his manager to the Relative Performance reaction curve $RC_1^{RP}$ and, by readjusting the managerial incentive parameter, optimally readjusts its slope until $RC_1^{RP*}$. Compared with $RC_1^{PR}$, manager 1 under the $RC_1^{RP*}$ has now become a more aggressive seller, because given $q_2$ he now produces more. However, the manager of the deviant owner sets output at a level equal to the one he would set under the candidate equilibrium ($E^{PR}$). Only at point $E^{PR}$, both of the aforementioned equilibrium conditions hold. More specifically, only at point
The reaction curve $RC_{1}^{PR^*}$ is intersected with $RC_{2}^{PR}$ and also, the firm 1’s (firm 2’s) isoprofit curve $\Pi_1$ ($\Pi_2$) is tangent to the manager 2’s (manager 1’s) reaction curve $RC_{2}^{PR}$ ($RC_{1}^{PR}$). Note that neither point $C_1$ nor point $C_2$ can emerge in the subgame perfect equilibrium because the slope of the firm 1’s isoprofit curve $\Pi_1$ is non-zero, i.e. points $C_1$ and $C_2$ do not belong in the manager 1’s reaction curve $RC_{1}^{PR^*}$.

In the same lines, examining the second deviation, i.e., the case where owner 1 deviates from the Universal Profits-Revenues Contracts candidate equilibrium towards compensating his manager with a Market Share contract, one finds that the magnitude of owner 1’s incentive to deviate from the Universal Profits-Revenues Contracts candidate equilibrium towards compensating his manager with a Market Share contract, is zero. With respect to the subgame equilibrium of the game, the results are summarized in the following Proposition:

**Proposition 3** Under Cournot competition and no ex-ante commitment over the type of contract that each firm’s owner chooses, each type of contract, namely, a Profits-Revenues contract, a Relative Performance contract, a Market Share contract, is owner i’s best response to owner j’s choice.

This result is driven by the fact that in equilibrium, two conditions must be fulfilled: firstly, managers’ reaction curves must be intersected and secondly, each firm’s isoprofit curve must be tangent to the competing manager’s reaction curve. The above conditions do hold for all the different types of contracts that owners can offer to their managers, regardless the functional forms of cost and demand. Therefore, assuming no ex-ante commitment over any type of contract that owners will offer to managers, even a contract of different type could be an owner’s best response to the competing owner’s choice.

Note that the two-staged game is characterized by multiplicity of equilibria. Subsequently, the following question is addressed: which types of the alternative managerial incentive contracts will finally emerge in equilibrium? Using the results for equilibrium profits from Section 3, by employing focal point analysis, it is found that rival owners would realize that it is in their
mutual interest to move towards the equilibrium that ensures them the highest profits. Therefore it is expected that each owner will finally choose to compensate his manager with a Relative Performance contract.

7 Conclusion

In this paper we have investigated the endogenous structure of incentive contracts that firms’ owners offer to their managers, when these contracts can be either of a “Profits-Revenues” type, or of a “Relative Performance”, or, finally, of a “Market Share” one. The analysis suggested that each firm’s owner dominant strategy is to reward his manager with a Relative Performance type of contract. Intuitively, a Relative Performance contract makes a manager’s behavior less susceptible to strategic manipulation by rivals. Less scope for strategic manipulation gives to the rival owner less reason to provide incentives for aggressive behavior to his manager and thus, competition remains softer, implying higher profits. Considering the case in which there is no ex-ante commitment over any type of contract that each firm’s owner offers to his manager, the analysis revealed that each type of contract is a best response to the competing owner’s choice. This result is driven by the fact that in equilibrium, two conditions must be fulfilled: firstly, managers’ reaction curves must be intersected and secondly, each firm’s isoprofit curve must be tangent to the competing manager’s reaction curve. By employing focal point analysis, it was found that each owner will finally choose to compensate his manager with a Relative Performance contract.

Regarding the societal consequences of the alternative delegation schemes, the use of managerial incentive contracts is found to increase total welfare. Moreover, the Universal Profits-Revenues Contracts equilibrium leads to the highest total welfare, since it is the most competitive.

Our results have been derived in the context of a duopolistic market with a linear demand and cost model. We are of the opinion that the duopolistic market reveals all essential differences between the alternative types of contracts. We are also aware of the limitations of our analysis in assuming specific functional forms. However, the properties of the equilibrium condi-
tions that drive our results that allows us to argue that these results will also hold under general demand and cost functions. The use of more general forms would jeopardize the clarity of our findings, without significantly changing their qualitative character.

