Cournot is more competitive than Bertrand! Upstream Monopoly with Two-part Tariffs.*

Maria Alipranti† Emmanuel Petrakis‡

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Abstract

The present paper compares the Cournot and Bertrand equilibrium outcomes and social welfare in vertically related markets with upstream monopolistic market structure, where the trade between the upstream monopolist and the downstream firms is conducted via two-part tariffs contracts. We show that the equilibrium quantities, the profits of the downstream firms, the consumers’ surplus and the social welfare are always higher under Cournot final market competition than under Bertrand final market competition. On the contrary the equilibrium profits of the upstream monopolist under Bertrand market competition always exceed those obtained under Cournot market competition.

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†Department of Economics, University of Crete, e-mail: alipranti@econ.soc.uoc.gr.

‡Corresponding author. Department of Economics, University of Crete, Univ. Campus at Gallos, Rethymnon 74100, Greece, Tel: +302831077409, Fax: +302831077406, e-mail: petrakis@uoc.gr.
1 Introduction

It is well established in the oligopoly theory that the mode of the market competition crucially affects the market’s equilibrium outcomes and the social welfare. Singh and Vives (1984) were the first to show that the equilibrium market outcomes and the social welfare differ significantly under alternative types of market competition, namely Cournot (quantity) and Bertrand (price) competition, with the equilibrium prices and profits (quantities, respectively) being higher (lower, respectively) under Cournot competition than under Bertrand, while the Bertrand competition being more efficient in terms of social welfare than the Cournot. Thus, given the significant market and policy implications that the mode of the market competition has, a substantial economic literature has been developed that examines the robustness of these traditional results under alternative market frameworks. (see among, Cheng, 1985; Vives, 1985; Okuguchi, 1987; Dastidar, 1997; Häckner, 2000). Cheng (1985) provides the geometrical proof of the Singh and Vives (1984) results, while Okuguchi, 1987 provides the general conditions for comparing the prices under Cournot and Bertrand competition. Further, Vives (1985), examining the Cournot-Bertrand differences in a n-firms oligopoly market with general demand functions, shows that the price-cost margin under Cournot competition is higher than under Bertrand. Dastidar (1997) points out the sensitivity of the Singh and Vives (1984) results on the market shares rules and demonstrates that they may not be valid under equal sharing and cost asymmetries. More recently, Häckner (2000) investigates the robustness of the Singh and Vives (1984) results under a n-firm oligopoly market structure with vertical product differentiation and shows that the results can not be generalized to the n-firm case when the products are of sufficiently different quality (i.e., a high quality firm may obtain higher profits under Bertrand competition than under Cournot).

Further, the robustness of these cornerstone results in the presence of R&D and innovation firms’ strategic investments has been extensively investigated in the economic literature (see among, Delbono and Denicolo, 1990; Qiu, 1997; Symeonidis, 2003; Mukherjee, 2011; Chang and Peng, 2012). In particular, Delbono and Denicolo (1990) show that, under a symmetric and homogenous product duopoly, the Singh and Vives (1984) results over the welfare can be reversed when firms undertake R&D investments. Furthermore, Qiu (1997) demonstrates that when firms invest in cost-reducing R&D the relevant efficiency of the Cournot versus the Bertrand competition crucially depends on the R&D productivity, the extent of the
R&D spillover and the degree of the final market competition. In more details, he shows that Bertrand competition is more efficient than the Cournot, if either R&D productivity is low, or spillovers are weak, or products are sufficiently differentiated, while the opposite holds when R&D productivity is high, spillovers are strong and the products are close substitutes. More recently, Symeonidis (2003) comparing the Bertrand and Cournot equilibria in a differentiated duopoly with substitute goods and product R&D, shows that prices and firms’ profits are always higher under quantity competition than under price competition, while, the output, the consumer surplus and the social welfare are higher under Bertrand than under Cournot competition when the R&D spillovers are weak or when the products are sufficiently differentiated. The opposite holds when the R&D spillovers are strong and the products are less differentiated.

