(In)efficient trading forms in competing vertical chains

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Abstract

We study competing vertical chains where upstream and downstream firms bargain over their form and terms of trading. Both (conditionally) inefficient wholesale price contracts and efficient contracts that take the form of price-quantity bundles (and not of two-tariffs) arise in equilibrium under different parameter configurations. Changes in bargaining power distribution affect market outcomes by altering the trading terms and, more importantly, the trading form. As a result, a firm might benefit by a reduction in its bargaining power and consumers could benefit from an increase in the downstream “countervailing power” or from a more uneven bargaining power distribution.

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1. Introduction

We study transactions between firms that operate at different stages of a vertical supply chain, such as input producers and final good manufacturers or wholesalers and retailers - “upstream” and “downstream” firms in general. A standard assumption in the existing literature is that downstream firms play a passive role in these transactions. We assume instead that the downstream firms are large players that participate actively not only in setting their final market prices, but also in determining how they trade with their upstream suppliers. Within this framework, a number of fundamental issues arise: How is vertical trading organized? How do bilateral negotiations between large upstream and downstream firms affect their form and terms of trading? How do final market outcomes depend on the distribution of bargaining power between the upstream and the downstream firms? Do final consumers benefit from the “countervailing” power of large downstream firms? What is the role of strategic competition across chains? This set of questions is not only of theoretical interest but also of practical importance.

Downstream firms are in many cases large players, actively involved in shaping their contracts with their suppliers, as the widespread evidence of increased concentration in the downstream sectors of many industries suggests. The food industry, in which large “supermarkets” become increasingly stronger in trading with their suppliers, is one of the examples that have recently received significant attention. The picture is similar in many other industries: large tour operators trading with airlines and hotels, car manufacturers purchasing car parts, large book retailers (e.g. Barnes & Noble) dealing with publishers, large clothing retailers or, indeed, general retailers in both the U.S. (e.g. Wal-Mart) and Europe (e.g. Carrefour).

We employ the simplest model that allows us to study transactions in competing vertical chains when both the upstream and the downstream firms participate actively in the determination of their form and terms of trade. There are two vertical chains, each consisting of one upstream and one downstream firm. The firms play a three-stage game with observable actions. At the first stage, within each vertical chain the

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1 See e.g. European Commission report (1999), and OECD report (1999). In 1996, the five largest food retailers had a 64% market share in the U.K., 61% in the Netherlands, 52% in France, and 41% in Germany, while the concentration in food retailing in Europe exceeded the concentration in food manufacturing and there was evidence of increased net margins for the top retailers (Dobson and Waterson, 1999).

upstream and downstream firm bargain in order to determine their form of trading. The possible trading forms, or contract types, are: a (linear) wholesale price contract, a two-part tariff contract, and a price-quantity “bundle” (or “package,” specifying the total input quantity and its corresponding total price). At the second stage, the upstream and downstream firm in each chain bargain over the contract terms of their previously selected contract types. Finally, at the third stage, downstream firms produce differentiated final goods and compete in quantities.

We show that upstream-downstream bargaining plays an important role in vertical trading not only because it affects the terms of trade but, more importantly, because it can also affect the form of trade emerging in different industries. Since different trading forms can appear in the presence of bargaining than in its absence, our analysis highlights the role of bargaining power distribution for vertical supply chains outcomes. We show that, while in the absence of bargaining, i.e. when all power is either upstream or downstream, “conditionally efficient” contracts are dominant, in the presence of bargaining, “conditionally inefficient” contracts such as (linear) wholesale price contracts may arise. Also, within the set of conditionally efficient contracts, two-part tariffs are always dominated by price-quantity bundles, and thus, never arise in equilibrium (though socially optimal). Our analysis thus reveals that price-quantity bundle contracts, which have been largely ignored in the literature, consist a significant form of trading within vertical chains.3

The intuition for these results lies on the features of the different contract types. As mentioned above, both price-quantity bundles and two-part tariffs are conditionally efficient contracts, that is, lead to the maximization of the “pie” (i.e. the vertical chain’s joint profits), given the rival chain’s strategy. Yet, price-quantity bundles are preferred to two-part tariffs, due to their superior commitment value. In particular, a vertical chain, by using a price-quantity bundle contract, is able to commit to a certain final output level before reaching the final market competition stage. On the other hand, wholesale price contracts do not lead to the maximization of a chain’s joint profits (conditional on rival behavior), since in the absence of fixed fees there is double marginalization. However, they may arise in equilibrium, as they turn out to be an attractive choice for non-very powerful downstream firms. Due to the absence of transfers, a wholesale price plays a double role: not only controls how

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3 Björnerstedt and Stennek (2004), Kolay and Shaffer (2003), and Rey and Tirole (2003) are some exceptions that also consider such contracts, however, their focus is very different from ours.
aggressive the downstream firm in the final good market is, but it also determines how the surplus is shared within the chain. As a result, a downstream firm with little power is allowed to keep a larger share of the (otherwise smaller) pie under a wholesale price contract than under a price-quantity bundle contract. An implication of the above is that product differentiation, by shifting the emphasis from strategic competition vis-à-vis the rival chain to how the surplus is divided within the chain, makes wholesale price contracts more likely to appear.

Since a change in the distribution of bargaining power may lead to the adoption of different trading forms, it can also lead to drastic changes in the levels of firm’s profits, consumers’ surplus, and total welfare. Interestingly, we find that a firm - upstream or downstream - might benefit from a reduction in its own bargaining power. The intuition comes from the fact that a change in the allocation of the bargaining power between the upstream and the downstream firms can affect the equilibrium outcomes not only through changes in the contract terms but, more importantly, through changes in the contract types. In particular, from the viewpoint of a downstream firm, although a reduction in its bargaining power means that it captures a smaller share of the pie, it can also mean a more favourable way of dividing the pie due to the possible appearance of wholesale price contracts. From the viewpoint of an upstream firm, while an increase in its bargaining power leads to an increase in its share of the pie, it can also lead to a smaller pie and a less favourable way of dividing it, through the appearance of conditionally inefficient contracts.

An analysis along the above lines also allows us to address the following important question: since in an increasing number of markets “countervailing” power of large retailers becomes a significant factor, does such a force operate in the benefit of consumers and total welfare? We find that an increase in the “countervailing” power of downstream firms can, under certain conditions, be beneficial both for the consumers and welfare. This is so because wholesale price contracts, that imply high final market prices, do not appear in equilibrium when the downstream bargaining power is sufficiently high. Interestingly enough, we also find that a more even distribution of bargaining power may turn out to be harmful both for the consumers and total welfare. When the distribution of bargaining power within chains is extreme, conditionally efficient contracts, that due to the absence of double marginalization lead to lower final prices, tend to arise.
Some important modifications of our basic model are also considered in the paper. First, we enlarge the set of possible trading forms, by allowing each vertical chain to vertically integrate. We show that this option is strategically weaker than trading via a price-quantity bundle contract (because such contracts have a commitment value). Second, we relax the assumption made in the main body of the analysis that the price-quantity bundles have direct “downstream quantity commitment” (that is, the downstream firm’s final output is directly dictated by the input quantity specified in the contract). We show that our main results hold true, independently of whether the price-quantity bundles are with or without downstream quantity commitment, at least as long as the marginal production cost of the input is not too low. Intuitively, a vertical chain can still commit (indirectly) to aggressive downstream behavior by employing such a contract, because it can induce its downstream firm to act as a zero marginal cost competitor (up to the specified input quantity) during the final market competition stage. Third, we exclude by assumption price-quantity bundles (as most of the related literature has also done) and analyze the case in which the choice of contracts is only between two-part tariffs and wholesale prices. We find that, in the absence of price-quantity bundles, the appearance of wholesale price contracts becomes more likely.

Our paper is related to an extensive and influential literature on strategic vertical contracting. In particular, it is more closely related to work that has focused on how vertical contracts influence downstream competition. An important difference between our paper and the existing literature is that the latter has ignored the role of bargaining in the endogenous organization of vertical trading. For instance, while Gal-Or (1991) and Rey and Stiglitz (1995) have endogenized the choice among two-part tariff and wholesale price contracts, they have analyzed only the unilateral incentives of the upstream firms. In particular, in a setting in which the contract types used by two competing vertical chains are chosen before the terms, and the

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4 Thus, the fact that we focus in the basic model on the case where the price-quantity bundles are with downstream quantity commitment is, to a great extent, for expositional simplicity.
5 We also discuss a number of other extensions, such as downstream price competition, unobservability of contract terms, etc.
6 For a review see e.g. Tirole (1988, ch.4), Katz (1989), Dobson and Waterson (1996), Irmen (1998), and Rey and Tirole (2003). The set of important contributions is large and includes the influential papers of Vickers (1985) and Fershtman and Judd (1987).
7 Different strands of the literature have focused on other important aspects of the issue, such as the role of uncertainty in the selection of contracts - see e.g. Rey and Tirole (1986), Martimort (1996), and Kühn (1997).
downstream competition is in prices, they have shown that the wholesale price contracts are always dominated by the two-part tariff contracts. By allowing the downstream firms’ participation in the contracting procedure, we show that the distribution of bargaining power may affect not only the contract terms and final markets prices but also the contract types that emerge. In particular, we show that linear pricing may emerge endogenously; this is important, because, while such contracts are observed in reality and have been studied in the literature, in previous work on strategic contracting they do not emerge in equilibrium.

Some recent papers have also considered settings with upstream-downstream bargaining. In particular, Dobson and Waterson (1997), Chen (2003), and Marx and Shaffer (2003, 2004), using different models, have offered important insights as to how the increasing downstream bargaining power can affect the terms of vertical trading and the final consumers. However, these papers differ significantly from ours. In addition to various differences in modelling firms’ behavior, they consider a single upstream firm and do not deal with the endogenous choice of the trading form in competing chains, which is our main focus.

We should also stress that, in contrast to much of the previous work on vertical contracting where transactions are restricted to follow either linear or two-part tariff pricing schemes, we also consider price-quantity bundles. As, during upstream-downstream bargaining over contracts, the two parties cannot be restrained from putting both dimensions of the transaction (total payment and quantity) on the table, one should include contracts that specify a single point in the price-quantity space in the feasible set of contracts. Our analysis suggests that price-quantity bundle contracts play a key role and they should be observed frequently in reality. This finding is consistent with how vertically-linked firms with market power trade in many industries: e.g. tour operators make lump-sum payments to airlines (or hotels) before the beginning of a tourist season for a given number of seats (or rooms); and airlines agree with manufacturers to purchase a given number of aircrafts for a given total payment and so on. Thus, the a priori exclusion of the price-quantity bundles from the analysis is, on the one hand, inconsistent with some real-world cases, and on the other, may lead to flawed inferences about firms’ profits, consumers’ surplus and

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8 Other papers such as Horn and Wolinsky (1988), Inderst and Wey (2003), and Inderst and Shaffer (2004) have examined the incentives for mergers, investment decisions and product variety in settings with upstream-downstream bargaining.
welfare: assuming that wholesale prices prevail would overestimate final prices, whereas assuming two-part tariff competition would underestimate them.

The remainder of the paper is as follows. In Section 2, we set up the basic model. In Section 3, we examine the final two stages of the game, for given choices of contract types: downstream competition and equilibrium contract terms. In addition, we emphasize the main strategic characteristics of the different contractual configurations. In Section 4, we analyze the first stage equilibrium, that is, the choice of contract types. In Section 5, we examine the effect of a change in the distribution of bargaining power. In Section 6, we extend our model in order to examine first the possibility of vertical integration, then the case when price-quantity bundles are without direct downstream quantity commitment, and finally the choice only among two-part tariffs and wholesale price contracts. In Section 7, we discuss some of the model’s assumptions and possible directions for future research. Section 8 concludes. All proofs are relegated to the Appendix.

2. The Basic Model

We consider a two-tier industry consisting of two upstream firms and two downstream firms (e.g. input suppliers and final good producers). Each upstream firm, denoted by $U_i, i=1,2$, produces an input facing a constant marginal cost equal to $c$. Each downstream firm, denoted by $D_i, i=1,2$, produces a final good transforming one unit of input into one unit of final product. Each downstream firm has an exclusive relationship with one of the two upstream firms. In terms of notation, we assume that $U_i$ has an exclusive relationship with $D_i, i=1,2$, and refer to each $(U_i, D_i)$ pair as a vertical chain. We assume that a downstream firm faces no other costs than the total cost of obtaining the input from its upstream supplier.

The inverse demand function for the final product of downstream firm $D_i$ is:

$$p_i = a - q_i - \gamma q_j; \quad i, j = 1,2; \quad i \neq j; \quad 0 \leq c < a; \quad 0 < \gamma \leq 1$$

where $q_i$ and $p_i$ are respectively the quantity and the price of firm $D_i$’s final product. The parameter $\gamma$ measures the substitutability between the two final products, namely the higher is $\gamma$, the closer substitutes the two final goods are.

The terms of trade within each vertical chain are determined by a contract, prior to any productive activity. Each vertical chain can select among three different trading forms or contract types. The first, denoted in what follows by $W$, is a linear pricing
contract, consisting simply of a wholesale price $w_i$ that $D_i$ has to pay per unit of input. The second, denoted by $T$, is a two-part tariff contract, consisting of a wholesale price plus a fixed fee - transfer, $(w_i, F_i)$. The third type, denoted by $B$, is a price-quantity bundle (or package) contract, specifying the total input quantity along with its corresponding total price, $(q_i, T_i)$. In the main body of the analysis, we assume that the total input quantity specified in the price-quantity bundle directly dictates the final good quantity. In Section 6, we relax this assumption, that is, we allow the final good quantity to be lower than the total input quantity specified by the $B$ contract.

