Undeclared Labour in Cournot Oligopoly

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Abstract

In a duopoly where firms are competing by adjusting their quantities and the wages are exogenously determined, we analyze the undeclared labour phenomenon and its side effects in product market. Our analysis focuses on the opportunity cost between the taxation and the contributions for social security. The findings of our analysis indicate that there is a strong relationship between the tax rate, the rate of contributions for social insurance and undeclared labour. It is furthermore determined that any combination of tax (t) / contributions (k) rates under the curve, will lead firms to practice undeclared labour, in order to avoid paying contributions for social security, since the alternative choice is more costly.

Keywords: Undeclared Labour, Cournot Duopoly, Labour Unions, Unionisation, Endogenous Objectives

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Introduction

Undeclared work is defined as "any paid activities that are lawful as regards their nature but not declared to public authorities". It is a complex phenomenon associated with tax evasion and social security fraud. Undeclared labour concerns various types of activities, ranging from informal household services to clandestine work by illegal residents, but excludes criminal activities.

It is a process that may engage both employers and employees voluntarily, because of the potential gain in avoiding taxes and social security contributions, social rights and the cost of complying with regulations.

From a macroeconomic point of view, undeclared labour reduces tax revenues (since employees declare no income and then no taxes are imputed) and undermines the financing of social security systems. To the extent that undeclared work competes with and even crowds out activities that comply with regulations, it is the main source of social dumping. In the case of undeclared work performed by individuals who are receiving benefits compensating their inactivity, there is also a dimension of social fraud.

From a microeconomic perspective, undeclared labour distorts fair competition among firms and causes productive inefficiencies, as informal businesses typically avoid access to formal services and inputs (e.g. credit) and prefer to stay small.

Undeclared labour is a decomposite phenomenon, that is influenced by a great range of economic, social, structural and cultural factors, tending to comprise a constraint to economic, fiscal, and social policies applied for the economic growth of an economy.

The fact that undeclared labour on one hand cannot be observed and on the other hand may be otherwise defined among countries, makes it even more difficult to establish credible evaluations about the growth of this phenomenon. However, a research, conducted on behalf of European Committee at 2004, while it accented important differences among countries regarding the qualitative characteristics as
well as the size of undeclared labour, estimated undeclared labour’s maximum values at 20% at some countries of Eastern and South Europe.

Given the complexity and the heterogeneity of the phenomenon, there is no simple solution to confront it. Nevertheless, the resolution of the European Union’s Council of 29 October 2003 on transforming undeclared work into regular employment proposed the following policies:

- Reducing the financial attractiveness of undeclared work stemming from the design of tax and benefit systems, and the permissiveness of the social protection system with regard to the performing of undeclared work;
- Administrative reform and simplification, with a view to reducing the cost of compliance with regulations;
- Strengthening the surveillance and sanction mechanisms, with the involvement of labour inspectorates, tax offices and social partners;
- Trans-national cooperation between Member States, and
- Awareness raising activities.

Regarding the first policy group of meters, European Committee concluded that there is still a great deal of actions to be done in order to balance both the motives and the disincentives offered by the social security systems. In particular, proposed policies concern the reservation of adequate income levels (taking into account the relation between benefits and contributions), the enforcement of exercising control over the labour market and over the persons entitled to social benefits and the imposition of proper economic penalties for tax and contribution evasion.

To gain all the above, policies should emphasize in:

(i) Proper taxation of overtime work;
(ii) Maintaining the institutional minimum wages;
(iii) Regulating tax distortions between tax systems applied in wage earners and those applied to self-employed;
(iv) Reducing the taxation of low productivity activities.

Even though during the past decades a broad range of methods has been developed to analyze the undeclared labour phenomenon, to understand its dimensions and causes, to formulate an appropriate policy to constrain its spread, neither this phenomenon has been examined with any available method, nor the discussion about which methodology is the most appropriate has still not come to an end. In particular, there has been an extended use of econometrics and applied statistics in the relevant researches. Surveys from international organizations (such as OECD, ILO, EU etc) based mostly on evidence and results of state audits also consist a notable framework. However, undeclared labour has not yet been approached or analyzed using the framework of industrial organization and game theoretic analytical toolkit.

With this research, we aspire to deliver a different approach, using the industrial’s organization framework. Moreover, one of the main goals of this work is to propose a different policy for restraining the phenomenon of undeclared labour. As it is shown, the use of proper tax rates relative to those of social insurance could – under certain circumstances – restrain the economic attractiveness of this phenomenon.

1. The model

Consider a homogeneous good sector where two firms, $f_1$ and $f_2$, compete by adjusting their quantities. We also assume a production function $q_i = L_i$ for both firms ($q_i$: the production of $i$ firm, $L_i$: the workers used in $i$ firm to produce $q_i$, $i$: 1, 2). The first firm insures its personnel and faces $1 \cdot w$ unit labour cost$^1$, including

$^1$ We normalize production per unit cost to zero.
contributions for social insurance, where \( w \) stands for wage and \( k \) for the percentage of the wage for social insurance contributions. The second firm decides not to insure its personnel and faces \( w \) unit labour cost (just the wage).