In a different vein, Mukherjee (2011) and Chang and Peng (2012) introducing into the analysis technology licensing show that the traditional results over the Cournot-Bertrand can be efficiently reversed with regard to the innovation levels. However, all of the aforementioned literature has restricted its attention in the classic one-tier market structure, while the relevant research under more complex market structures such as markets with vertical relations is still limited. Correa-Lopez and Naylon (2004) were the first to extend the Cournot-Bertrand debate, taking into account the effects of vertical relations. In particular, comparing the Cournot and Bertrand equilibrium profits under a unionized duopoly model with differentiated products, they show that the profits of the downstream firms are higher under Bertrand competition when the unions are sufficiently powerful, place considerable weight over the wages on their utility function and the products are imperfect substitutes, the reversed result holds for all the other cases. In the same direction, Lopez (2007) shows that if the upstream firms are profit maximizers and the products are close substitutes, then the downstream firms’ profits are higher under Cournot competition. Furthermore, Fanti and Meccheri (2011), investigating the Cournot-Bertrand profits differential in a differentiated duopoly market with unions and decreasing labour returns, demonstrate that the traditional result that the profits are higher under Cournot competition, holds when the labour has decreasing returns and the wages are unilaterally fixed by a total wage bill maximizing union. More recently, Mukherjee et al. (2012), examining the Cournot-Bertrand profits in a vertically related market with homogenous final goods, where the upstream monopolist sets the input price for the downstream firm and the downstream firms differ in terms of production technologies, show that under uniform upstream monopolist’s pricing strategy
and downstream firms’ sufficiently asymmetric production technologies the downstream firms’ profits are higher under Bertrand than under Cournot competition. Furthermore, they show that the standard Cournot-Bertrand profits result holds under price discrimination of the upstream monopolist. However, all of the aforementioned works share a common feature, they examine the differences of the equilibrium outcomes under Cournot and Bertrand competition in vertically related markets where the upstream firms/unions negotiates with the downstream firms over the per unit input price/wages, while they ignore the alternative trading forms, such as two-part tariffs trading contracts. Thus, given that the two-part tariffs trading contracts are widely practiced in vertically related industries, the objective of the present paper is to extend the existing literature over the Cournot-Bertrand debate by investigating the robustness of the standard results on the ranking of Cournot and Bertrand equilibrium outcomes under vertically related markets, where trade between the upstream and the downstream firms is conducted via two-part tariffs contracts.

To do so, we have considered a two-tier industry with differentiated final goods, consisted by an upstream monopolistic firm and two downstream firms. The upstream monopolist provides to the downstream firms a necessary input for the final good’s production, while the downstream firms transform the input into the final good and then sells it to the final consumers. The trade between the upstream monopolist and the downstream firms is conducted via two-part tariffs contracts, that means that the upstream and the downstream firms bargain over both the per -unit of input price, or else the wholesale price, and the fix fees. The timing of the game is given as follows. In the first stage of the game the upstream monopolist bargains simultaneously and separately with each of the downstream firms over the contract terms (i.e., the wholesale price and the fix fees), while, in the second stage, the downstream firms compete by setting either their prices or their quantities.

We show that under vertically related markets with upstream monopolistic market structure, where the trade between the firms is conducted via two-part tariffs contracts, the standard result that the equilibrium quantities are lower under Cournot final market competition than under Bertrand is reversed. In other words, we demonstrate that the equilibrium Cournot quantities always exceed the respective ones obtained under Bertrand final market competition and thus, the Cournot market is more competitive in terms of output production than the Bertrand one. The intuition behind this result is based on the effect that the alternative modes of final market competition have on the wholesale prices. In particular, we show that when the firms
compete in quantities, the upstream monopolist sets wholesale prices that are lower than its marginal production cost, while this does not hold under Bertrand final market competition. Thus, the equilibrium wholesale prices under Cournot market competition are always lower than under Bertrand market competition, that means that under Cournot final market competition the per unit of input price is lower and thus, the downstream firms’ marginal cost of production is lower. Put in other words, when the downstream firms compete by setting their quantities the upstream monopolist via lower wholesale prices that he sets, turns to subsidize the downstream firms’ production.

Furthermore, in line with the Sighn and Vives (1984), we confirmed that the downstream firms' profits under Cournot competition are always higher than those under Bertrand competition. Clearly, the lower wholesale prices that the downstream firms pay to the upstream monopolist under Cournot final market competition along with their higher output production, increase the downstream firms' profitability and thus the equilibrium Cournot profits always exceed the respective ones obtained under Bertrand final market competition. On the contrary, we demonstrate that the upstream monopolist profits under Bertrand competition are higher than under Cournot competition. The intuition behind the latter comes straightforward from the fact that the wholesale prices that the upstream monopolist sets under Cournot final market competition are always higher than those under Cournot competition, since, under the former type of final market competition, the "commitment problem" that the upstream monopolist faces is less severe.1

Interestingly enough, we also show that contrary to the conventional wisdom that suggests that the consumers’ surplus and the social welfare is higher under Bertrand competition, in

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1 As standard in the literature, we have shown that the upstream monopolist, when the trade is conducted via two-part tariffs contracts and the final market competition takes place in quantities, can not fully excerpt its market power and thus, instead of charging the wholesale price that allows him to obtain the monopoly profits, it sets wholesale prices lower than its marginal production cost. This is so, due to the so called "commitment problem" that arises when the contracts negotiations are not fully observable, since the upstream monopolist could not commit to the downstream firms that it is not going to behave opportunistically and secretly offers a lower wholesale price to the rival downstream firm. Thus, none of the downstream firms is going to agree to a wholesale price higher than the upstream monopolist cost of production. However, in vertically related markets with Bertrand final market competition, the wholesale prices that the upstream monopolist sets are always higher than its marginal production cost, which means that the upstream does not subsidize the downstream production via the wholesale prices. This is so because under Bertrand final market competition the upstream monopolist faces a less severe commitment problem, since the prices, contrary to quantities, are strategic complements, and thus the upstream does not wish its downstream partners to behave aggressively in the final market competition. For detailed analysis of the commitment problem see among, McAfee and Schwartz, 1995; Rey and Vergi, 2004; de Fontenay and Gans, 2005; Milliou and Petrakis, 2007.
vertically related markets with upstream monopolistic market structure and two-part tariffs 
trading contracts both the consumers’ surplus and the social welfare is higher under Cournot 
competition than under Bertrand competition. This finding has significant policy implications, 
since it reveals that in vertically related markets with two-part tariffs contracts and 
monopolistic upstream sector the Cournot competition is preferable in terms of welfare than 
the Bertrand one.