To capture the idea that trading forms are often strategic decisions with “longer-run” characteristics than the choice of the exact contract terms, we postulate that each vertical chain first selects its contract type and then chooses its specific terms. This is a standard assumption in the literature (see e.g. Irmen, 1998). Indeed, the contract type may be viewed as representing the form of the relationship between the firms in the vertical chain that is manifested in the particular form of organization and communication among the parties, and hence cannot be changed very often or easily.9

We assume that both the upstream and downstream firms possess some power over setting both the type and terms of the vertical contracts. To keep the analysis simple, we restrict attention to the case where the distribution of power is identical across vertical chains and, within each vertical chain, across the contract type and contract terms negotiations.10 In particular, we assume that the bargaining power of each upstream firm is $\beta$ and of each downstream firm $1-\beta$, with $0 \leq \beta \leq 1$.

In particular, we analyze a three-stage game with observable actions. The timing of the game is depicted in Fig. 1. In the first stage, the type of contract that will be subsequently signed within each vertical chain is selected. We assume that, within each vertical chain, with probability $\beta$ the contract is chosen by the upstream firm and with probability $1-\beta$ by the downstream firm. As these probability draws are independent across chains, while a contract type is chosen in one chain, it is not

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9 In Section 7, we further discuss this assumption as well as the implications of considering the case of simultaneous bargaining over both the contract type and terms.
10 In principle, the power that a firm possesses in setting the contract type does not have to be equal to its bargaining power over the contract terms as the two procedures often involve distinct layers of the firm’s management. Whereas the assumption of constant bargaining power across both negotiation stages is adopted here for simplicity (its generalization is straightforward), it can be justified on the grounds that the firms’ relative power cannot differ too much across bargaining stages.
known whether it is the upstream, or the downstream, firm which gets the initiative to choose the contract type in the rival chain.\textsuperscript{11}

In the second stage, bargaining over the specific terms of the selected contracts takes place within each vertical chain. For instance, if the contract type employed by the \((U_i, D_i)\) chain is a two-part tariff then, in the second stage, \(U_i\) and \(D_i\) negotiate over the value of both the wholesale price \(w_i\) and the fixed fee \(F_i\). As is standard in the literature, we use the generalized Nash bargaining solution to determine the negotiations outcome - the contract terms - within each vertical chain.\textsuperscript{12} We assume moreover that, during their bargaining, each vertical chain takes as given the outcome of the negotiations in the rival chain; that is, the solution concept employed is Nash equilibrium between the two Nash Bargaining problems.\textsuperscript{13}

In the third stage, each downstream firm chooses its \textit{final product quantity}, unless it is engaged in a price-quantity bundle contract which directly dictates its final good quantity.

We derive the subgame perfect Nash equilibria of the above three-stage game.

3. Contract Terms and Downstream Competition

- Third Stage: Downstream Competition

If neither vertical chain has signed a \(B\) contract, the last stage corresponds to a standard (differentiated goods) Cournot game. Each downstream firm \(D_i\), given its input price \(w_i\) and the quantity of its rival \(q_j\), chooses \(q_i\) to maximize its profits:

\textsuperscript{11} This is the simplest way of capturing the participation of both the upstream and downstream firms in the contract type selection and in particular, in negotiations over a discrete choice variable such as the contract type. Bargaining has been modeled in a similar way in different settings - see e.g. De Fraja and Sákovics (2001), Chemla (2003), and Rey and Tirole (2003).

\textsuperscript{12} This way of modelling the bargaining procedures across stages (over the contract type and contract terms) is not only for analytical convenience, but is also natural since while the contract types are discrete choice variables, the contract terms are continuous variables. Qualitatively similar results would also be obtained in the following two scenarios: (i) If the contract terms negotiations were modelled in line with the stage one bargaining, i.e. within each vertical chain, the contract terms are chosen with probability \(\beta\) by the upstream firm and with probability \(1-\beta\) by the downstream firm, and (ii) if one uses the generalized Nash Bargaining solution to solve the “convexified” contract type bargaining problem where the parties are negotiating over the \textit{probability} with which the upstream firm will be chosen to set the contract type.

\textsuperscript{13} Note however that, as we are dealing with a compound problem that encompasses two synchronous bargaining processes, applying the Nash bargaining solution is not entirely straightforward, since one should account for the dynamic interdependencies between the simultaneous bargaining sessions. Nevertheless, if there is no exchange of information among sessions while negotiations last and if downstream competition occurs only after bargaining has been terminated in both sessions, one can show that Binmore’s (1987) observation that the Nash solution is essentially implemented by non-cooperative, alternating offer and counter-offer bargaining games à la Rubinstein (1982) can be extended to this case too.
The reaction functions of the downstream firms are:
\[ R_i(q_j, w_i) = \frac{a - \gamma q_j - w_i}{2}. \]  

Clearly, a decrease in the wholesale price charged to \( D_i \) shifts its reaction function upwards and makes the downstream firm more aggressive in the final goods market. From (3), we obtain the Cournot equilibrium quantities:
\[ q_i(w_i, w_j) = \frac{a(2 - \gamma) - 2w_i + \gamma w_j}{4 - \gamma^2}. \]

If only one vertical chain has signed a \( B \) contract, the quantity produced by that chain’s downstream firm has been determined in the previous stage during the contract terms negotiations. Hence, the chain’s downstream firm simply transforms all the purchased input quantity to output, while the downstream firm of the rival chain, which employs either a \( T \) or a \( W \) contract, reacts optimally to that quantity according to equation (3). This corresponds to a standard (Stackelberg) leader-follower game.

Finally, if both chains have signed \( B \) contracts, since the terms of a \( B \) contract dictate also the quantity of the final product, in the last stage downstream firms simply produce the quantities that have been specified in the previous stage; hence downstream firms make no strategic decisions in the market competition stage.

**Second Stage: Contract Terms**

In the second stage, within each vertical chain, the upstream and downstream firms negotiate over the terms of their already selected contracts. There are nine possible subgames. In what follows we will use the notation \([X, Y]\) for the subgame where the \((U_1, D_1)\) vertical chain employs contract type \( X \) and the \((U_2, D_2)\) chain employs \( Y \), with \( X,Y \in \{W, T, B\} \). Rather than going through the cumbersome derivation of equilibria for each of the nine possible subgames, we will selectively present key intuitive arguments that are required for the determination of the equilibrium contracts in the next section. In Tables 1 and 2 the equilibrium wholesale prices, quantities and profits for all the possible subgames are reported.

We start our analysis by stating the following Lemma.

**Lemma 1:** (a) Whenever a vertical chain employs a \( B \) or a \( T \) contract, contract terms negotiations lead to the maximization of the chain’s joint (upstream plus downstream)
profits, \( \pi_{(U_i,D_i)} = \pi_{U_i} + \pi_{D_i} \), given the contract terms of the rival chain. Moreover, the chain’s profits are distributed to the upstream and downstream firm according to their respective bargaining powers, \( \beta \) and \((1 - \beta)\).

(b) Whenever a vertical chain employs a \( W \) contract, the chain’s joint profits are not maximized (given the rival chain’s strategy) and are distributed so that the ratio of upstream to downstream firm’s profits is lower than their relative bargaining power, \( \beta / (1 - \beta) \).

An important implication of Lemma 1 is that while both the price-quantity bundles and the two-part tariffs are conditionally efficient – i.e. they maximize the chain’s joint profits given the rival chain’s strategy – the wholesale price contracts are not. Moreover, while under both the price-quantity bundles and the two-part tariffs the “pie” (i.e. the chain’s joint profits) is shared according to the firms’ bargaining power, under the wholesale price contracts the downstream firms enjoy a larger share (of the smaller “pie”) than the one corresponding to their bargaining power. The intuition for this last result is as follows. An increase in the wholesale price raises the upstream profits by less than the final good output, because such an increase in the marginal cost of the downstream firm has a negative effect on final market production and, consequently, on the input quantity demanded by the downstream firm. On the other hand, the downstream profits decrease by more than the final good output, because an increase in its marginal cost makes the rival downstream firm more aggressive and this negative strategic effect adds up with the (negative) own-costs effect. In addition, maximization of the chain’s Nash product implies that the optimal wholesale price is such that the weighted - by the respective bargaining powers - percentage decrease in upstream profits and percentage increase in downstream profits should be equal. Therefore, the ratio of upstream to downstream profits under the optimal wholesale price is lower than their relative bargaining power.

Using Lemma 1, we can make a number of observations regarding the equilibrium outcomes under alternative contractual configurations. If both vertical chains employ a \( B \) contract, each chain maximizes its joint profits, given the rival chain’s input, and thus output quantity:

\[
\max_{q_i} \pi_{(U_i,D_i)}^{BB} = (a - q_i - \gamma q_j)q_i - cq_i.
\]
Thus, in the \([B, B]\) subgame, the two vertical chains play a standard Cournot game with marginal costs equal to the “true” marginal cost of input \(c\). The equilibrium input, and thus final good, quantities are \(q_i^{BB} = \frac{\nu}{2 + \gamma}\), where we define \(\nu = a - c\).

If one vertical chain - say chain \((U_1, D_1)\) - employs a price-quantity bundle and the other chain \((U_2, D_2)\) a two-part tariff, their interaction is as follows. \((U_1, D_1)\) chooses its input quantity, \(q_1\), and simultaneously \((U_2, D_2)\) selects its wholesale price, \(w_2\), each in order each to maximize its joint profits. Since the \((U_1, D_1)\) chain, through its input quantity choice, can commit to an equal final good production by its downstream firm, it acts as a Stackelberg leader in setting its quantity, to which the rival chain’s downstream firm will react as Stackelberg follower in the last stage according to (3).

Formally, the two vertical chains’ maximization problems are:

\[
\max_{q_1} \pi_{(U_1,D_1)}^{BT}(q_1, w_2) = (a - q_1 - \gamma R_2(q_1, w_2) - c)q_1, \quad \text{and} \\
\max_{w_2} \pi_{(U_2,D_2)}^{BT}(q_1, w_2) = (a - R_2(q_1, w_2) - \gamma q_1 - c)R_2(q_1, w_2).
\]

By inspection of (7), the higher the negotiated wholesale price \(w_2\) is, the lower is the downstream \(D_2\)’s output (since from (3), \(\partial R_2 / \partial w_2 < 0\)) and the lower are the joint profits of the \((U_2, D_2)\) chain. Therefore, the chain employing the \(T\) contract optimally sets its wholesale price equal to marginal cost, \(w_2^{BT} = c\). As the rival downstream firm’s quantity is taken as given when the wholesale price is negotiated, the \((U_2, D_2)\) chain knows that, in the last stage its downstream firm will act as a monopolist on the residual demand. It has, thus, no incentive to manipulate the wholesale price \(w_2\) in order to commit its downstream firm to a more aggressive behavior in the final product market. As a consequence, the \([B, T]\) case reduces to a standard Stackelberg game with both marginal costs equal to \(c\) and the equilibrium quantities are equal to \(q_1^{BT} = \frac{\nu(2 - \gamma)}{2(2 - \gamma^2)}\) and \(q_2^{BT} = \frac{\nu(4 - 2\gamma - \gamma^2)}{4(2 - \gamma^2)}\).

In contrast to the previous case, when both vertical chains employ a two-part tariff contract, the negotiated wholesale price of each will not be equal to the marginal cost of input \(c\). In this case, each chain chooses \(w_i\) to maximize its joint profits, taking as given the wholesale price of its rival, i.e.

\[
\max_{w_i} \pi_{(U_i,D_i)}^{TT}(w_i, w_j) = [a - q_k(w_i, w_j) - \gamma q_j(w_i, w_j) - c]q_k(w_i, w_j), \quad \text{where} \quad q_k(w_i, w_j), \quad k = i, j, \quad \text{are given by (4)}.\]

From (8), we find that the equilibrium wholesale prices satisfy: \(w_i^{TT} = [2c(2 + \gamma) - a\gamma^2] / (4 + 2\gamma - \gamma^2) < c\). Thus, in the \([T,
T] case, wholesale prices reflect a subsidy from the upstream firms to their respective downstream firms. The intuition is that, by lowering its wholesale price, a chain allows its downstream firm to commit to more aggressive behavior. Technically, it shifts its downstream firm’s reaction curve out and, as the reaction curves are downward-sloping, this results in lower quantity for the rival downstream firm, and higher quantity and profit for the own firm. A similar result has been obtained in the “strategic delegation” literature, where the upstream firms (firms’ owners) unilaterally set two-part tariffs, in anticipation of their downstream firms’ (managers’) quantity competition (see e.g. Vickers, 1985, Fershtman and Judd, 1987, and Sklivas, 1987).\footnote{This holds when there is a single downstream firm associated with each upstream. Otherwise, issues of “intra-brand” competition arise and, when endogenizing the number of downstream rivals, the incentives may be reversed; see Baye et al. (1996), and Saggi and Vettas (2002).} Here, we extend this result to the case where both the upstream and the downstream firms participate actively in the determination of the contract terms. In this regard, notice that the equilibrium level of \( w^{TT}_{i} \) is independent of the bargaining power distribution \((\beta, 1 - \beta)\). This is because, by Lemma 1, when a \( T \) contract is employed, a chain maximizes its joint profits (given the rival chain’s behavior). Further, note that, while each vertical chain chooses to unilaterally commit to more aggressive behavior by setting its input price below its true marginal input cost, in equilibrium, the two chains’ profits are lower than those in the \([B, B]\) case, in which the chains maximize joint profits on the basis of their true marginal input cost \(c\).\footnote{Loosely speaking, one can say that the two chains are trapped into a “prisoners’ dilemma”; while illustrative and often used in similar contexts, such a description is not entirely accurate in one respect, that the wholesale prices levels do not represent dominant strategies.} This is also reflected by the fact that the equilibrium output under \( T \) contracts is larger than under \( B \) contracts, i.e. \( q^{TT}_{i} = 2\nu/(4 + 2\gamma - \gamma^2) > q^{BB}_{i} \).