Appendix

Appendix A1: Universal Market Share Contracts

\[
\frac{\partial q^M_1}{\partial a_1^M} = \frac{2q^M_2 (q^M_1 + q^M_2)^3 + a^M_2 q^M_1}{\partial q^M_1 + q^M_2}\left[3(q^M_1 + q^M_2)^4 + (q^M_1 + q^M_2)(q^M_1 + 3q^M_2) \alpha^M_1 + ((q^M_1 + q^M_2)(3q^M_1 + q^M_2) + a^M_2) a^M_2\right]
\]

\[
\frac{\partial q^M_2}{\partial a_1^M} = -\frac{q^M_1 (q^M_1 + q^M_2)^3 + a^M_2 q^M_2}{(q^M_1 + q^M_2)^4 + (q^M_1 + q^M_2)(q^M_1 + 3q^M_2) \alpha^M_1 + ((q^M_1 + q^M_2)(3q^M_1 + q^M_2) + a^M_2) a^M_2}
\]

\[
\frac{\partial q^M_1}{\partial a_2^M} = \frac{2q^M_2 (q^M_1 + q^M_2)^3 + a^M_2 q^M_1}{(q^M_1 + q^M_2)^4 + (q^M_1 + q^M_2)(q^M_1 + 3q^M_2) \alpha^M_1 + ((q^M_1 + q^M_2)(3q^M_1 + q^M_2) + a^M_2) a^M_2}
\]

\[
\frac{\partial q^M_2}{\partial a_2^M} = \frac{q^M_1 (q^M_1 + q^M_2)^3 + a^M_2 q^M_2}{(q^M_1 + q^M_2)^4 + (q^M_1 + q^M_2)(q^M_1 + 3q^M_2) \alpha^M_1 + ((q^M_1 + q^M_2)(3q^M_1 + q^M_2) + a^M_2) a^M_2}
\]

Appendix A2: Coexistence of Profits-Revenues and Market Share Contracts

\[
\frac{\partial q^{pr}_1}{\partial a^{pr}_1} = \frac{2a^{pr}_2 (q^{pr}_1 + q^{pr}_2)}{3(q^{pr}_1 + q^{pr}_2)^4 + (3q^{pr}_1 + q^{pr}_2) a^{pr}_1}
\]

\[
\frac{\partial q^{pr}_2}{\partial a^{pr}_1} = -\frac{q^{pr}_1 (q^{pr}_1 + q^{pr}_2)^4 + (3q^{pr}_1 + q^{pr}_2) a^{pr}_1}{3(q^{pr}_1 + q^{pr}_2)^4 + (3q^{pr}_1 + q^{pr}_2) a^{pr}_1}
\]

\[
\frac{\partial q^{pr}_1}{\partial a^{pr}_2} = \frac{c(q^{pr}_1 + q^{pr}_2)^4 - (3q^{pr}_1 + q^{pr}_2) a^{pr}_2}{3(q^{pr}_1 + q^{pr}_2)^4 + (3q^{pr}_1 + q^{pr}_2) a^{pr}_1}
\]

\[
\frac{\partial q^{pr}_2}{\partial a^{pr}_2} = \frac{2c \left[1 + \frac{a^{pr}_2 q^{pr}_1}{(q^{pr}_1 + q^{pr}_2)^4}\right]}{3 + \frac{(3q^{pr}_1 + q^{pr}_2) a^{pr}_2}{(q^{pr}_1 + q^{pr}_2)^4}}
\]

Appendix A3: Coexistence of Market Share and Relative Performance Contracts

\[
\frac{\partial q^m_1}{\partial a^m_1} = \frac{2a^m_2 (q^m_1 + q^m_2)}{(q^m_1 + q^m_2)^3 + a^m_2 q^m_1}
\]

\[
\frac{\partial q^m_2}{\partial a^m_1} = -\frac{q^m_1 (q^m_1 + q^m_2)^3 + a^m_2 q^m_2}{(q^m_1 + q^m_2)^3 + a^m_2 q^m_1}
\]

\[
\frac{\partial q^m_1}{\partial a^m_2} = \frac{2a^m_2 (q^m_1 + q^m_2)}{(q^m_1 + q^m_2)^3 + a^m_2 q^m_1}
\]

\[
\frac{\partial q^m_2}{\partial a^m_2} = -\frac{q^m_1 (q^m_1 + q^m_2)^3 + a^m_2 q^m_2}{(q^m_1 + q^m_2)^3 + a^m_2 q^m_1}
\]

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\[
\frac{\partial q^n}{\partial a^n} = -\frac{q^n_1 \left[ 1 + \left( \frac{q^n_p - q^n_m}{q^n_p + q^n_m} \right)^{-m} \right]}{3 + a^n_2 + a^n_1 \frac{q^n_1(1-a^n_1)}{(q^n_p + q^n_m)^3} + 2 a^n_1 q^n_p (3 + a^n_2)}
\]

\[
\frac{\partial q^n_p}{\partial a^n} = -\frac{q^n_1 \left[ -2 - \frac{2 a^n_1 q^n_p}{(q^n_p + q^n_m)^3} \right]}{3 + a^n_2 + a^n_1 \frac{q^n_1(1-a^n_1)}{(q^n_p + q^n_m)^3} + 2 a^n_1 q^n_p (3 + a^n_2)}
\]

Appendix B: Total Welfare

\[
TW^{PR} = \frac{12(A - c)^2}{25} \quad TW^{RP} = \frac{15(A - c)^2}{32}
\]

\[
TW^M = \frac{7(A - c)^2}{19 - 3\sqrt{2}} \quad TW^{(pr-rp)} = \frac{15(A - c)^2}{32}
\]

\[
TW^{(pr-m)} = \frac{16(A - c)^2}{13 + 5\sqrt{17}} \quad TW^{(m-rp)} = \frac{15(A - c)^2}{32}
\]

References