The rest of the paper is organized as follows. In Section 2, we present our model. In Section 
3, we present the equilibrium analysis of our model under both Cournot and Bertrand final 
market competition and we compare the equilibrium results under the two cases. In Section 4, 
we examine the robustness of our results in vertically related markets with upstream separate 
firms market structure. In Section 5, we conclude.

2 The Model

We consider a two-tier industry consisting of an upstream monopolist and two downstream 
firms denoted by $U$ and $D_i$, respectively, with $i = 1, 2$. The upstream monopolist is an input 
provider with its marginal production cost, for sake of simplicity, to be normalized at zero. The 
downstream firms are final good manufactures, with one unit of input being transformed into 
one unit of final good. Following, Singh and Vives (1984), we assume that the utility function of 
the representative consumer in the market is given by, $V_i(q_i, q_j) = a(q_i + q_j) - (q_i^2 + q_j^2 + \gamma q_i q_j)/2$. 
Thus, maximizing the consumer’s utility function, we have that each downstream firm $D_i$ sells 
its product to the final market facing the following demand and inverse demand functions:

$$q_i = \frac{(\alpha - p_i) - \gamma(\alpha - p_j)}{1 - \gamma^2}, \quad i, j = 1, 2; \quad i \neq j; \quad 0 < \gamma \leq 1$$ \hspace{1cm} (1)

$$p_i = a - q_i - \gamma q_j, \quad i, j = 1, 2; \quad i \neq j; \quad 0 < \gamma \leq 1$$ \hspace{1cm} (2)

where $q_i, p_i$ and $q_j, p_j$ denote the $D_i$’s and $D_j$’s output and price, respectively. The parameter 
$\gamma$ denotes the degree of product substitutability. The higher the $\gamma$, the closer substitutes the 
final products are, or in other words, the fiercer the final market competition is (Vives, 1985).

We assume that the trade between the upstream monopolist and the downstream firms is 
conducted via two part tariffs contracts $(w_i, F_i)$, where $w_i$ denotes the per-unit of input 
price, or else, the wholesale price that each downstream firm pays to the upstream monopolist
and $F_i$ denotes the fixed fees. Downstreams are endowed with the same constant returns to scale production technology with their marginal production cost given by $c_i = c + w_i$, where $c, 0 < c < \alpha$, is an exogenous constant marginal cost. A two stage game has been considered, where the sequence of the moves are given as follows. In the first stage of the game, the upstream monopolist bargains, simultaneously and separatively, with each downstream firm over the trading contract terms $(w_i, F_i)$, where the bargaining power of the upstream firm in the market is given by $\beta$, while $1 - \beta$ corresponds to the bargaining power of the downstream firms. In the second stage, the downstream firms compete by setting either their outputs (Cournot final market competition) or their prices (Bertrand final market competition). We solve the above game backwards by employing the Subgame Perfect Nash Equilibrium (SPNE) solution concept.

Further, in order to ensure that all the participants in the market are active under all the configurations considered, the following assumption should hold throughout the paper:

**Assumption 1.** $\beta \geq \beta(\gamma)$, where $\beta(\gamma) = \gamma^3/4 - 2\gamma - 2\gamma^2 + \gamma^3$

Assumption 1 is a necessary and sufficient condition in order to ensure the existence of pure strategy pairwise proof equilibria under the case of the upstream monopolist. Non-existence of pure strategy equilibria may occur because pairwise proofness leads to negative profits for the upstream monopolist. This is so, since, if for given $\gamma$ the upstream monopolist power is low enough, the upstream is being subject to opportunism and is unable to cover its losses from the input subsidization via the fixed-fees.

### 3 Equilibrium Outcomes and Mode of Competition

#### 3.1 Cournot Competition

Starting with the case where the downstream firms compete in the final market by setting their outputs, we have that in the second stage of the game, each downstream firm $D_i$ chooses its output $q_i$, taking as given the decision over the output of the rival downstream firm $q_j$, in order to maximize its (gross) profits,

$$\max_{q_i} \pi_i^C (.) = (a - q_i - \gamma q_j)q_i - (c + w_i)q_i$$

(3)
The first order conditions give rise to the following reaction functions,

\[ q_i(q_j) = \frac{a - c - w_i - \gamma q_j}{2} \]  

(4)

Observe here that, as \( \gamma > 0 \), the reaction functions are downward sloping, that means that, as standard in the literature, under Cournot final market competition, the quantities are strategic substitutes. Also notice, that a reduction in the wholesale price that \( D_i \) pays to the upstream monopolist, shifts out its reaction function and leads to higher own output’s production and to lower rival’s output production, or in other words, makes the \( D_i \) more aggressive in the final market competition.