The previous analysis leads to the following Lemma.

**Lemma 2:** The equilibrium joint profits of the vertical chains satisfy

(a) \( \pi^{BB}_{(U_1,D_1)} > \pi^{TT}_{(U_1,D_1)}, \) and

(b) \( \pi^{BT}_{(U_2,D_2)} > \pi^{BB}_{(U_1,D_1)} > \pi^{TB}_{(U_1,D_1)} (= \pi^{BT}_{(U_2,D_2)}). \)

Now, let us turn to the rest of the cases where at least one vertical chain employs a wholesale price contract. When e.g. chain \((U_2, D_2)\) employs a \( W \) contract, its negotiated wholesale price solves:
\[ \max_{w_2} \left[ \pi_{U_2}^{\beta} \left[ \pi_{D_2}^{\beta} \right]^{1-\beta} \right] = \left[ (w_2 - c) q_2(.) \right]^{\beta} \left[ (a - \gamma q_1(.) - q_2(.) - w_2) q_2(.) \right]^{1-\beta}, \]  \hspace{1cm} (9)

where \( q_i(.) = q_i(w_1, w_2) \) is given by (4) when the rival chain \((U_1, D_1)\) employs a \(W\) or a \(T\) contract. While if \((U_1, D_1)\) employs a \(B\) contract, \(q_1(.) = q_1\) is taken as given by \((U_2, D_2)\) and \(q_2(.) = R_2(q_1, w_2)\) (see (3)). The following Lemma compares a chain’s joint profits under a price-quantity bundle and a two-part tariff in case that the rival chain employs a wholesale price contract.

**Lemma 3:** The joint profits of a vertical chain are higher under a \(B\) than under a \(T\) contract, when the rival chain employs a \(W\) contract, \(\pi_{(U_2, D_2)}^{BW} > \pi_{(U_1, D_1)}^{TW}\).

The intuition is as follows. When the \((U_2, D_2)\) chain employs a \(W\) contract, the \((U_1, D_1)\) chain can achieve higher joint profits under a \(B\) than under a \(T\) contract, as long as the wholesale price of \((U_2, D_2)\) is not lower in the \([B, W]\) than in the \([T, W]\) case. Again, this is so because \((U_1, D_1)\)’s downstream firm acts as a Stackelberg leader in the former case, while as a Cournot competitor in the latter case. Indeed, the wholesale price of \((U_2, D_2)\) is lower when the rival chain employs a \(T\) than a \(B\) contract. In the former case, \((U_2, D_2)\) has an incentive to lower its input price \(w_2\) to make its downstream firm more aggressive in the final good market, while in the latter case this strategic incentive is absent, because \(D_2\) is a Stackelberg follower acting as a monopolist on the residual demand. In addition, a decrease in \(w_2\) has a stronger positive effect on \(D_2\)’s output in the \([T, W]\) than the \([B, W]\) case, because in the former case the \((U_2, D_2)\) chain expects \(D_1\) to optimally adjust its quantity along its downwards sloping reaction function, while in the latter case it takes \(D_1\)’s output as given.

It should be noticed that in all the subgames in which at least one vertical chain employs a \(W\) contract, the equilibrium outcome depends on the bargaining power distribution. For instance, in the \([W, W]\) case, the equilibrium wholesale price and output are (see Table 1):

\[ w_{1w} = \frac{2(2-\beta)c + a\beta(2-\gamma)}{4-\beta\gamma} > c \quad \text{and} \quad q_{1w} = \frac{2\gamma(2-\beta)}{(2+\gamma)(4-\beta\gamma)}. \]  \hspace{1cm} (10)

It is straightforward to check that, as the bargaining power of the upstream firm, \(\beta\), tends to zero, the wholesale price tends to the marginal input cost \(c\). Moreover, the higher \(\beta\) is, the higher is the wholesale price and the lower the final good quantity.
This is in sharp contrast to the other subgames, where maximization of the chains’ joint profits implies that the bargaining power distribution simply dictates how the chain’s maximum joint profits are shared between the upstream and downstream firm.

Clearly, as \( w_{IT}^W < c < w_{IT}^W \), wholesale prices are higher under \( W \) contracts than under \( T \) contracts. Finally, since under \( B \) contracts competition is based on true marginal input costs \( c \), the “imputed” wholesale price under \( B \) contracts lies in between the other two cases. An immediate consequence is that aggregate final output is the highest under two-part tariffs and the lowest under wholesale price contracts, with that of price-quantity bundle contracts lying in between.

4. Equilibrium Contractual Configurations

We now determine the first stage equilibrium. Since the strategy set of each \( U_i \) and \( D_i \) has three elements, \( \{ W, T, B \} \), there exist nine possible contractual configurations within each vertical chain, and thus eighty-one first stage candidate equilibria. The following Proposition simplifies the subsequent analysis by substantially reducing the number of candidate equilibria:

**Proposition 1:** For each upstream firm \( U_i \) and each downstream firm \( D_i \), \( i=1,2 \), a two-part tariff contract \( T \) is strictly dominated by a price-quantity bundle contract \( B \), for all values of \( \beta \) and \( \gamma \).

According to Proposition 1, price-quantity bundle contracts always dominate two-part tariffs contracts. This holds both for the upstream and the downstream firm within a chain, regardless of the contract type chosen by the rival chain. The intuition for this result is as follows. Recall from Lemma 1(a) that, under both \( B \) and \( T \) contracts, the interests of both the upstream and downstream firm are aligned with the interests of the vertical chain. Moreover, recall that both \( B \) and \( T \) contracts are conditionally efficient. Still, the \( B \) contracts are preferred to the \( T \) contracts, because they have an additional advantage that is absent in the case of the \( T \) contracts. In particular, the \( B \) contracts have a commitment value since they allow the chain to commit to a certain output level before reaching the final market competition stage. If the rival chain employs either a \( T \) or a \( W \) contract, a vertical chain obtains higher joint profits with a \( B \) than a \( T \) contract, because with the former it can transform its downstream firm to a Stackelberg leader in the final good market. Moreover, in case
that the rival chain employs a $B$ contract, a vertical chain again attains higher profits with a $B$ than a $T$ contract, because while with a $T$ contract its downstream firm is a Stackelberg follower, with a $B$ contract it is a Cournot competitor in the final good market. Therefore, in the first stage, the vertical chain always “expects” to attain higher profits with a $B$ contract than with a $T$ contract.\(^{16}\)

The next Proposition also contributes to the reduction of the number of candidate equilibria by stating that the $W$ contracts are also always dominated for the upstream firms (as we show below the same does not always hold for the downstream firms).

**Proposition 2:** For each upstream firm $U_i$, $i = 1,2$, a wholesale price contract $W$ is strictly dominated by a price-quantity bundle contract $B$, for all values of $\beta$ and $\gamma$.

The intuition of Proposition 2 stems directly from Lemma 1. An upstream firm prefers a $B$ to a $W$ contract, because under a $B$ contract both the size of the pie and its own share of the pie are larger than under a $W$ contract.

In view of Propositions 1 and 2, the only equilibria that remain feasible are: $[(B, B), (B, B)]; [(B, W), (B, W)]; [(B, B), (B, W)]$ and $[(B, W), (B, B)]$. Note that in the notation we use here, the first entry within each bracket refers to the contractual configurations proposed by the upstream and downstream firm, respectively, within the $(U_i, D_i)$ chain and the second entry to the ones proposed, respectively, within the $(U_2, D_2)$ chain.

The equilibrium contractual configurations are stated in Proposition 3.

**Proposition 3:** There exist continuous functions $\beta_W(\gamma)$ and $\beta_B(\gamma)$, increasing in $\gamma$, with $\lim_{\gamma \to 0} \beta_W(\gamma) = 0, \lim_{\gamma \to 0} \beta_B(\gamma) = 0, \beta_W(1) = 0.791, \beta_B(1) = 0.882,$ and $\beta_W(\gamma) < \beta_B(\gamma)$ such that:

(a) The contractual configuration $[(B, W), (B, W)]$ is an equilibrium for $\beta \geq \beta_W(\gamma)$.

(b) The contractual configuration $[(B, B), (B, B)]$ is an equilibrium for $\beta \leq \beta_B(\gamma)$.

(c) The asymmetric contractual configurations $[(B, B), (B, W)]$ and $[(B, W), (B, B)]$ never arise in equilibrium.

\(^{16}\)The expectation here refers to the uncertain outcome of the negotiations over the contract type in the rival chain. The chain’s upstream and downstream firm rationally expect that these negotiations will lead with probability $\beta$ to the contract preferred by the rival upstream firm and with probability $1 - \beta$ to the contract preferred by the rival downstream firm.
According to Proposition 3, different contractual configurations can emerge in equilibrium under different distributions of the bargaining power. Clearly, the \( W \) contract is not always dominated for the downstream firms and thus the configuration in which a vertical chain employs a \( W \) contract can arise in equilibrium. In particular, the configuration \([(B, W), (B, W)]\) is the unique equilibrium if, for given degree of product differentiation, the upstream bargaining power is not too low, i.e. \( \beta > \beta_b(\gamma) \). The configuration instead in which both vertical chains always employ \( B \) contracts is the unique equilibrium if the upstream bargaining power is sufficiently low, i.e. \( \beta < \beta_w(\gamma) \). Note that for intermediate values of \( \beta, \beta_w(\gamma) \leq \beta \leq \beta_b(\gamma) \), we have two equilibrium configurations: \([(B, W), (B, W)]\) and \([(B, B), (B, B)]\). Fig. 2 illustrates the respective regions in the \((\beta, \gamma)\) space.

The intuition of Proposition 3 is as follows. A downstream firm, while comparing a \( B \) to a \( W \) contract (recall that \( T \) contracts are dominated), faces the following trade-off: although with a \( W \) contract the pie is smaller, its share of the pie is larger. When the upstream power is not too low (\( \beta \) high enough), a downstream firm prefers a \( W \) contract because its share of the pie under a \( B \) contract (reflected in its power, \( 1 - \beta \)) is not large enough. Moreover, when the upstream power is high enough, it is likely that a \( B \) contract is the outcome of the rival chain’s first stage negotiations, and thus the size of the pie size that the chain expects to enjoy by also using a \( B \) contract is not that big. This may provide an additional incentive for the downstream firm to opt for a \( W \) contract.

A number of testable implications regarding the type of contracts one should expect to observe in different industries can be derived from the above analysis. First, only price-quantity bundles are expected to be observed in industries in which there is no bargaining. This can be seen from Fig. 3, where the bold line represents the likelihood that a \( W \) contract will be used by at least one of the chains \( (1 - \beta)^2 + 2\beta(1 - \beta) \) for \( \beta \geq \beta_w(\gamma) \), and zero otherwise), and the dashed line the respective likelihood of a \( B \) contract \( \beta^2 + 2\beta(1 - \beta) \) for \( \beta \geq \beta_w(\gamma) \), and one otherwise). Formally:

**Remark 1:** Price-quantity bundles in both vertical chains is the unique equilibrium contractual configuration when \( \beta = 1 \) or \( \beta = 0 \), for all values of \( \gamma \).
By contrast, in industries in which bargaining over the contract type and contract terms takes place (0<β<1), wholesale price contracts may also be observed in equilibrium. Clearly then, bargaining plays a crucial role in vertical trading since different forms of trading may appear under its presence than under its absence. Interestingly, the likelihood of a \( W \) contract is the highest for “intermediate” values of upstream bargaining power (e.g. for \( \gamma = 0.4 \) when \( \beta = 0.129 \) and for \( \gamma = 0.8 \) when \( \beta = 0.547 \); see Fig. 3). The second point to make by comparing the two graphs included in Fig. 3 is that, the more differentiated the products are in an industry (the lower is \( \gamma \)), the more likely is the appearance of wholesale price contracts. This occurs because, when the products are not close substitutes, the role of strategic commitment vis-à-vis the rival chain becomes less important and that of intra-chain bargaining dominates.

Finally, one might wonder what is the optimal from a social point of view contractual configuration. As Proposition 4 states below, the configuration in which both chains employ two-part tariffs is the socially preferred one. In particular, welfare (measured as the sum of consumers’ and producers’ surplus) takes its highest value under two-part tariff contracts and its lowest value under wholesale price contracts (with the price-quantity bundles being in between).

**Proposition 4:** Welfare takes its highest value when both vertical chains employ two-part tariffs and its lowest value when both chains employ wholesale prices, with all the other cases lying in between.

The above result is a straightforward consequence of the fact that the equilibrium quantities are at the highest level under \([T, T]\) and at the lowest under \([W, W]\) since the respective equilibrium wholesale prices are lower than the marginal input cost \( c \) in the \([T, T]\) case and higher than \( c \), in the \([W, W]\) case (see Section 3). As two-part tariffs do not arise in equilibrium, the market does not deliver the socially optimal outcome.

5. The Effect of a Change in the Distribution of Bargaining Power

We have established that the equilibrium contractual configuration differs depending on the distribution of bargaining power and the degree of product differentiation. We have also seen that the firms’ profits as well as the consumers’ surplus and the total welfare could substantially differ across the two equilibrium contractual configurations, \([B, B), (B, B)]\) and \([(B, W), (B, W)]\). Thus, a change in the
distribution of bargaining power can affect firm’s profits, consumers’ surplus and welfare not only through changes in the contract terms but more importantly through changes in the contract types. Keeping this in mind, an interesting question to ask is what is the effect of a change in the distribution of bargaining power on the firm’s profits, the consumers’ surplus and the total welfare?