Additionally, both firms pay taxes of rate \( t \) on their declared net profits. Notice that, since the first firm declares and insures its workers, the whole payroll costs (meaning both wages and contributions for social security) should decrease the final net profits; while the second firm doesn’t have this option, since undeclared labour cannot be shown at any public authority, including tax office. The tax functions form as follows\(^2\):

\[
\begin{align*}
\text{f}_1 \text{ profit’s taxation } &= t \cdot \left(p - (1 + k) \cdot w\right) \cdot q_1 \\
\text{f}_2 \text{ profit’s taxation } &= t \cdot (p \cdot q_2)
\end{align*}
\]

Therefore, the first firm will pay contributions for social insurance and fewer taxes (since declared profits will be fewer), while the second firm will pay nothing for social security but more taxes (since declared profits will be significantly higher). It is clearly shown that there is an opportunity cost for firms, between taxation and contributions for social insurance.

Notice that at this stage of our early analysis, any choice of the firms to declare their workers or not, as well as the wage determination, are both considered exogenously. We assume that one firm acts in reverse to the other and examine which one is finally in better position. On the other hand, the wages are considered to be institutionally announced and apply for all firms in the economy (i.e., \( w_1 = w_2 = w \)). Our analysis does not, also, include any governmental surveillance or compliance penalties. We simply examine the equilibrium of the market, when it is auto-regulated, without any further interventions.

\(^2\) Quantities, profits and taxation for each firm must be a positive argument. Thus, in order our model to have internal solutions, we set \( t < t_{cr} = \frac{1 + (-1 + k)w}{1 + w + kw} \) and \( 0 < w < 0.5 \).
2. Solving the Model

Let for tractability the reverse demand function be normalized to \( P(Q) = 1 - Q \), where \( Q = q_1 + q_2 \). Then, given our setup, the firms’ profit functions are as follows:

\[
\Pi_1 = \left[ P(Q) - (1+k) \cdot w \right] \cdot q_1 - t \cdot \left[ P(Q) - (1+k) \cdot w \right] \cdot q_1 \\
\Pi_2 = \left[ P(Q) - w \right] \cdot q_2 - t \cdot \left[ (P(Q) - w) \cdot q_2 \right]
\]

Taking the first order conditions and solving the model, we conclude that the quantities of each firm have as follows:

\[
q_1 = \frac{1}{3} \left[ 1 - \frac{1 + 2k - 2t(1+k)}{(1-t)} \cdot w \right] \\
q_2 = \frac{1 - t - (1-k)(1-t) + t}{3(1-t)} \cdot w
\]

Therefore, \( q_1 - q_2 = \frac{(t \cdot (1+k) - k)}{1-t} \cdot w \), that is, if \( t > \frac{k}{1+k} \) then \( q_1 > q_2 \), while if \( t < \frac{k}{1+k} \) then \( q_1 < q_2 \). It is clear that if the implied tax rate is high enough (greater than \( \frac{k}{1+k} \)), then the firm that declares its personnel will enjoy higher market share. Otherwise, if the tax rate is low enough (less than \( \frac{k}{1+k} \)), then firm 2 enjoys higher market share. So, in terms of market share, we can illustrate the above with the following diagram:

![Diagram showing the relationship between declared and undeclared labour with market share](image-url)
It reveals that each combination of t & k above the curve \( t^* = \frac{k}{1+k} \) obliterates any competitive advantage of the second firm, derived from the practice of undeclared labour, since in that case the first firm will enjoy greater market share. On the other hand, if any combination of t and k below the curve is applied, then the second firm will have an incentive to practice undeclared labour, since in this manner it will obtain greater market share.

The same result also applies with profit analysis. The profits of each firm, as they are derived, have as follows:

\[
\Pi_1 = \frac{(-1 + t + w + 2kw - 2(1 + k)tw)^2}{9(1-t)} \quad (5)
\]

\[
\Pi_2 = \frac{(-1 + t + (1-k(1-t)+t)w)^2}{9(1-t)} \quad (6)
\]

Abstracting (5)-(6) we have:

\[
\Pi_1 - \Pi_2 = \frac{(k(-1+t)+t)w(2-(2+k)w+t(-2+w+kw))}{3(1-t)} \quad (7)
\]

The roots of the above expression are \( t_1^* = \frac{k}{1+k} \) and \( t_2^* = \frac{-2+2w+kw}{-2+w+kw} \). Since \( t_{cr} = \frac{1+(1+k)w}{1+w+kw} < t_2^* = \frac{-2+2w+kw}{-2+w+kw} \) for \( 0 < w < 0.5 \), we reject \( t_2^* \) as a critical value\(^3\) and we conclude to the same results, as for the market share analysis; i.e. if \( t < t_1^* = \frac{k}{1+k} \) then the firm that practices undeclared labour will gain more profits than the other one which declares its personnel. If, on the other hand, \( t > t_1^* \), then the firm that declares its workers will gain more profits. Preposition 1 summarizes.

**Proposition 1:**

*In the case of exogenous wage, the greater the tax rate than \( \frac{k}{1+k} \) is, the less strong is the incentive for undeclared labour. In other words, comparatively low enough taxation (less than \( \frac{k}{1+k} \)) will create incentives for undeclared labour and conversely.*

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\(^3\) As already mentioned, \( t \) should be less than \( t_{cr} \).
3. Conclusions

Interpreting the results above, a comparatively low tax rate will enforce the phenomenon of undeclared labour. As a matter of fact, firms face an opportunity cost – dilemma:

- Either they practice undeclared work, pay no contributions for social insurance, but they state more profits and thus pay more taxes
- or they declare their personnel and pay the relevant contributions for social insurance, but they pay fewer taxes due to the fewer profits resulting for taxation.

Any combination of tax / contributions rates under the $t_1^* = \frac{k}{1+k}$ curve will indeed lead firms to practice undeclared labour, in order to avoid paying contributions for social security, since the alternative choice is more costly.
References

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