Solving the system of the reaction functions, given in equation (4), the equilibrium quantities in the second stage of the game when the final market competition takes place in quantities, are given by,

\[ q_i^C(w_i, w_j) = \frac{(a - c)(2 - \gamma) - 2w_i + \gamma w_j}{4 - \gamma^2} \]  

(5)

Consequently, the equilibrium prices and \( D_i \)’s gross profits in the second stage under Cournot final market competition are given respectively by,

\[ p_i^C(w_i, w_j) = \frac{[a + c(1 + \gamma)](2 - \gamma) + (2 - \gamma^2)w_i + \gamma w_j}{4 - \gamma^2} \]  

(6)

\[ \pi_{D_i}^C(w_i, w_j) = \frac{[(a - c)(2 - \gamma) - 2w_i + \gamma w_j]^2}{[4 - \gamma^2]^2} \]  

(7)

In the first stage of the game, the upstream monopolist \( U \) and each of the downstream firms \( D_i \) negotiate over the contract terms \((w_i, F_i)\), taking as given the contract terms of the rival pair \((w_{MC}^j, F_{MC}^j)\). Thus, each \( U \) and \( D_i \) pair chooses the trading contract terms, \( w_i \) and \( F_i \) in order to maximize the generalized Nash product:

\[ \max_{w_i, F_i} \left[ \pi_U^C(w_i, w_{MC}^j) + F_i + F_{MC}^i - d(w_{MC}^j, F_{MC}^j) \beta \pi_{D_i}^C(w_i, w_{MC}^j) - F_i \right]^{1-\beta}. \]  

(8)

where, \( \pi_U^C(w_i, w_{MC}^j) \) denotes the profits of the upstream monopolist that are given by the sum of its’ sales to both downstream firms, that is, \( \pi_U^C(w_i, w_{MC}^j) = w_i q_i^C(w_i, w_{MC}^j) + w_{MC}^j q_j^C(w_i, w_{MC}^j) \), while \( d(w_{MC}^j, F_{MC}^j) = w_{MC}^j q_{MC}^j + F_{MC}^j \) is the disagreement payoff that the upstream monopolist faces, if an agreement between a \((U, D_i)\) pair is not reached. In more details, if an agreement between a \((U, D_i)\) pair can not be reached, the upstream monopolist is
expected to obtain the revenues by its input sales on the remaining downstream firm, $D_j$, that is, $\pi_U^C(w_{jMC}) = w_{jMC}q_{jMON}$, where $q_{jMON} = \frac{a-c-w_{jMC}}{2}$ is the output produced by the remaining downstream firm, that monopolizes the final market, plus the fixed fees, $F^MC_j$. Notice that a disagreement between the $U$ and $D_i$, does not give rise to new negotiations over the contract terms of the remaining $(U, D_j)$ pair.

Maximizing (8) with respect to the fix fees, $F_i$, we have that,

$$F_i = \beta \pi_D^C(w_i, w_{jMC}) - (1 - \beta)[\pi_U^C(w_i, w_{jMC}) - w_{jMC}q_{jMON}]$$

(9)

Substituting (9) into (8), we obtain that the net profits of the upstream monopolist, above its disagreement payoff, and the net profits of each downstream firm, $D_i$, are proportional to their joint surplus, $S^C = \pi_U^C(w_i, w_{jMC}) + \pi_D^C(w_i, w_{jMC}) - w_{jMC}q_{jMON}$, with the coefficients of the proportionality to be given by the bargaining powers, $\beta$ and $1 - \beta$, respectively. Thus, the wholesale prices, $w_i$, are chosen in order to maximize this surplus:

$$\text{Max}_{w_i} S^C = [a - q_i^C(w_i, w_{jMC}) - \gamma q_j^C(w_i, w_{jMC})]q_i^C(w_i, w_{jMC}) + w_{jMC}q_j^C(w_i, w_{jMC}) - q_j^{MON}$$

(10)

From the first order conditions of (10), we obtain that the equilibrium wholesale prices under Cournot final market competition are given by,

$$w_i^C = w_j^C = w^C = -\frac{(a - c)\gamma^2}{2(2 - \gamma)}$$

(11)

Observe here, that in vertically related markets with upstream monopolistic market structure and Cournot final market competition, the equilibrium wholesale prices are always lower than the upstream monopolist’s marginal production cost (i.e., $w^C < 0$). That means that the upstream monopolist, when trade is conducted via two-part tariffs contracts and the final market competition takes place in quantities, can not fully excerpt its market power and thus, instead of charging the wholesale price that it allows him to obtain the monopoly profits, it subsidizes via the wholesale prices the downstream production. This is so, due to the so called "commitment problem" that arises when the contracts negotiations are not fully observable. In more details, as standard in the relevant literature (see e.g, McAfee and Schwartz, 1995; Rey and Vergi, 2004; de Fontenay and Gans, 2005; Milliou and Petrakis, 2007), when the
contracts negotiations are not fully observable, the upstream monopolist could not commit to the $D_i$ firm that it is not going to behave opportunistically and secretly offers a lower wholesale price to the rival downstream firm, $D_j$. Clearly, when the contract terms negotiations are not observable, the downstream firms are aware of the upstream monopolist’s incentives to behave opportunistically and to secretly offer a lower wholesale price to the rival downstream firm, in order to make the latter more aggressive in the final market and to be benefited by the higher gross profits that the favored downstream partner will obtain (the upstream monopolist could extracts the benefit of the higher downstream firm’s profits by transferring part of these higher profits upstream via the fix fees). Therefore, none of the downstream firms will agree to a wholesale price higher than the upstream monopolist’s cost of production.