To answer this question, we first determine the equilibrium outcomes corresponding to the two equilibrium contractual configurations. We then perform a comparative statics analysis with respect to local changes in the distribution of bargaining power. Under the contractual configuration \([B, B], (B, B)\], the equilibrium outcomes are the ones reported in Tables 1, 2 and 3, in the boxes corresponding to the \([B, B]\) case. Under the contractual configuration \([B, W], (B, W)\], with probability \((1 - \beta)^2\) we end up in the \([W, W]\) case, while with probabilities \(\beta^2\) and \(\beta(1 - \beta)\) and \((1 - \beta)\beta\) we end up in the \([B, B]\), \([B, W]\) and \([W, B]\) cases, respectively. The expected equilibrium outcomes can then be obtained on the basis of these probabilities and the equilibrium values reported in the respective boxes in Tables 1, 2, and 3. We start by examining, in the next Proposition, the effect of a change in the distribution of bargaining power on the firm’s profits.

**Proposition 5:** The expected equilibrium profits of both a downstream and an upstream firm may decrease with their own bargaining powers, \(1 - \beta\) and \(\beta\) respectively.

Proposition 5 implies that, contrary to basic intuition or conventional wisdom, a firm may benefit from a reduction in its own bargaining power. Thus an upstream firm may enjoy higher profit in an industry where the upstream firms’ bargaining power is lower; similarly for a downstream firm.\(^{17}\) Fig. 4 illustrates this point by presenting the expected profits of the upstream and the downstream firm as functions of \(\beta\). As can been seen, an increase in the upstream firm’s bargaining power from the critical point \(\beta_B\), would lead to an upward “jump” in the downstream firm’s expected profits and to a downward “jump” in the upstream firm’s expected profits. The explanation for this interesting finding stems from the fact that such an increase in \(\beta\), besides leading to a change in the firm’s share of the pie, may have the more important implication of altering the equilibrium contractual type. As a result, it may

\(^{17}\) See also Marx and Shaffer (2004) who obtain a similar result in a different vertical contracting context.
alter both the size of the pie and the way this is divided. In particular, from the viewpoint of a downstream firm, although a reduction in its bargaining power means that it captures a smaller share of the pie, it can also imply a more favourable way of dividing the pie, due to the possible appearance of wholesale price contracts. From the viewpoint of an upstream firm, while an increase in its own bargaining power leads to an increase in its share of the pie, it can also lead to a smaller pie and a less favourable way of dividing it, through the appearance of conditionally inefficient contracts.

Turning to the consumers’ surplus and total welfare, Fig. 5 illustrates that they do not only “jump” at the critical value $\beta_B$, but that they are also not monotonic in $\beta$ for $\beta > \beta_B$. An analysis along these lines allows us to address an important question: since in an increasing number of markets “countervailing” power of large retailers becomes a significant factor, does such a force operate in the benefit of the consumers and total welfare? As Fig. 5 illustrates, the consumers’ surplus not only increases when the upstream power decreases at $\beta_B$, but it also takes its highest value when the downstream firms’ bargaining power is high enough. This is so because in the presence of wholesale price contracts (which could appear when $\beta > \beta_B$), double marginalization leads to higher final good prices. Fig. 5 also illustrates that welfare behaves in a similar way with the consumers’ surplus. In other words, we find that the recently observed increase in the “countervailing” power of downstream firms in some sectors can, under some circumstances, be beneficial both for the consumers and total welfare. The following Proposition summarizes.

**Proposition 6:** Consumers’ surplus and total welfare are not monotonic in the downstream firms’ bargaining power and may “jump up” as the downstream “countervailing” power increases.

Interestingly, as Fig. 5(b) shows, a more even distribution of bargaining power can harm both the consumers and total welfare. This is so, because a move from an uneven distribution of power to a more even one, may lead to the appearance of wholesale price contracts (see Fig. 2). This implies that in industries in which the power is asymmetrically distributed among the different production stages, market outcomes can be more competitive, than in industries characterized by a symmetric

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18. The only exception is when the two products are almost perfect substitutes, in which case consumers’ surplus (as well as total welfare) takes its highest value for $\beta = 1$.
19. As long as the downstream power, $1-\beta$, was not initially too low.
distribution of power among the firms that operate at different production stages. Nevertheless, the opposite could also happen when the goods are close enough substitutes (see Fig. 5(a)).

6. Extensions

In this section we consider a number of modifications of the basic model in order to examine the robustness of our main results.

6.1 Vertical Integration

Vertical integration (VI) is an alternative way in which upstream and downstream firms can be linked. According to Tirole (1988, p.170) “an upstream firm is vertically integrated if it controls (directly or indirectly) all the decisions made by the vertical structure”. Therefore, VI is an additional long-term contractual relationship option for the upstream and the downstream firm, besides the price-quantity bundles, the two-part tariffs or the wholesale price contracts. One should wonder whether the price-quantity bundles, that are conditionally efficient contracts, are equivalent to VI. As we will see, in our setting, the answer is no. A price-quantity bundle contract is a different strategic option than VI, because of its distinct commitment value.

We modify our basic model by allowing VI to be a feasible alternative at the time the contract type is chosen within each vertical chain. An issue that arises, then, is how profits would be divided following a VI. A reasonable assumption is that when VI takes place, the integrated firm’s profits are divided between the (previously independent) upstream and downstream units according to their relative bargaining power. Both options, VI and B, are conditional efficient. Moreover, in the event that both chains have chosen VI, the final market equilibrium outcome is the same as when both chains have chosen a B contract, that is, competition between the chains leads to a standard Cournot outcome with marginal costs $c$. What significantly simplifies our analysis at this point is that in all the other cases a chain’s joint profits under a B contract are higher than the profits under vertical integration. This is due to the B contract’s commitment value. When employing a B contract, a chain can effectively commit to a certain final good quantity level in the second stage of the game, while a VI firm chooses its output only at the last stage. Under downstream quantity competition, this commitment has value, and thus, VI leads to lower joint profits than
a price-quantity bundle contract. Therefore, since under a $B$ contract equilibrium profits are also shared according to the firms’ relative bargaining power (Lemma 1), we conclude that $VI$ is dominated by a price-quantity bundle contract for both firms and thus will not arise in equilibrium. The main result is as follows.

**Proposition 7:** Vertical integration ($VI$) leads to strictly lower joint profits for a chain than a price-quantity bundle ($B$), regardless of the rival chain’s long-term contractual relationship ($VI$, $B$, $T$, or $W$).

Clearly, in some industries additional considerations (such as informational or contractual problems) may make $VI$ a desirable choice. Such considerations have been exposed in the literature – here, we raise the point that, if there are contracts with commitment value, the choice of $VI$ may not be selected for strategic reasons.

6.2 Price-Quantity Bundles without Downstream Quantity Commitment

In this subsection, we relax our assumption that under a $B$ contract, there is downstream 'quantity commitment', i.e. that the final good’s quantity is necessarily equal to the input quantity specified in the price-quantity bundle. Instead, we assume that there is 'free disposal', that is, a downstream firm is free to produce any final good quantity up to the input quantity specified in the $B$ contract. In this sense, the input quantity specified in the price-quantity bundle is a *capacity constraint* for the downstream firm. Moreover, as the total input price has been paid in the second stage of the game, it is a *sunk cost* for the downstream firm in the last stage. As a result, in the market competition stage, the downstream firm faces a zero marginal production cost up to the specified capacity (and infinite thereafter). This reveals an alternative commitment mechanism inherent in the price-quantity bundle contracts. The vertical chain, through the use of a $B$ contract, can commit to *an aggressive downstream competition up to the capacity level specified during the contract terms negotiations*.

Whether or not a contract between an upstream and a downstream firm can directly dictate the quantity to be supplied in the downstream market depends on the specificities of the market under consideration. In some cases, technological, legal or other institutional factors imply that a downstream retailer automatically forwards to the final consumers the quantity of the final good that it receives from an upstream manufacturer. In some other cases, the downstream firm may be receiving intermediate inputs from an upstream supplier and after making the total payment
required for all the input units may have the option to simply not use some of them ('free disposal').

We show that under 'free disposal', all our previous analysis holds with no need for any modification, provided that the marginal production cost of the input \( c \) is not too low. Thus, our results turn out to be robust with respect to the nature of commitment inherent in the price-quantity bundles. In fact, the marginal input cost \( c \) is a measure of the effectiveness of the alternative commitment mechanism. The higher is \( c \), the more valuable is for the vertical chain to be able to commit to an aggressive downstream behavior by inducing its downstream firm to act as a zero marginal cost competitor. In contrast, when the marginal input cost is low, a price-quantity bundle loses a great part of its commitment value. A modified analysis would be required in order to determine the equilibrium contractual configurations in this case, a task that is out of the scope of the present paper. The following Proposition states our main result.

**Proposition 8:** Propositions 1-6 hold also in case that a vertical chain, through a price-quantity bundle \( B \), can commit only to a specific downstream capacity, if the marginal production cost of input \( c \) is not too low, i.e. if \( c \geq a\hat{c_n}(\gamma) \), where

\[
\hat{c_n}(\gamma) = \frac{(2-\gamma)(8-3\gamma^2)\sqrt{4-\gamma^2} - 2(4-\gamma^3)(4-2\gamma-\gamma^2)}{2(8-3\gamma^2)\sqrt{4-\gamma^2} - 2(4-\gamma^3)(4-2\gamma-\gamma^2)}
\]

with \( d\hat{c_n}/d\gamma > 0 \), \( \lim_{\gamma \to 0} \hat{c_n}(\gamma) = 0 \) and \( \hat{c_n}(1) = 0.235 \), independently of the distribution of power between the upstream and downstream firm \( (\beta, 1-\beta) \).\(^{20}\)

The intuition is as follows. A vertical chain, say \((U_1, D_1)\), through the negotiations over the terms of the price-quantity bundle, can transform its downstream firm \( D_1 \) to a capacity constrained competitor with zero marginal production costs up to capacity. The \((U_1, D_1)\) would never select an input quantity in excess of the output that its downstream firm will actually produce in the final good market, because by eliminating the excess downstream capacity, the chain can save on input production costs and increase its joint profits. At the same time, the rival chain \((U_2, D_2)\) has two options if it employs a \( T \) or a \( W \) contract: Either, to select a relatively high wholesale price and abide with its downstream firm being a Stackelberg follower in the final

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\(^{20}\) The proof of this Proposition is cumbersome and is thus omitted. It is though available by the authors upon request.
good market; or, to select a relatively low wholesale price and transform its downstream firm to a Cournot competitor in the final good market. It turns out that this latter option cannot be profitable unless the marginal cost of input is too low. The reason is that the higher is $c$, the more strategically “distorted” the downstream competition becomes and thus the lower are both chains joint profits. In particular, $(U_2, D_2)$’s joint profits are lower under the (strategically induced) fierce downstream competition than those obtained by abiding to a Stackelberg follower role for its downstream firm. Finally, if the rival chain $(U_2, D_2)$ also employs a $B$ contract, both downstream firms produce at capacity, resulting thus in a standard Cournot game with marginal costs equal to $c$.

6.3 Wholesale Prices vs. Two-Part Tariffs

In some markets, technological or institutional considerations may make the price-quantity bundle contracts non-feasible. Accordingly, and for the completeness of the analysis, in this section we constrain the choice of contracts to that only between wholesale prices and two-part tariffs. Our main findings are summarized in the following Proposition.

**Proposition 9:** If only $T$ and $W$ contracts are feasible, then there exist continuous functions $\beta_T(\gamma)$ and $\beta_{WT}(\gamma)$, increasing in $\gamma$, with $\lim_{\gamma \to 0}\beta_T(\gamma) = 0$, $\lim_{\gamma \to 0}\beta_{WT}(\gamma) = 0$, $\beta_T(1) = 0.694$, $\beta_{WT}(1) = 0.495$, and $\beta_{WT}(\gamma) < \beta_T(\gamma) < \beta_W(\gamma)$ such that:

(a) The contractual configuration $[(T, W), (T, W)]$ is an equilibrium for $\beta > \beta_{WT}(\gamma)$.

(b) The contractual configuration $[(T, T), (T, T)]$ is an equilibrium for $\beta < \beta_T(\gamma)$.

(c) The asymmetric contractual configurations $[(T, T), (T, W)]$ and $[(T, W), (T, T)]$ never arise in equilibrium.

A $W$ contract is always dominated by a $T$ contract for an upstream firm. This is due to the fact that the $T$ contracts, unlike the $W$ contracts, are conditionally efficient, and lead to a higher share of the pie for the upstream firms than that under the $W$ contracts. Proposition 9 implies that the same does not always hold for the downstream firms. Indeed, the appearance of wholesale price contracts in equilibrium is possible even when the set of feasible contracts does not include the $B$ contracts. Under the same restricted contract set, albeit with downstream price competition, Gal-Or (1991) and Rey and Stiglitz (1995) have shown that $W$ contracts never arise in
equilibrium when the upstream firms have all the bargaining power. Here, we extend their result to the case of quantity competition since we find that only $T$ contracts arise in equilibrium when the upstream firms have all the power. But more importantly, we show that their result would not always hold in the presence of bargaining and we thus highlight the crucial role that bargaining can play in the contractual procedure.

We should also point out that, according to Proposition 9, the appearance of $W$ contracts is more likely when $B$ contracts are non-feasible. More precisely, the critical value of $\beta$ in the absence of $B$ contracts is lower than the respective one in their presence, $\beta_{WT}(\gamma) < \beta_{W}(\gamma)$. This is so, because as we know from Proposition 1 $B$ contracts are always preferred to $T$ contracts, due to the former type’s commitment value. Thus, the wholesale price contracts are less desirable when the price-quantity bundle contracts are feasible than when they are not.