Further, observe that the wholesale prices do not depend on the bargaining power distribution in the market, $\beta$, since they are chosen such to maximize the joint surplus of each $(U, D_i)$ pair, while they are decreasing in the degree of the final market competition, $\gamma$. Clearly, the closer substitutes the products, the fiercer the final market competition is, that in turn, leads the upstream monopolist to decrease the wholesale prices in order to make the downstream firms more aggressive in the final market.

**Lemma 1** In a vertically related market with monopolistic upstream structure and Cournot final market competition the equilibrium wholesale prices are always lower than the upstream monopolist’s marginal production cost, while they are independent of the bargaining power distribution, $\beta$, and decreasing in the degree of the final market competition, $\gamma$.

Using (11), (5) and (9) it follows that the equilibrium output and fix fees when the final market competition takes place in quantities, are given respectively by,

$$q^C(.) = \frac{(a-c)(2-\gamma)}{2(2-\gamma^2)}$$

$$F^C(.) = \frac{(a-c)^2(\gamma-2)^2[2\beta + (1-\beta)\gamma^2]}{8(\gamma^2-2)^2}$$

Observe here that the equilibrium fix fees depend on the bargaining power distribution, $\beta$, while they always exceed the upstream monopolist’s marginal production cost.

Moreover, the equilibrium downstream firms’ profits and the upstream monopolist’s profits
under Cournot final market competition are given respectively by,

$$\pi^C_{Di}(.) = \frac{(1 - \beta)(a - c)^2(\gamma - 2)^2}{8(2 - \gamma)^2}$$  \hspace{1cm} (14)$$

$$\pi^C_{U}(.) = \frac{(a - c)^2(2 - \gamma)[\beta(2 - \gamma)(2 - \gamma^2) - \gamma^3]}{4(\gamma^2 - 2)^2}$$  \hspace{1cm} (15)$$

### 3.2 Bertrand Competition

We turn now to discuss the case where the downstream firms compete by setting their prices (i.e., Bertrand final market competition). In the second stage of the game, each downstream firm $D_i$, chooses its price $p_i$, taking as given the decision over the price of the rival downstream firm $p_j$, in order to maximize its (gross) profits,

$$\text{Max}_{p_i} \pi^B_i(.) = (p_i - c - w_i)\frac{(\alpha - p_i) - \gamma(\alpha - p_j)}{1 - \gamma^2}$$  \hspace{1cm} (16)$$

The first order conditions give rise to the following reaction functions,

$$p_i(p_j) = \frac{(1 - \gamma)\alpha + c + \gamma p_j + w_i}{2}$$  \hspace{1cm} (17)$$

Observe here that for $\gamma > 0$, the reaction functions are upward sloping and thus, prices in the Bertrand final market competition are strategic complements. Further, observe that a reduction in the wholesale price that $D_i$ pays to the upstream monopolist, shifts its reaction function downwards and thus, given the negative price-output relationship, makes $D_i$ firm more aggressive in the final market competition.

Solving the system of the reaction functions given in (17), the equilibrium prices in the second stage of the game when the final market competition takes place in prices, are given by,

$$p^B_i(w_i, w_j) = \frac{(2 + \gamma)[(1 - \gamma)\alpha + c] + 2(w_i + w_j)}{4 - \gamma^2}$$  \hspace{1cm} (18)$$

Consequently, the equilibrium output and $D_i$’s gross profits in the second stage under Bertrand final market competition are given respectively by,

$$q^B_i(w_i, w_j) = \frac{(\alpha - c)(2 + \gamma)(1 - \gamma) - (2 - \gamma^2)w_i + \gamma w_j}{4 - 5\gamma^2 + \gamma^4}$$  \hspace{1cm} (19)$$
\[
\pi^B_{D_i}(w_i, w_j) = \frac{[(a - c)(\gamma^2 + \gamma - 2) + (2 - \gamma^2)w_i + \gamma w_j]^2}{(1 - \gamma)(\gamma^2 - 4)^2} \tag{20}
\]