7. Discussion and Further Extensions

We now discuss briefly some of the model’s assumptions, in order to highlight their role in the analysis and to suggest directions for future research.

Asymmetry in bargaining power across chains. To keep the analysis tractable we have assumed that the relative bargaining power of upstream and downstream firms is the same in the two chains. In principle, we could have situations where the bargaining power distribution differs across chains in the same industry. In such an extension of our model, the main results of our analysis would still hold. Specifically, asymmetries in the firms’ bargaining power would not alter our finding that price-quantity bundles are always preferred to two-part tariffs. They would neither change the result that only for the upstream firms the wholesale prices are always dominated. The only real difference would be that asymmetric contractual configurations might emerge in equilibrium, simply because the firms have asymmetric bargaining powers.

Contract type chosen together with the contract terms. Following most of the literature (see e.g. Gal-Or, 1991, and Rey and Stiglitz, 1995), we have assumed that the type of the contract is selected before the contract terms. Such an assumption allows us to capture the idea that the selection of the contract type is a choice with “longer-run” characteristics than the choice of its exact terms. Why this could be so? Because while the exact terms of trade are typically easier to change (perhaps as responding to marginal variations in market conditions), shifting from one contract
type to another may require a more complicated procedure, e.g. involvement of firms’ more senior management and legal departments, or changes in the monitoring and trading technology. In addition, such an assumption allows us to capture the contract types’ commitment value (see e.g. Irmen, 1998), due to their observability. While this assumption might not hold in all the real world cases, we believe that it is plausible in many of them, and that it captures an essential feature of firms’ behavior. In our setting, if this assumption were violated, bargaining would have to take place simultaneously over both the type and the terms of contract. In such a situation, conditionally inefficient contracts, like wholesale prices, would not be chosen.

- *Uncertainty in the downstream market.* We have shown that, in the absence of any uncertainty, price-quantity bundles are always preferable to two-part tariffs. However, this result may not always hold in the presence of uncertainty. This is so because price-quantity bundles pre-specify the final quantity, and thus, lack flexibility. In particular, if at the time that the contract is signed there is demand (or cost) uncertainty, more flexible contracts (e.g. two-part tariffs) that involve a marginal price may be preferable. Hence, introducing uncertainty in such a fashion into the model is expected to generate additional equilibria and to make price-quantity bundles less likely to appear.

- *Downstream price competition.* In our model, downstream firms produce differentiated goods and compete in quantities. Since the mode of downstream competition does affect the equilibrium contract terms (and thus, it could also affect the equilibrium contract types), one might wonder what would happen if the firms competed in prices instead. In the cases in which at least one of the chains would use a price-quantity bundle contract, the downstream competition would transform to one of (one-sided or two-sided) capacity constrained Bertrand competition. This is so because a chain employing a $B$ contract can commit to a certain capacity level (equal to the input quantity specified during the contract terms negotiations) before price competition in the downstream market takes place. Then if both chains employ $B$ contracts, competition in last two stages becomes equivalent to a standard differentiated goods Cournot game. This is in the spirit of Kreps and Scheinkman (1983), with the only difference that in our setting, capacities would be chosen by the vertical chains (through bargaining over the terms of the price-quantity bundles)

21 In some countries, the producers are required to publish their “general conditions” of trade, e.g. whether or not they use franchise fees (see e.g. Rey and Stiglitz, 1995, p. 445).
before the chains’ downstream firms choose their prices. However, since the price-quantity bundles are conditionally efficient contracts, it would still be true that a chain would behave in equilibrium, in a way that it gets transformed into a Cournot competitor. If only one chain employs a $B$ contract, that chain becomes a capacity constrained Stackelberg leader, while the rival chain (employing a $T$ or a $W$ contract) regards its downstream firm as a monopolist on the residual demand. This then leads to a situation which is equivalent to a Stackelberg game with quantity competition. Therefore, the price-quantity bundle contracts continue to have desirable features under downstream price competition too, since they transform the game to one of quantity instead of price competition. In the cases instead that the chains would use either $T$ or $W$ contracts, the downstream competition stage would correspond to a standard differentiated Bertrand game. It is known that in a differentiated Bertrand game the competition is stronger and the profits are lower than in a differentiated Cournot game. Thus, while the mechanics of the model would be somewhat different under downstream price competition, the basic qualitative features of our analysis would remain valid.\footnote{The analysis of this alternative formulation is not trivial and is complicated by the presence of product differentiation. Our results indicate that, under certain conditions, price-quantity bundles will be selected more often because the $B$ contracts have a stronger commitment value under downstream price competition (for details see Milliou et al., 2004)}

- \textit{Unobservable contract terms}. In our analysis, we have assumed that not only the contract types but also the contract terms are observed before the final good competition stage. This is a central assumption in the strategic contracting literature.\footnote{For the implications of relaxing the observability assumption, see e.g. Caillaud et al. (1995), and Fershtman and Kalai (1997).} If we relax it and assume, instead, that the contract terms are unobservable (“secret contracts”), then the price-quantity bundle contracts loose their commitment value because the input quantity specified in the contract terms is unobservable. Notice for instance that, under secret two-part tariffs, the chains would end-up playing a standard Cournot game with marginal costs equal to $c$. The same would happen under secret price-quantity bundles. It is easy to see that the two-part tariff contracts and the price-quantity bundle contracts become equivalent (they lead to the same equilibrium outcome) when they are secret. It follows, that under secret contracts, the contract choice would effectively be transformed into a binary choice, that among a price-quantity bundle (or, equivalently, a two-tariff) and a wholesale price contract. Given this, one can easily show that the equilibrium contractual configuration under secret
contracts would be qualitatively similar to that included in sub-section 6.3, where the choice is also binary. It is important to note that since the lack of observability implies that a \( B \) (or a \( T \)) contract loses part of its (strategic) value, in equilibrium, \( W \) contracts would tend to be selected more often.

8. Conclusions

It has been recently recognized that in an increasing number of industries, the downstream firms are either as large as, or even larger than, their upstream partners. In this paper, we study the implications of the above phenomenon for the organization of vertical trading and, through this, for firm’s profits, consumers’ surplus and overall welfare. In order to do so, we consider a setting in which both the form and terms of trading in competing vertical chains are determined through bilateral bargaining.

The existing vertical contracting literature has either examined strategic incentives taking the organization of vertical trading as given, or has ignored the role of bargaining for the endogenous organization of vertical trading. We demonstrate that bargaining has significant repercussions for the organization of vertical trading. First, conditionally efficient contracts do not necessarily arise in equilibrium, and when they arise are in the form of price-quantity bundles and not of two-tariffs, as the existing literature has suggested so far. Second, vertical integration is a dominated option and both the upstream and the downstream firm would prefer instead a price-quantity bundle contract. Third, linear wholesale price contracts that are often observed in practice may emerge endogenously; this is important, because in previous work on strategic contracting such conditionally inefficient contracts do not arise in equilibrium. Moreover, one testable implication of our analysis is that the likelihood of appearance of wholesale price contracts is higher in industries in which the allocation of bargaining power between the upstream and the downstream firms is not too skewed and/or in which the products are more differentiated.

Our analysis also allows us to put forward the important point that a change in the distribution of bargaining power may drastically affect firm’s profits, consumers’ surplus and welfare through changes in the form of trading (besides the changes in the terms of trading). Contrary to conventional wisdom, we find that a firm, upstream or downstream, might benefit from a reduction in its own bargaining power. Although such a reduction means that the firm will enjoy a smaller share of the pie.
Moreover, by pointing out that wholesale price contracts (that lead to high final prices) do not arise in equilibrium when downstream firms are fairly powerful, we provide support to the view that increased buyers’ “countervailing” power may sometimes be beneficial for the consumers and total welfare. Finally, our analysis suggests that a more extreme distribution of bargaining power is expected to increase the likelihood of conditionally efficient contracts and, generally, to lead to lower final market prices.

We have demonstrated that our main results are robust under various modifications of our basic model. In addition, while some of our results have been derived in the context of a linear demand model, the intuition behind them appears robust and of more general applicability. While this is, to the best of our knowledge, the first paper that examines the relation between firms’ bargaining power in competing vertical chains and the strategic organization of vertical trading, more work needs to be done on the topic. In addition to the extensions mentioned in the previous section, this work will hopefully include an empirical study of how the organization of vertical trading is influenced by the bargaining power of firms in oligopolistic industries.

Appendix

**Proof of Lemma 1:** (a) Let $A_i$ be the transfer specified in a $B$ or $T$ contract. This transfer does not affect the marginal conditions in the downstream competition stage and, thus, the downstream and upstream gross profits are independent of the transfer. Hence maximization of the generalized Nash product with respect to $A_i$,

$$
\max_{A_i} \left[ \pi^g_{U_i} + A_i \right]^\beta \left[ \pi^g_{D_i} - A_i \right]^{-\beta},
$$

(A1)

leads to a bargained transfer $A_i^* = \beta (\pi^g_{U_i} + \pi^g_{D_i}) - \pi^g_{U_i} = \beta \pi_{(U_i,D_i)} - \pi^g_{U_i}$. As a result, the net profits of the upstream and downstream firms become $\pi^*_{U_i} = \pi^g_{U_i} + A_i^* = \beta \pi_{(U_i,D_i)}$ and $\pi^*_{D_i} = \pi^g_{D_i} - A_i^* = (1-\beta) \pi_{(U_i,D_i)}$, respectively. Substituting these expressions into

---

it follows that the generalized Nash product reduces to an expression proportional to the chain’s joint profits $\pi_{(U_1, D_1)}$.

(b) It is easy to see that (9) does not lead to the maximization of the $(U_2, D_2)$ chain’s joint profits (except for the extreme cases where one firm has all the power).

Regarding the distribution of the chain’s joint profits, note that after taking the logarithm of (9), the first-order condition becomes,

$$\frac{\beta (\partial \pi_{U_2} / \partial w_2) - (1 - \beta) (\partial \pi_{D_2} / \partial w_2)}{\pi_{U_2}} = 0 \text{ or, } \frac{\pi_{U_2}}{\pi_{D_2}} = \frac{\beta}{1 - \beta} \left[ \frac{(\partial \pi_{U_2} / \partial w_2)}{(\partial \pi_{D_2} / \partial w_2)} \right].$$

(A2)

It remains to show that the term in brackets is smaller than 1, i.e. that an increase in wholesale price $w_2$ increases the upstream profits by less than it decreases the downstream profits in equilibrium. Note first that,

$$\frac{\partial \pi_{U_2}}{\partial w_2} = q_2() [1 + \frac{w_2 - c}{q_2()} \frac{\partial q_2()}{\partial w_2} ] < q_2(),$$

(A3)

because $w_2 > c$ and by (3) and (4), $\frac{\partial q_2()}{\partial w_2} < 0$ in all cases. Second, by the envelope theorem, we obtain

$$\frac{\partial \pi_{D_2}}{\partial w_2} = q_2() [1 - \gamma \frac{\partial q_1()}{\partial w_2} ] \leq -q_2(),$$

(A4)

because $\frac{\partial q_1()}{\partial w_2} > 0$ if $(U_1, D_1)$ employs a $W$ or a $T$ contract and $\frac{\partial q_1()}{\partial w_2} = 0$ if it employs a $B$ contract. By combining (A3) and (A4), we obtain the result. □

Proof of Lemma 2: (a) This is an immediate consequence of the discussion preceding Lemma 2.

(b) We will use two arguments. First, as we saw above, in the cases $[B, T], [B, B]$, and $[T, B]$, any output decision maker (i.e. the downstream firm or the vertical chain) faces the same marginal cost $c$. Second, it is well known that for the symmetric cost case, the Stackelberg leader’s profits, $(U_1, D_1)$’s profits under $[B, T]$, are larger than the profits of the Cournot competitors, profits under $[B, B]$, and those are larger than the profits of a Stackelberg follower, $(U_1, D_1)$’s profits under $[T, B]$. □

Proof of Lemma 3: We first show that $w_2^{TW} < w_2^{BW}$. Let chain $(U_2, D_2)$ employ a $W$ contract. If $(U_1, D_1)$ employs a $T$ (or, a $W$) contract, from (9) the first order condition for the $(U_2, D_2)$ chain can be written as:
\[
\frac{\beta}{w_2 - c} \frac{\partial q_2}{\partial w_2} - \frac{(1 - \beta)(\partial q_2 / \partial w_2 + 1)}{q_2} = \frac{(1 - \beta)\gamma}{q_2}, \tag{A5}
\]

where \(q_i = q_i(w_1, w_2), i=1,2\), are given by (4). While if \((U_1, D_1)\) employs a B contract, the first order condition can be written as:

\[
\frac{\beta}{w_2 - c} \frac{\partial R_2}{\partial w_2} - \frac{(1 - \beta)(\partial R_2 / \partial w_2 + 1)}{q_2} = 0, \tag{A6}
\]

where \(q_2 = R_2(q_1, w_2)\) is given by (3). In (A5) and (A6), it has been taken into account that \(q_2 = a - q_2 - \gamma q_1 - w_2\) from the first order conditions of the downstream firm \(D_2\) in the last stage.

Notice first that, since from (4) \(\partial q_1 / \partial w_2 > 0\), the RHS of (A5) is positive, while the RHS of (A6) is zero. Moreover, from (3) and (4), we have \(\partial q_2 / \partial w_2 < \partial R_2 / \partial w_2\). As a result, the sum of the last two terms in the LHS of (A5) turn out to be more negative than the respective sum of terms of (A6). Therefore, \(w_2^{TW} < w_2^{BW}\). (This result can also be obtained by a direct comparison of the equilibrium wholesale prices as given in Table 1.)