In the first stage of the game, the upstream monopolist \(U\) and the downstream firm \(D_i\) bargain over the contract terms \((w_i, F_i)\), taking as given the contract terms of the rival pair \((w_j^{MB}, F_j^{MB})\). Thus, each \(U\) and \(D_i\) pair chooses the trading contract terms, \(w_i\) and \(F_i\) in order to maximize the generalized Nash product:

\[
\max_{w_i, F_i} [\pi^B_U(w_i, w_j^{MB}) + F_i + F_j^{MB} - d(w_j^{MB}, F_j^{MB})] = [\pi^B_{D_i}(w_i, w_j^{MB}) - F_i]^{1-\beta} \tag{21}
\]

where \(\pi^B_U(w_i, w_j^{MB})\) denotes the upstream monopolist’s profits that are given by the sum of the upstream’s sales to both downstream firms, that is, \(\pi^B_U(w_i, w_j^{MB}) = w_i q_i^B(w_i, w_j^{MB}) + w_j^{MB} q_j^B(w_i, w_j^{MB})\), while \(d(w_j^{MB}, F_j^{MB}) = w_j^{MB} q_j^{MON} + F_j^{MB}\) is the disagreement payoff that the upstream monopolist will face, if an agreement between a \((U, D_i)\) pair is not reached.

Maximizing (21) with respect to the fix fees, \(F_i\), we have that,

\[
F_i = \beta \pi^B_{D_i}(w_i, w_j^{MB}) - \left[ (1 - \beta)[\pi^B_U(w_i, w_j^{MB}) - w_j^{MB} q_j^{MON}] \right] \tag{22}
\]

Substituting (22) into (21), we have that the net upstream monopolist’s profits above its disagreement payoff, and the net profits of the downstream firm, \(D_i\), are proportional to their joint surplus, \(S^B = \pi^B_U(w_i, w_j^{MB}) + \pi^B_{D_i}(w_i, w_j^{MB}) - w_j^{MB} q_j^{MON}\), with the coefficients of proportionality to be given by the bargaining powers, \(\beta\) and \(1 - \beta\), respectively. Hence, the wholesale prices, \(w_i\), are chosen in order to maximize this surplus:

\[
\max_{w_i} S^B = [a - q_i^B(w_i, w_j^{MB}) - \gamma q_j^B(w_i, w_j^{MB})] q_i^B(w_i, w_j^{MB}) + w_j^{MB} q_j^B(w_i, w_j^{MB}) - q_j^{MON} \tag{23}
\]

From the first order conditions of (23), we have that the equilibrium wholesale prices under Bertrand final market competition are given by,

\[
w^B_i = w^B_j = w^B = \frac{(a - c) \gamma^2}{4} \tag{24}
\]

Observe here, that in contrast to the vertically related markets with Cournot final market competition, under vertically related markets with upstream monopolistic market structure and Bertrand final market competition, the equilibrium wholesale prices are always higher than the
upstream monopolist’s marginal production cost (i.e., \( w^B > 0 \)). That means that, in vertically related markets with Bertrand final market competition, the upstream monopolist does not subsidize the downstream production via the wholesale prices. Intuitively, under Bertrand final market competition the upstream monopolist faces a less severe commitment problem, since the prices, contrary to quantities, are strategic complements. Thus, the upstream does not subsidize the downstream partners via the wholesale prices since, he does not wish its downstream partners to behave aggressively in the final market competition. Also note that, in line with the Cournot final market competition case, the equilibrium wholesale prices under the Bertrand final market competition are independent of the bargaining power distribution in the market, \( \beta \), since they are chosen in order to maximize the joint surplus of each vertical chain in the market. Yet, contrary to the Cournot final market competition case, under Bertrand final market competition the equilibrium wholesale prices are increasing in the degree of the final market competition, \( \gamma \). This is so, since, prices are strategic complements and thus, as the final market competition becomes fiercer, the upstream monopolist is willing by setting a higher wholesale price, to force the downstream partners to behave less aggressively.

**Lemma 2** In a vertically related market with monopolistic upstream structure and Bertrand final market competition, the equilibrium wholesale prices always exceed the upstream monopolist’s marginal production cost, while they are independent of the bargaining power distribution (\( \beta \)) and increasing in the degree of the final market competition (\( \gamma \)).

Using (24), (19) and (22), it follows that the equilibrium output and the fix fees when the final market competition takes place in prices, are given respectively by,

\[
q^B(.) = \frac{(a - c)(2 + \gamma)}{4(1 + \gamma)} \tag{25}
\]

\[
F^B(.) = \frac{(a - c)^2(2 + \gamma)[\beta(4 - 2\gamma - \gamma^3 + \gamma^4) - (2 - \gamma + \gamma^2)\gamma^2]}{32(1 + \gamma)} \tag{26}
\]

while the equilibrium downstream firms’ profits and the upstream monopolist’s profits, under Bertrand final market competition, are given respectively by

\[
\pi^B_{D_i}(.) = \frac{(1 - \beta)(a - c)^2(2 + \gamma)[4 - 2\gamma - \gamma^3 + \gamma^4]}{32(1 + \gamma)} \tag{27}
\]
\[
\pi^B_U(.) = \frac{(a-c)^2(2+\gamma)[2\beta(2-\gamma) - (1-\beta)(\gamma^4 - \gamma^3)]}{16(1+\gamma)}
\]

Note by the eq. (26), that the equilibrium fix fees turn to be negative when the upstream monopolist’s bargaining power in the market is low enough, that is, \( F^B < 0 \) if \( \beta < \beta_c = \frac{\gamma^2(2-\gamma+\gamma^2)}{4-2\gamma-\gamma^3+\gamma^4} \). That means that, in contrast to the Cournot final market competition case where the upstream monopolist subsidizes its downstream partners via the wholesale prices, under Bertrand final market competition the upstream monopolist’s subsidizes its downstream partners via the fix fees if its bargaining power in the market is relatively low. This is so, since when the upstream bargaining power in the market is low enough, the power to extract the fix fees is instead reversed and thus, it is the downstream firms that obtain the benefit of the fix rents.