Now \(w_2^{TW} < w_2^{BW}\) implies that \(R_2(q_1, w_2^{BW}) < R_2(q_1, w_2^{TW})\). As a result, the Stackelberg leader \((U_1, D_1)\)'s joint profits are: \(\pi_{(U_1, D_1)}(w_2^{BW}) > \pi_{(U_1, D_1)}(w_2^{TW})\). Further, the latter profits are higher than those that \((U_1, D_1)\) attains when \(D_1\) acts as a Cournot competitor in the final good market: \(\pi_{(U_1, D_1)}(w_2^{BW}) > \pi_{(U_1, D_1)}(w_2^{TW})\). \(\Box\)

**Proof of Proposition 1:** Lemma 1(a) says that, under both B and T contracts, the upstream and the downstream firm share the chain’s joint profits according to their bargaining powers. Hence, it is sufficient to compare the chain’s joint profits under these contracts. From Lemma 2 and 3 we have \(\pi_{(U_1, D_k)}^{BB} > \pi_{(U_1, D_k)}^{TB}, \pi_{(U_1, D_k)}^{BT} > \pi_{(U_1, D_k)}^{TT}\), and \(\pi_{(U_1, D_k)}^{BW} > \pi_{(U_1, D_k)}^{TW}\), i.e. a B contract leads to strictly higher joint profits than a T contract for chain \((U_1, D_1)\), regardless of whether the rival chain employs a B, a T or a W contract (or any convex combination of these). \(\Box\)

**Proof of Proposition 2:** It follows immediately from Lemma 1. \(\Box\)

**Proof of Proposition 3:** Given Propositions 1 and 2, in order for the contractual configuration \([(B, W), (B, W)]\) to be an equilibrium, it is sufficient to show that each of the downstream firms, e.g. \(D_1\), does not have a profitable deviation to \(B\), given that
the rival chain \((U_2, D_2)\) chooses \((B, W)\), that is, the negotiations in the competing vertical chain lead to a \(B\) contract with probability \(\beta\) and to \(W\) contract with probability \(1-\beta\). Taking the relevant profit difference and setting it equal to zero, we find that there exists a unique critical value, \(\beta_W(\gamma)\) in terms of \(\gamma\), such that:

\[
\beta_W^{BB} + (1-\beta)\pi_W^{BW} - \beta\pi_W^{BB} - (1-\beta)\pi_W^{WB} > 0 \text{ if } \beta < \beta_W(\gamma),
\]

and negative otherwise. We then establish that \(\partial\beta_W(\gamma)/\partial \gamma > 0\), \(\lim_{\gamma \to 0^+} \beta_W(\gamma) = 0\) and \(\beta_W(1) = 0.791\). It follows immediately that \([(B, W), (B, W)]\) is an equilibrium for all \(\beta \geq \beta_W(\gamma)\). This proves part (a).

In order for the contractual configuration \([(B, B), (B, B)]\) to be an equilibrium, it is sufficient to show that one of the downstream firms, e.g. \(D_1\), has no incentive to deviate from \(B\) to \(W\), given that the \((U_2, D_2)\) chain chooses \((B, B)\). Setting the difference \(\pi_W^{BW} - \pi_W^{BB}\) equal to zero, we find that there exists a unique critical value, \(\beta_B(\gamma)\) in terms of \(\gamma\), such that:

\[
\pi_W^{BW} - \pi_W^{BB} > 0 \text{ for } \beta > \beta_B(\gamma) \text{ and } \pi_W^{BW} - \pi_W^{BB} < 0 \text{ for } \beta < \beta_B(\gamma).
\]

We then establish that \(\partial\beta_B(\gamma)/\partial \gamma > 0\), \(\lim_{\gamma \to 0^+} \beta_B(\gamma) = 0\) and \(\beta_B(1) = 0.882\). It follows immediately that \([(B, B), (B, B)]\) is an equilibrium for all \(\beta \leq \beta_B(\gamma)\). This proves part (b). Further, it can be easily checked that \(\beta_W(\gamma) < \beta_B(\gamma)\) for all \(0 < \gamma \leq 1\) (see also Fig. 2).

Finally, in order to prove part (c) we proceed as follows. We know from (A7) that, given that chain \((U_2, D_2)\) chooses \((B, W)\), the profits of \(D_1\) from deviating from \(B\) to \(W\) are higher than under \(B\) for all \(\beta > \beta_W(\gamma)\). Moreover, we infer from (A8) that, given that chain \((U_1, D_1)\) chooses \((B, B)\), the profits of \(D_2\) from deviating from \(W\) to \(B\) are higher than under \(W\) for all \(\beta < \beta_B(\gamma)\). Since \(\beta_W(\gamma) < \beta_B(\gamma)\) for all \(0 < \gamma \leq 1\), it is clear that either \(D_1\) or \(D_2\) have an incentive to deviate for all parameter values; hence, \([(B, B), (B, W)]\) cannot be an equilibrium contractual configuration (and by symmetry \([(B, W), (B, B)]\) cannot be neither). This completes the proof. □

**Proof of Proposition 4:** Total welfare is given by (see e.g. Singh and Vives, 1984):

\[
W(q_1, q_2) = (a-c)(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2).
\]

(A9)

The welfare level that corresponds to each of the six possible second stage subgames is found by substituting into (A9) the respective equilibrium quantities from Table 1. Table 3 reports the welfare levels for all these cases.
We start with the comparison of the symmetric cases. Taking the respective differences, it can be easily shown that: $W_{TT} > W_{BB} > W_{WW}$. Similarly in the asymmetric cases, we have: $W_{TB} > W_{WT} > W_{WB}$. Next we find that $W_{WB} < W_{TB}$, and $W_{BB} < W_{TB}$. Further, taking the difference $W_{BB} - W_{WB}$, we find that, for any given $\beta$, there exists a critical value of $\gamma$, $\gamma' (\beta)$, such that $W_{BB} > W_{WB}$ if and only if $\gamma < \gamma' (\beta)$, with $\gamma' (\beta) = [\beta' (\gamma)]^{-1}$ and

$$
\beta' (\gamma) = \frac{2}{K} \frac{32 \gamma - 64 - 4 \gamma^4 - 8 \gamma^5 + 4 \gamma^5 + 2 \sqrt{K}}{64 + 12 \gamma^4 + 8 \gamma^5 - 32 \gamma^3 - 4 \gamma^2 + \gamma^6},
$$

with $K \equiv (\gamma^6 + 2 \gamma^5 - 6 \gamma^4 + 4 \gamma^2 - 16 \gamma + 16)(2 + \gamma)^2(4 - \gamma^2 - 2 \gamma)^2$.

Similarly, by taking the difference $W_{BB} - W_{WT}$, we find that for any given $\gamma$, there exists a critical value of $\gamma$, $\gamma'' (\beta)$, such that $W_{BB} > W_{WT}$ if and only if $\gamma < \gamma'' (\beta)$. The critical value of $\gamma$ is $\gamma'' (\beta) = [\beta'' (\gamma)]^{-1}$, where

$$
\beta'' (\gamma) = \frac{2}{L} \frac{128 \gamma^2 - 128 + 4 \gamma^5 - 48 \gamma^4 - 32 \gamma^3 - \gamma^8 - 2 \gamma^7 + 10 \gamma^6 + 64 \gamma + \sqrt{L}}{128 - 224 \gamma^2 - 32 \gamma^3 + 144 \gamma^4 - 36 \gamma^5 + 32 \gamma^6 + \gamma^9 - 12 \gamma^7 + 3 \gamma^8},
$$

with $L \equiv (\gamma^6 + 2 \gamma^5 - 6 \gamma^4 + 4 \gamma^2 - 16 \gamma + 16)(2 - \gamma)^2(4 - \gamma^2 - 2 \gamma)^2 (2 + \gamma)^4$.

By comparing (A10) to (A11), it follows that $\gamma' (\beta) > \gamma'' (\beta)$. Thus, for $\gamma < \gamma'' (\beta)$, $W_{BB} > W_{WT} > W_{WB}$, while for $\gamma > \gamma' (\beta)$, we have $W_{WT} > W_{WB} > W_{BB}$ and for $\gamma'' (\beta) < \gamma < \gamma' (\beta)$, $W_{WT} > W_{NN} > W_{WB}$. □

Proof of Proposition 5: Using the second stage equilibrium profits reported in Table 2, one can check that:

$$
\pi_{UB}^{BB} > \beta^2 \pi_{UB}^{BB} + \beta (1 - \beta) \pi_{UB}^{BW} + \beta (1 - \beta) \pi_{UB}^{WB} + (1 - \beta)^2 \pi_{UB}^{WW}, \quad \text{and} \quad (A12)
$$

$$
\pi_{DB}^{BB} < \beta^2 \pi_{DB}^{BB} + \beta (1 - \beta) \pi_{DB}^{BW} + \beta (1 - \beta) \pi_{DB}^{WB} + (1 - \beta)^2 \pi_{DB}^{WW}. \quad \text{A}\text{13}
$$

That is, the (expected) profits of an upstream firm are strictly higher in the $[(B, B), (B, B)]$ than in the $[(B, W), (B, W)]$ equilibrium for all parameter values, while the opposite is true for a downstream firm. Now for any given $\gamma$, pick $\beta = \beta_0 (\gamma)$. Then an infinitesimal increase in the upstream power $\beta$ necessarily leads to lower expected profits for the upstream firm. Similarly, an infinitesimal increase in the downstream power $(1 - \beta)$, starting from $\beta = \beta_0 (\gamma)$, necessarily leads to lower expected profits for the upstream firm (see Fig. 4). (Similar arguments apply for all $\beta_0 (\gamma) \leq \beta \leq \beta_0 (\gamma)$, provided that a slight perturbation of the distribution of bargaining power leads to a change in the contractual configuration equilibrium). □
Proof of Proposition 6: Using the welfare levels reported in Table 3 and the second stage equilibrium profits reported in Table 2, one can calculate the expected welfare and consumer surplus under the two equilibrium contractual configurations \([(B, B), (B, B)]\) and \([(B, W), (B, W)]\). For instance, total welfare in the former case is given in the \((B, B)\) box of Table 3, while in the latter case is equal to:

\[
\beta^2 W_{BB} + 2\beta(1-\beta)W_{BW} + (1-\beta)^2 W_{WW}.
\]

On the other hand, consumers’ surplus equals total welfare minus the sum of the chains’ equilibrium profits that could be obtained by adding the upstream and downstream profits of the two vertical chains reported in Table 2. Remember that under \([(B, B), (B, B)]\) the equilibrium outcome is independent of the bargaining power \(\beta\) for all \(\beta \leq \beta_0(\gamma)\), provided that we are in the \([(B, B), (B, B)]\) equilibrium. This is not any more true in the other equilibrium contractual configuration, i.e. under \([(B, W), (B, W)]\) both welfare and consumers’ surplus depend on \(\beta\) in a non-monotonous way (see Fig. 5). It can also be checked that both welfare and consumers’ surplus are strictly lower under the \([(B, W), (B, W)]\) than under the \([(B, B), (B, B)]\) equilibrium for all \(\beta_W(\gamma) \leq \beta \leq \beta_B(\gamma)\). This implies that an infinitesimal increase in the countervailing power \(1-\beta\), starting e.g. from \(\beta=\beta_W(\gamma)\), leads to a “jump up” in both the welfare level and the consumers’ surplus (see Fig. 5). 

Proof of Proposition 7: We proceed by presenting four claims; as the basic intuition is given earlier in the proofs of Lemmas 2 and 3, some details are omitted here. First, we have \(\pi_{VI}^{B,U,D} > \pi_{VI}^{VI,U,D}\). This is because \(\pi_{VI}^{B,U,D} = \pi_{VI}^{BT,U,D} > \pi_{VI}^{BB,U,D} = \pi_{VI}^{VI,U,D}\), since when both chains choose either \(B\) or \(VI\), they play a standard Cournot game with marginal costs \(c\), while when one chain chooses \(B\) and the other \(VI\) or \(T\) we have a Stackelberg game. Second, we have \(\pi_{VI}^{BB,U,D} > \pi_{VI}^{BT,U,D}\). This is because \(\pi_{VI}^{BB,U,D} > \pi_{VI}^{VI,U,D}\) (same intuition as for the case above). Third, \(\pi_{VI}^{BB,U,D} > \pi_{VI}^{VI,U,D}\). This is because \(\pi_{VI}^{BB,U,D} > \pi_{VI}^{VI,U,D}\); the first inequality by the same logic as above, Stackelberg leadership, and the second by the properties of the reaction function in the wholesale prices space – there are strategic substitutes. Finally, \(\pi_{VI}^{BB,U,D} > \pi_{VI}^{VI,U,D}\). This, again, is due to the fact that a chain’s wholesale price
(under a $W$ contract) is lower when the rival chain has chosen $VI$ rather than a $B$ contract. Collecting the four claims presented above, we obtain the proof. □

Proof of Proposition 9: It follows from Lemma 1 that $W$ contracts are strictly dominated by $T$ contracts for the upstream firms. Thus, the only remaining candidate equilibria are $[(T, T), (T, T)], [(T, W), (T, W)], [(T, W), (T, T)]$ and $[(T, T), (T, W)]$. The rest of the proof is along the lines of Proposition 3 proof with the $T$ contracts substituting the $B$ contracts. □