### 3.3 Welfare Analysis

In this subsection, we present the social welfare implications of Cournot and Bertrand final market competition. In vertically related markets with upstream monopolistic market sector, the social welfare is defined as the sum of the consumers surplus plus the profits of the upstream monopolist and the downstream firms

\[
SW = CS + \Pi_U + \Pi_{Di} + \Pi_{Dj}
\]

where the consumers surplus is given by,

\[
CS(.) = a(q_i + q_j) - \frac{(q_i^2 + q_j^2 + \gamma q_i q_j)}{2} - p_i q_i - p_j q_j
\]

In the equilibrium, imposing symmetry we have that, \( q_i = q_j = q \), \( p_i = p_j = p \), and thus, the social welfare can be written as,

\[
CS = (1 + \gamma)[q]^2
\]

Substituting (12) into (31) we have that the consumers’ surplus in vertically related markets with upstream monopolistic market structure, where the final market competition takes place
in quantities, is given by,

\[ CS^C = \frac{(1 + \gamma)(a - c)^2(2 - \gamma)^2}{4(2 - \gamma^2)^2} \] (32)

while, substituting (32), (14) and (15) into (29), we have that the social welfare under Cournot final market competition is given by,

\[ SW^C = \frac{(a - c)^2(2 - \gamma)(6 - \gamma - 3\gamma^2)}{4(2 - \gamma^2)^2} \] (33)

Correspondingly, substituting (25) into (31) we have that the consumers’ surplus under Bertrand final market competition is given by,

\[ CS^B = \frac{(a - c)^2(2 + \gamma)^2}{16(1 + \gamma)} \] (34)

while, substituting (34), (27) and (28) into (29), we have that the social welfare under Bertrand final market competition is given by,

\[ SW^B = \frac{(a - c)^2(2 + \gamma)(6 - \gamma)}{16(1 + \gamma)} \] (35)

3.4 Comparison

We turn now to compare the equilibrium outcomes and the social welfare obtained under Cournot and Bertrand final market competition, in order to examine how the alternative modes of final market competition affect the output production, the upstream monopolist’s and the downstream firms’ profits and the social welfare.

Starting with the comparison over the output production under the alternative modes of competition, using the equations (12) and (25), we have that the Cournot-Bertrand output differential is given by,

\[ \Delta q = q^C - q^B = \frac{(a - c)\gamma^3}{4(2 + 2\gamma - \gamma^2 - \gamma^3)} > 0 \] (36)

from which the following result derives,

**Proposition 1** The equilibrium output production under Cournot final market competition is always higher than under Bertrand final market competition.

Interestingly enough, the above result reveals that the product market competition is more
intense under Cournot competition than under Bertrand competition, since the equilibrium quantity in the Cournot market is always higher than that in the Bertrand market. The intuition behind this result is based on the lower marginal production cost that the downstream firms face under Cournot final market competition, since the wholesale prices (or else, the per unit of input prices) under Cournot final market competition are always lower than under Bertrand final market competition. In other words, according to our discussion over Lemma 1 and Lemma 2, we show that, contrary to the Bertrand competition market, when the final market competition takes place in quantities, the upstream monopolist subsidizes the downstream production by setting wholesale prices lower than its marginal production cost. This means that the downstream firms under Cournot final market competition behave more aggressively and thus the final market competition is more intense under Cournot competition than under Bertrand competition.

Furthermore, using the equations (14) and (27), we have that Cournot-Bertrand downstream firms’ profits differential is given by,

\[
\Delta \pi_D = \pi^C_D - \pi^B_D = \frac{(1 - \beta)(a - c)\gamma^2 \Xi}{32(1 + \gamma)(2 - \gamma^2)} > 0
\]  

(37)

where, \( \Xi = 8 - (2 - \gamma)(2 + \gamma)\gamma(1 + \gamma) \). From the above equation the following proposition derives

**Proposition 2** The equilibrium downstream firms’ profits under Cournot final market competition are always higher than under Bertrand final market competition.

Clearly, the lower wholesale prices that the upstream monopolist sets under Cournot final market competition along with the higher downstream firms’ output production, increase the downstream firms’ profitability in the Cournot market and thus, the downstream firms’ profits in the Cournot competition market always exceed those of the Bertrand competition market.