References


## Table 1: Equilibrium Wholesale Prices and Final Market Quantities

<table>
<thead>
<tr>
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<th>$W$</th>
<th>$T$</th>
<th>$B$</th>
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<tbody>
<tr>
<td>$W$</td>
<td></td>
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<tr>
<td>$w_i^{W}$</td>
<td>$w_i^{W} = \frac{(2 - \beta)c + a\beta(2 - \gamma)}{4 - \beta\gamma}$</td>
<td>$w_i^{W_T} = \frac{a\beta(16 - 8\gamma - 8\gamma^2 + 2\gamma^3 + \gamma^4)}{32 - 16\gamma^2 + \beta\gamma^4}$</td>
<td>$w_i^{W_B} = \frac{a\beta(4 - 2\gamma - \gamma^2)}{8 - (4 - \beta)\gamma^2}$</td>
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<tr>
<td></td>
<td>$q_i^{W}$</td>
<td>$q_i^{W} = \frac{2\nu(2 - \beta)}{(2 + \gamma)(4 - \beta\gamma)}$</td>
<td>$q_i^{W_T} = \frac{a\gamma(4 - 2\gamma - \gamma^2) + 2c(4 - 2\gamma^2 - \beta(2 - \gamma - \gamma^2))}{32 - 16\gamma^2 + \beta\gamma^4}$</td>
</tr>
<tr>
<td>$T$</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>See $[W, T]$</td>
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<tr>
<td></td>
<td></td>
<td>$w_i^{T} = \frac{2\nu(2 + \gamma) - a\gamma^2}{4 + 2\gamma - \gamma^2}$</td>
<td>$w_i^{T_B} = c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$q_i^{T} = \frac{2\nu}{4 + 2\gamma - \gamma^2}$</td>
<td>$q_i^{T_B} = \frac{v(4 - 2\gamma - \gamma^2)}{4(2 - \gamma^2)}$</td>
</tr>
<tr>
<td>$B$</td>
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<td>See $[W, B]$</td>
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<td></td>
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<td>$q_i^{B} = \frac{\nu}{2 + \gamma}$</td>
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</table>
Table 2: Second Stage Equilibrium Profits

<table>
<thead>
<tr>
<th></th>
<th>( W )</th>
<th>( T )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>( \pi_{W^W} = \frac{4v^2(2 - \beta)^2(4 - 2\gamma - \gamma^2)}{(2 + \gamma)(4 - \beta\gamma)^2} )</td>
<td>( \pi_{T^W} = \frac{4v^2(1 - \beta)(2 - \gamma)^2(4 + \beta\gamma)^2(2 - \gamma^2)}{(32 - 16\gamma^2 + \beta\gamma^4)^2} )</td>
<td>( \pi_{B^W} = \frac{(2 - \beta)^3v^2(4 - 2\gamma - \gamma^2)^2}{4(8 - (4 - \beta)\gamma^2)} )</td>
</tr>
<tr>
<td>( \pi_{W^U} = \frac{2v^2\beta^2(2 - \beta)(2 - \gamma)}{(2 + \gamma)(4 - \beta\gamma)^2} )</td>
<td>( \pi_{T^U} = \frac{2v^2(2 - \beta)(2 - \gamma)^2(4 + \beta\gamma)^2(2 - \gamma^2)}{(32 - 16\gamma^2 + \beta\gamma^4)^2} )</td>
<td>( \pi_{B^U} = \frac{\beta v^2(4 - 2\beta)\gamma^4(2 - \gamma^2)}{2(8 - (4 - \beta)\gamma^2)} )</td>
<td></td>
</tr>
<tr>
<td>( \pi_{U^W} = \frac{2v^2\beta^2(2 - \gamma)^2(4 + \beta\gamma)^2(2 - \gamma^2)}{(32 - 16\gamma^2 + \beta\gamma^4)^2} )</td>
<td>( \pi_{U^T} = \frac{2v^2(2 - \gamma)^2(4 + \beta\gamma)^2(2 - \gamma^2)}{(32 - 16\gamma^2 + \beta\gamma^4)^2} )</td>
<td>( \pi_{U^B} = \frac{\beta v^2(4 - 2\beta)\gamma^4(2 - \gamma^2)}{2(8 - (4 - \beta)\gamma^2)} )</td>
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<tr>
<td>( \pi_{W^T} = \frac{2v^2(1 - \beta)(2 - \gamma^2)}{(4 + 2\gamma - \gamma^2)^2} )</td>
<td>( \pi_{T^T} = \frac{2v^2(1 - \beta)(2 - \gamma^2)}{(4 + 2\gamma - \gamma^2)^2} )</td>
<td>( \pi_{T^B} = \frac{(1 - \beta)v^2(4 - 2\gamma - \gamma^2)^2}{16(2 - \gamma^2)^2} )</td>
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<tr>
<td>( \pi_{U^T} = \frac{2v^2(2 - \gamma^2)}{(4 + 2\gamma - \gamma^2)^2} )</td>
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<td>( \pi_{U^B} = \frac{(1 - \beta)v^2}{(2 + \gamma)^2} )</td>
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<td>( \pi_{W^B} = \frac{2v^2\beta^2(2 - \gamma)^2(4 + \beta\gamma)^2(2 - \gamma^2)}{(32 - 16\gamma^2 + \beta\gamma^4)^2} )</td>
<td>( \pi_{T^B} = \frac{\beta v^2(4 - 2\beta)\gamma^4(2 - \gamma^2)}{2(8 - (4 - \beta)\gamma^2)} )</td>
<td>( \pi_{U^B} = \frac{\beta v^2(4 - 2\beta)\gamma^4(2 - \gamma^2)}{2(8 - (4 - \beta)\gamma^2)} )</td>
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</table>

See \([W, T]\)

See \([W, B]\)

See \([T, B]\)
<table>
<thead>
<tr>
<th></th>
<th>$W$</th>
<th>$T$</th>
<th>$B$</th>
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</thead>
<tbody>
<tr>
<td>$W$</td>
<td>$W_{ww} = 4v^2 (2 - \beta) \frac{6 - \beta \gamma + 2 \gamma - \beta \gamma^2 + \beta}{(2 + \gamma)^2 (4 - \beta \gamma)^2}$</td>
<td>$W_{tt} = 4v^2 \frac{3 + \gamma - \gamma^2}{(4 + 2 \gamma - \gamma^3)}$</td>
<td>$W_{wb} = v^2 \frac{96 - 48\gamma^2 + 3\gamma^4 - 64\gamma + 28\gamma^3}{32(2 - \gamma^3)^2}$</td>
</tr>
<tr>
<td>$T$</td>
<td>See $[W, T]$</td>
<td>$W_{tt} = 4v^2 \frac{3 + \gamma - \gamma^2}{(4 + 2 \gamma - \gamma^3)}$</td>
<td>$W_{tb} = v^2 \frac{96 - 48\gamma^2 + 3\gamma^4 - 64\gamma + 28\gamma^3}{32(2 - \gamma^3)^2}$</td>
</tr>
<tr>
<td>$B$</td>
<td>See $[W, B]$</td>
<td>See $[T, B]$</td>
<td>$W_{bb} = v^2 \frac{3 + \gamma}{(2 + \gamma)^2}$</td>
</tr>
</tbody>
</table>
Bargaining over contract type

Bargaining over contract terms

Downstream competition in quantities

Fig. 1: Timing in the basic model

Fig. 2: Equilibrium Contractual Configurations

Fig. 3: Likelihood of W and B contracts
(a) For $\gamma = 0.8$

(b) For $\gamma = 0.4$

Fig. 4: Expected Equilibrium Profits

(a) For $\gamma = 0.8$

(b) For $\gamma = 0.4$

Fig. 5: Expected Equilibrium Consumers’ Surplus ($ECS$) and Welfare ($EWe$)
Appendix for referees – Not for Publication

*Proof of Proposition 8:* To prove this result, we need to show that in all the subgames where a $B$ contract is employed by at least one vertical chain, the equilibrium when the chain is unable to commit to a specific downstream quantity during the contract terms negotiations stage remains the same as in the case that in which the chain can commit to a specific downstream quantity. In the former case, the chain can instead commit to a capacity (equal to the input quantity specified by the price-quantity bundle) up to which its downstream firm produces at zero marginal cost, since the total input price is a sunk cost for the downstream firm at the downstream competition stage. We consider the cases $[B, B], [B, T]$ and $[B, W]$ separately in order to find sufficient conditions for the equilibrium to be robust under the alternative commitment assumption.

The $[B, B]$ case: Let $K_i$ be the input quantity specified by the $(U_i, D_i)$ chain’s contract terms negotiations. Without loss of generality we can restrict attention to $K_i < \bar{K} = a - c$, where $\bar{K}$ is an input quantity so large that even if the rival chain’s capacity is zero, the profits of the $(U_i, D_i)$ chain are nil when its downstream firm $D_i$ produces at capacity. Indeed, as the $(U_i, D_i)$ chain’s profits are negative for all $q_i = K_i > \bar{K}$, the chain cannot credibly commit to a downstream production equal to capacity in this case. Now since $D_i$’s marginal cost equals zero, it is easy to see from (3) that its reaction function is given by $R_i(K_j, q_j) = \min[K_i, (a - \gamma q_j)/2], i, j = 1, 2$. That is, the $D_i$’s reaction function is kinked at its capacity level $K_i$, after which it becomes perpendicular to the $q_i$ axis. Clearly, if $K_1$ and $K_2$ are large enough, i.e. $K_i \geq a/(2 + \gamma)$ and $K_2 \geq a/(2 + \gamma)$, the third stage equilibrium is $q_i^* = q_2^* = a/(2 + \gamma)$. On the other hand, if $K_i$ is small enough and $K_j$ is large enough, i.e. $2K_j + \gamma K_i < a$ and $2K_j + \gamma K_i > a$, the equilibrium is $q_i^* = K_i$ and $q_j^* = (a - \gamma K_i)/2$. Finally, if $K_i$ and $K_2$ are small enough, i.e. $2K_1 + \gamma K_2 \leq a$ and $2K_2 + \gamma K_1 \leq a$, the equilibrium is $q_1^* = K_1$ and $q_2^* = K_2$. The latter implies that, for any permissible $K_i$, the $(U_i, D_i)$ chain can induce, if it wishes, a two-sided capacity constraint equilibrium, i.e. $q_i^* = K_i$ and $q_j^* = K_j$. Since in this case the $(U_i, D_i)$ chain’s profits are maximized along its
reaction function, $R_i^{(U_i, D_i)}(K_j) = (a - \gamma K_j - c)/2$, it is clear that the $(U_i, D_i)$ chain has an incentive to induce such an equilibrium by properly selecting its input quantity. An immediate consequence is that both vertical chains have incentives to induce the capacity constraint equilibrium and by doing so we end up in the standard Cournot equilibrium where $q_i^* = K_i^* = (a - c)/(2 + \gamma)$.

The $[B, T]$ case: Let $K_1$ be the input quantity specified by the $(U_1, D_1)$ chain’s contract terms negotiations and $w_2$ the wholesale price specified by the $(U_2, D_2)$ chain’s negotiations. As $D_1$’s marginal cost is zero, its reaction function from (3) is $R_1(K_1, q_2) = \min[K_1, (a - \gamma q_2)/2]$, while the reaction function of the rival firm $D_2$ is $R_2(q_1, w_2) = (a - \gamma q_1 - w_2)/2$. For small $K_1$, i.e. $K_1 < [a(2 - \gamma) + \gamma w_2]/(4 - \gamma^2)$, the third stage equilibrium is $q_i^* = K_i$ and $q_j^* (w_2) = (a - \gamma K_i - w_2)/2$; otherwise, the third stage equilibrium is an asymmetric Cournot, $q_i^C (0, w_2) = [a(2 - \gamma) + \gamma w_2]/(4 - \gamma^2)$ and $q_j^C (0, w_2) = [a(2 - \gamma) - 2w_2]/(4 - \gamma^2)$. Now for any given $K_1$, the $(U_2, D_2)$ chain has two options. First, to induce an one-sided capacity constrained third stage equilibrium, in which case $(U_2, D_2)$ will optimally set a wholesale price $w_2 = c$ in order to maximize the chain’s joint profits

$$\pi_{(U_2, D_2)}(K_1, w_2) = (a - \gamma K_1 - q_i^*(w_2) - c)q_j^*(w_2).$$

And second, to induce an asymmetric Cournot equilibrium by setting a low enough wholesale price, i.e. $w_2 < \bar{w}_2(K_1) = [(2 - \gamma)/\gamma][K_1(2 + \gamma) - a]$, in which case the chain’s profits will be

$$\pi_{(U_2, D_2)}^C(0, w_2) = [a - \gamma q_i^C (0, w_2) - q_j^C (0, w_2) - c]q_j^C (0, w_2),$$

or else

$$\pi_{(U_2, D_2)}(0, w_2) = \frac{(a(2 - \gamma) - 2w_2)[a(2 - \gamma) - c(4 - \gamma^2) + w_2(2 - \gamma^2)]}{(4 - \gamma^2)^2}$$  \hspace{1cm} (a1)

Note further that, if $w_2 = c$, the $(U_1, D_1)$ chain can induce its most-preferred equilibrium (i.e. the equilibrium that maximizes the chain’s joint profits given the reaction function of the rival downstream firm $D_2$, $R_2(q_1, c)$) by selecting $K_1^S = (a - c)(2 - \gamma)/2(2 - \gamma^2)$, provided that its downstream firm $D_1$ will do produce at capacity at the third stage, that is, if $q_i^C (0, c) \geq K_1^S$. It is easy to check that this
occurs if \( c/a > \hat{c}_n^1(\gamma) \equiv (2 - \gamma)\gamma^2 / (8 - 2\gamma^2 + \gamma^3) \), with \( \lim_{\gamma \to 0} \hat{c}_n^1(\gamma) = 0 \), \( \hat{c}_n^1(1) = 0.2 \) and \( \partial \hat{c}_n^1 / \partial \gamma > 0 \).