Further, using the equations (15) and (28) we have that Cournot-Bertrand upstream monopolist’s profits differential is given by,

\[
\Delta \pi_U = \pi^C_U - \pi^B_U = -\frac{(a - c)^2 \gamma^3 [16 - \beta(1 - \gamma^2)\Xi - \gamma^2(16 - 4\gamma - 6\gamma^2 + \gamma^3 + \gamma^4)]}{16(1 + \gamma)(\gamma^2 - 2)^2} < 0
\]  

(38)

from which the following proposition derives,
Proposition 3 The equilibrium upstream monopolist’s profits under Bertrand final market competition are always higher than under Cournot final market competition.

The intuition behind this result comes straightforward from the fact that the wholesale prices under Bertrand final market competition always exceed those of the Cournot final market competition. This is so because the "commitment problem", that the upstream monopolist faces when the contract negotiations are not fully observable under Bertrand final market competition, is less severe than under Cournot final market competition, since prices, contrary to quantities, are strategic complements.

We turn now to compare the consumers’ surplus and the total welfare the Cournot and Bertrand cases. As far as the consumers’ surplus is being concerned, it is straightforward that, since \( CS = (1 + \gamma)q^2 \) and \( q^C > q^B \) always holds, then \( CS^C > CS^B \). Clearly, the consumers’ surplus is higher under Cournot final market competition, that means that the consumers are better off when the downstream firms compete by setting their quantities. This is so since, according to our discussion over the Proposition 1, the Cournot market is more competitive than the Bertrand and thus, the consumers’ are being benefited by higher output production that Cournot competition implies.

Further, using the equations (33) and (35) we have that the social welfare differential is given by,

\[
\Delta TW = TW^C - TW^B = \frac{(a - c)^2\gamma^3[8 - \gamma(4 + (4 - \gamma)\gamma)]}{16(1 + \gamma)(\gamma - 2)^2} > 0
\]  
(39)

Thus, contrary to the standard result that the social welfare is higher under Bertrand competition, we show that the social welfare in vertically related markets with upstream monopolistic market structure, two-part tariffs contracts and Cournot final market competition is higher than that under Bertrand final market competition. Obviously, the beneficial effect that the higher consumers’ surplus and the higher upstream monopolist’s profits have on the social welfare dominates the diminishing effect of the lower downstream firms’ profits and thus, Cournot final market competition is more socially desirable than Bertrand competition.

3.5 Extensions/Discussion

- Upstream Separate Firms Market Structure. In our basic model we have assumed that the upstream market sector is being monopolized by a single firm. Here we relax this assumption and we briefly discuss the Cournot-Bertrand comparison results in vertically related markets
with upstream separate firms market structure. In particular, we extend our analysis considering a vertically related market consisted by two upstream and two downstream firms where the trade relations between each upstream and each downstream firm are exclusive, while the trading is conducted via two-part tariffs contracts. Note here, that given the assumption of exclusive trade relations between the upstream and the downstream firms, none of the upstream and the downstream firms could achieve an agreement with an alternative trading partner and thus, the disagreement payoffs under this configuration is null. Keeping all the other modeling specifications fixed, we reconfirmed the standard Singh and Vives (1984) results, that is, the upstream firms’ profits and the downstream firms’ profits are always higher under Cournot competition than under Bertrand competition, while the equilibrium output, the consumers’ surplus and the social welfare are higher under Bertrand final market competition. This is so, since in vertically related markets with upstream separate firms market structure, the effect of the lower wholesale prices that the upstream firms set under Cournot competition can not compensate the fact that the Cournot competition is by its nature more "monopolistic" than the Betrand competition.

3.6 Conclusions

In the present paper we investigate how the alternative modes of market competition, namely Cournot and Bertrand competition affect the equilibrium outcomes and the social welfare in vertically related markets with monopolistic upstream market structure, where the trade is conducted via two part tariffs contracts.

Interestingly enough, we show that the downstream firms’ output production is higher under Cournot final market competition than under Bertrand final market competition, or in other words, the Cournot market is more competitive in terms of output production than the Bertrand one. Intuitively, the lower wholesale prices that the upstream monopolist sets under Cournot final market competition, make the downstream firms to behave more aggressively in the Cournot market than in the Bertrand one. Further, in line with the standard Singh and Vives (1984) result, that claims that profits are higher under Cournot competition than under Bertrand, we demonstrate that the profits in the downstream market under Cournot final market competition always exceed those obtained under Bertrand final market competition. On the contrary, we show that the upstream monopolist always obtains higher profits when the final market competition takes place in prices.
Moreover, regarding the societal effects of the different modes of final market competition, we argue that the consumers’ surplus is higher under Cournot final market competition than under Bertrand competition, since the consumers are benefiting by the higher output production of the former mode of market competition. Last but not least, contrary to the conventional wisdom that suggests that the social welfare is higher under Bertrand competition, we found that in vertically related markets with upstream monopolistic market structure where the trade is conducted via two-part tariffs contracts, the Cournot competition dominates the Bertrand competition in terms of welfare. That means that Cournot competition in such markets is more socially desirable than the Bertrand.
References