Let \( c/a > \hat{c}_n^1(\gamma) \) and \( K_1 = K_1^S \). If the \((U_2, D_2)\) chain’s joint profits are not higher when it follows its second option (i.e. to induce an asymmetric Cournot game in the third stage) then the equilibrium in the \([B, T]\) case coincides with that under commitment to downstream quantity. This would occur if there does not exist a

\[
\pi'(u_2, d_2)(0, w_2) > \pi'(u_2, d_2)(K_1^S, c),
\]

where

\[
\pi'(u_2, d_2)(K_1^S, c) = (a - c)^2 (4 - 2\gamma - \gamma^2)^2 / 16(2 - \gamma^2)^2
\]

are the profits of the Stackelberg follower. Note first from \((a1)\) that, for the \((U_2, D_2)\) chain’s price-cost margin and \(w_2 < \tilde{w}_2(K_1^S) = (2 - \gamma)[a\gamma^2 - c(4 - \gamma^2)] / 2\gamma(2 - \gamma^2)\) s.t.

\[
\pi'(u_2, d_2)(0, w_2) > \pi'(u_2, d_2)(K_1^S, c),
\]

we have \( \tilde{w}_2(K_1^S) > w \) only if \( c/a > \gamma / (2 + \gamma) > \hat{c}_n^1(\gamma) \), in which case the \((U_2, D_2)\) chain has no incentive to induce an asymmetric Cournot downstream game.

Further, maximizing \((a1)\) w.r.t. \( w_2 \) we obtain the (unrestricted) optimal wholesale price for the \((U_2, D_2)\) chain, \( w_2^n = (2 - \gamma)[2c(2 + \gamma) - a\gamma^2] / 4(2 - \gamma^2) \), in which case its (unrestricted) maximum profits are \( \pi^n_{(u_2, d_2)} = [2c - a(2 - \gamma)]^2 / 8(2 - \gamma^2) \). However, we have \( \tilde{w}_2(K_1^S) > w_2^n \) only if \( c/a < \gamma^2 / 4 < \gamma / (2 + \gamma) \). Moreover,

\[
\pi^n_{(u_2, d_2)} \leq \pi'(u_2, d_2)(K_1^S, c) \text{ if } c/a > \hat{c}_n^2(\gamma) \equiv \frac{8 - 4\gamma - \gamma^3 - (4 - 2\gamma - \gamma^2)\sqrt{2(2 - \gamma^2)}}{16 - \gamma^2(2 + \gamma)^2}
\]

with \( \lim_{\gamma \to 0} \hat{c}_n^2(\gamma) = 0 \), \( \hat{c}_n^2(1) = 0.2265 \) and \( \partial \hat{c}_n^2 / \partial \gamma > 0 \). It can be also checked that

\( \hat{c}_n^1(\gamma) < \hat{c}_n^2(\gamma) < \gamma^2 / 4 \). Clearly, \( \pi'(u_2, d_2)(w_2 < w_2^n) < \pi^n_{(u_2, d_2)} \leq \pi'(u_2, d_2)(K_1^S, c) \) for all \( \gamma^2 / 4 < c/a < \gamma / (2 + \gamma) \). Therefore, if \( c/a \geq \hat{c}_n^2(\gamma) \), the \((U_2, D_2)\) chain has no incentive to induce an asymmetric Cournot downstream game. Finally, as \( \hat{c}_n^2(\gamma) > \hat{c}_n^1(\gamma) \), we conclude that the equilibrium in the \([B, T]\) subgame coincides with that under no commitment to downstream quantity if \( c/a \geq \hat{c}_n^2(\gamma) \).
An implication of the above analysis is that in the \([B, T]\) subgame, for all 
\(c/a > \gamma/(2 + \gamma)\), there exists a unique equilibrium which is equivalent to a standard 
Stackelberg equilibrium with both marginal equal costs equal to \(c\). In contrast, for all 
\(c/a < \hat{c}_2^2(\gamma)\), there exists also a unique equilibrium which is equivalent to a Cournot 
asymmetric costs equilibrium with downstream costs zero for \(D_1\) and \(w_2^c\) for \(D_2\). For 
all \(\hat{c}_2^2(\gamma) < c/a < \gamma/(2 + \gamma)\), the above two equilibria coexist and are Pareto ranked 
with the Stackelberg equilibrium leading to higher profits for both chains than the 
Cournot one. A focal point argument can be used in the latter case for selecting the 
Pareto superior Stackelberg equilibrium.

The \([B, W]\) case: As in the \([B, T]\) case, the downstream reaction functions are 
\(R_1(K_1, q_2) = \min[K_1, (a - \gamma q_2)/2]\) and \(R_2(q_1, w_2) = (a - \gamma q_1 - w_2)/2\); hence, for small 
\(K_1, i.e. K_1 < [a(2 - \gamma) + \gamma w_2]/(4 - \gamma^2)\), the third stage equilibrium is \(q_1^* = K_1\) and 
\(q_2^*(w_2) = (a - \gamma K_1 - w_2)/2\); otherwise, it is \(q_1^C(0, w_2) = [a(2 - \gamma) + \gamma w_2]/(4 - \gamma^2)\) and 
\(q_2^C(0, w_2) = [a(2 - \gamma) - 2w_2]/(4 - \gamma^2)\). Again, for any given \(K_1\), the \((U_2, D_2)\) chain can 
induce (i) a capacity constrained equilibrium, in which case it will optimally set a 
wholesale price \(w_2^C(K_1) = [a\beta + (2 - \beta)c - \beta K_1]/2 > c\) to maximize the chain’s Nash 
product \(B_2^C(K_1, w_2) = [(w_2 - c)q_2^*(w_2)]^{\beta} [(a - \gamma K_1 - q_2^*(w_2) - w_2)q_2^*(w_2)]^{1-\beta}\); or (ii) an 
asymmetric Cournot equilibrium by setting \(w_2 < \tilde{w}_2(K_1) = [(2 - \gamma)/\gamma][K_1(2 + \gamma) - a]\), 
in which case the chain’s Nash product, after substituting \(q_i^C(0, w_2), i = 1, 2\), becomes:

\[
B_2^C(0, w_2) = \frac{(w_2 - c)^\beta [a(2 - \gamma) - 2w_2]^2 - \beta}{(4 - \gamma^2)^{1/\beta}}
\]

(a3)

Note further that, for any \(w_2 > c\), the \((U_1, D_1)\) chain can induce the equilibrium 
that maximizes the chain’s joint profits given \(D_2\)’s reaction function 
\(R_2(q_1, w_2) < R_2(q_1, c)\) by selecting \(K_1^s(w_2) = [a(2 - \gamma) - 2c + \gamma w_2]/2(2 - \gamma^2)\), provided 
that its downstream firm \(D_1\) will do produce at capacity at the third stage, that is, if 
\(q_1^C(0, w_2) \geq K_1^s(w_2)\). From the reaction functions in the \((K_1, w_2)\)-space, i.e. \(K_1^s(w_2)\) 
and \(w_2^s(K_1)\), we obtain the (candidate) one-sided capacity constrained equilibrium,

\[
K_1^s = \frac{(a - c)[4 - (2 - \beta)\gamma]}{8 - (4 - \beta)\gamma^2}; w_2^s = \frac{(4 - 2\gamma - \gamma^2)a\beta + 2c(4 - 2\gamma^2 - \beta(2 - \gamma - \gamma^2))}{8 - (4 - \beta)\gamma^2}
\]

(a4)
Note from (a4) that if $\beta = 0$, $w_2^s = c$ and $K_1^s = (a - c)(2 - \gamma)/2(2 - \gamma^2)$, which are the same as in the $[B, T]$ case. Moreover, that $\partial w_2^s / \partial \beta > 0$ and $\partial K_1^s / \partial \beta > 0$. Finally, it is can be checked that $q_1^c(0, w_2^s) \geq K_1^s$ if

$$c / a > c_n^3(\gamma, \beta) = [4 - (2 - \beta)\gamma]^{\gamma^2}/[16 - (2 - \beta)(2\gamma^2 + \gamma^3)]$$

with $\partial c_n^3 / \partial \gamma > 0$, $\partial c_n^3 / \partial \beta > 0$, $\lim_{\gamma \to 0} c_n^3(\gamma, \beta) = 0$, $c_n^3(1,0) = 0.2$ and $c_n^3(1,1) = 0.23077$.

Let $c / a > c_n^3(\gamma, \beta)$ and $K_1 = K_1^s$. If the $(U_2, D_2)$ chain’s Nash product is not higher when it induces an asymmetric Cournot game in the third stage, then the equilibrium in the $[B, W]$ case coincides with that under commitment to downstream quantity. This would occur if there does not exist a

$$w_2 < \tilde{w}_2(K_1^s) = \frac{(2 - \gamma)[2a\gamma(\beta + \gamma) - c(2 + \gamma)(4 - (2 - \beta)\gamma)]}{(8 - (4 - \beta)\gamma^2)}$$

s.t. $B_2^C(0, w_2) > B_2^s(K_1^s, w_2^s) = 2^{-2+\beta} \beta^\beta(2 - \beta)^{-2-\beta}(a - c)^2(4 - 2\gamma - \gamma^2) /[8 - (4 - \beta)\gamma^2]^2$

Note first from (a3) that, for the $(U_2, D_2)$ chain’s Nash product $B_2^C(0, w_2)$ to be positive, $w_2 > c$. However, $\tilde{w}_2(K_1^s) > c$ only if $c / a > \frac{(2 - \gamma)\gamma(\beta + \gamma)}{8 + 2\beta\gamma - 2\gamma^2 - \gamma^3} \equiv \sigma_n^h(\gamma, \beta)$, with $\sigma_n^h(\gamma, \beta) > c_n^3(\gamma, \beta)$, in which case the $(U_2, D_2)$ chain has no incentive to induce an asymmetric Cournot downstream game.

Further, maximizing (a3) w.r.t. $w_2$ we obtain the (unrestricted) optimal wholesale price for the $(U_2, D_2)$ chain, $w_2^s = [2c(2 - \beta) + a\beta(2 - \gamma)]/4$, in which case its (unrestricted) maximum Nash product is

$$B_2^{Cu} = (2 - \beta)^{-2-\beta} \beta^\beta[2c - a(2 - \gamma)]^2 / 2^{2+\beta}(4 - \gamma^2)^{-2-\beta}$$

However, $\tilde{w}_2(K_1^s) > w_2^s$ only if $c / a < \frac{(2 - \gamma)^2(8 - 4\beta\gamma - \beta^2\gamma)}{2(32 - 8\gamma^2 - (2 - \beta)^2\gamma^3)} \equiv \sigma_n^i(\gamma, \beta)$, with $\sigma_n^i(\gamma, \beta) < \sigma_n^h(\gamma, \beta)$. Moreover, $B_2^{Cu} \leq B_2^s(K_1^s, w_2^s)$ if

$$c / a > \hat{c}_n^4(\gamma, \beta) = \frac{2^\beta(4 - \beta)^2(4 - 2\gamma - \gamma^2) - (2 - \gamma)(4 - \gamma^2)^\beta[8 - (4 - \beta)\gamma^2]}{2^\beta(4 - \gamma^2)(4 - 2\gamma - \gamma^2) - 2(4 - \gamma^2)^\beta[8 - (4 - \beta)\gamma^2]}$$

with $\partial \hat{c}_n^4 / \partial \gamma > 0$, $\partial \hat{c}_n^4 / \partial \beta > 0$, $\lim_{\gamma \to 0} \hat{c}_n^4(\gamma, \beta) = 0$, $\hat{c}_n^4(1,0) = 0.2$ and $\hat{c}_n^4(1,1) = 0.235$. It can be further checked that $\hat{c}_n^4(\gamma, \beta) < \hat{c}_n^4(\gamma, \beta) < \sigma_n^i(\gamma, \beta)$ for all $(\gamma, \beta)$. Clearly,
\[ B_2^c(w_2 < w_2^u) < B_2^x \leq B_2^s(K_1^s, w_2^s) \] for all \( \sigma^l_n(\gamma, \beta) < c/a < \sigma^u_n(\gamma, \beta) \). Therefore, if \( c/a \geq \hat{c}_n^4(\gamma, \beta) \), the (U2, D2) chain has no incentive to induce an asymmetric Cournot downstream game. Finally, as \( \hat{c}_n^4(\gamma, \beta) > \hat{c}_n^3(\gamma, \beta) \), we conclude that the equilibrium in the \([B, W]\) subgame coincides with that under commitment to downstream quantity if \( c/a \geq \hat{c}_n^4(\gamma, \beta) \).

An implication of the above analysis is that in the \([B, W]\) subgame, for all \( c/a > \sigma^4_n(\gamma, \beta) \), there exists a unique equilibrium that is equivalent to a Stackelberg equilibrium. In contrast, for all \( c/a < \sigma^4_n(\gamma, \beta) \), there exists also a unique equilibrium that is equivalent to a Cournot asymmetric costs equilibrium with downstream costs zero for \( D_1 \) and \( w_2^u \) for \( D_2 \). While for all \( \hat{c}_n^4(\gamma, \beta) < c/a < \sigma^4_n(\gamma, \beta) \), the above two equilibria coexist and are Pareto ranked with the Stackelberg equilibrium leading to higher surplus for both chains than the Cournot one. A focal point argument can be used in the latter case for selecting the Pareto superior Stackelberg equilibrium.

Finally, let \( \hat{c}_n(\gamma) = \max[\hat{c}_n^2(\gamma), \max_{\beta} \hat{c}_n^4(\gamma, \beta)] \). It can be checked that \( \hat{c}_n(\gamma) = \hat{c}_n^4(\gamma, 1) \) for all \( \gamma \). The previous analysis implies that for all \( c/a \geq \hat{c}_n(\gamma) \) all three subgames have the same equilibrium as under downstream quantity commitment. \( \square \)