TEMPORAL AGGREGATION EFFECTS IN CHOOSING THE OPTIMAL LAG ORDER IN STABLE ARMA MODELS.
SOME MONTE CARLO RESULTS.

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ABSTRACT

A crucial aspect of empirical research based on ARIMA(p,q) model is the choice of the appropriate lag order. Several criteria have been used in order to identify the appropriate order of a ARIMA(p,q) process. In this paper we investigate the effects of using a variation of selection criteria under different temporal aggregation levels. We don’t spend our attention in determining the appropriate order but on the effects of using the above selection criteria on the dynamic characteristics (impulse responses) and the forecasting properties of the ARIMA(p,q) process. The conducted Monte Carlo simulation experiments show that the use of temporally aggregated data can affect seriously the impulse responses and the forecasting properties of the ARIMA model.

Keywords: Stable ARMA process, temporal aggregation and stochastic simulation.
JEL: C32,C43,C51

INTRODUCTION

Temporal aggregation poses many interesting questions which have been explored in time series analysis and which yet remain to be explored. An early example of research in this area is Quenouille (1957), where the temporal aggregation of ARIMA processes is studied. Amemiya and Wu (1972), and Brewer (1973) review and generalize Quenouille's result by including exogenous variables. Zellner and Montmarquette (1971) discuss the effects of temporal aggregation on estimation and testing. Engle (1969) and Wei (1978) analyze the effects of temporal aggregation on parameter estimation in a distributed lag model. Granger (1987) discusses the implications of aggregation on systems with common factors. Other contributions in this area include Tiao (1972), Stram and Wei (1986), Weiss (1984), Granger and Lee (1999), and Marcellino (1999), to name but a few.

In this paper, we examine the impact of temporal aggregation on the dynamic analysis and the forecasting properties of a stable ARIMA(p,q) process. There is a sizable theoretical literature that investigates the impact of temporal aggregation on ARIMA models (see Wei, 1990, and references therein). These studies are theoretical and focus on the effects of temporal aggregation on the orders p and q of the ARIMA process. This study using Monte Carlo simulation techniques, focus on the effects of using different selection criteria on the dynamic impulse responses and the forecasting properties of a stable ARMA process estimated at 16 different aggregation levels.
This article is organized as follows. Section II presents some selection criteria used to determine the appropriate p and q order of the ARIMA process. Section III introduces the design of the simulation procedure. Section IV provides the simulation results and the last section concludes.

THE OPTIMAL LAG ORDER IN AN ARIMA MODEL

We now review some selection criteria for the order of an ARIMA(p,q) process. These selection criteria are used very often for the selection of the appropriate order of the p and q dimensions of an ARIMA process. These criteria are:

Akaike Information Criterion (AIC) Test (Akaike 1969)

\[ AIC(p, q) = T \log \sigma^2_e + 2(p + q) \]

Corrected Akaike Information Criterion \( AIC_c \) Test (Hurvich, C. M. and Tsai, C. 1993)

\[ AIC_c(p, q) = T \log \sigma^2_e + 2(p + q - 1) + T(2 + 2 \log(2\pi) + 2 \log T) - 2 \log(t - 2(p + q) - 1) \]

Bayesian Information Criterion (BIC) Test (Amemiya, T. and R.Y. Wu 1972)

\[ BIC(p, q) = T \log \sigma^2_e - (T - p - q) \log \left[ 1 - (p + q)/T \right] + (p + q) \log T + (p + q) \log \left( (p + q)^2 / \sigma^2_e \right) - 1) \]

Schwarz (SC) Test (Schwarz 1978)

\[ SC(p, q) = T \log \sigma^2_e + (p + q) \log T \]

Hannan – Quinn Criterion Test (Hannan and Quinn 1979)

\[ HQ(p, q) = \log \sigma^2_e + (p + q) c \log \left[ \log T / (T - L) \right] \quad c > 2 \]

Bayesian Estimation Method (BEC) Test (Geweke and Meese 1981)

\[ BEC(p, q) = \sigma^2_e + (p + q) \sigma^2_e \log \left[ T / (T - L) \right] \quad L = \text{maximum number of p and q} \]

In addition to the above six criteria we used as well the RootMeanSquareError, the \( R^2 \) and the adjusted \( adj - R^2 \).

THE DESIGN OF THE MONTE CARLO SIMULATION

The simulations are conducted using the following stationary ARIMA(p,q) process:

\[ y_t = 3 + 0.315 y_{t-1} - 0.8453 y_{t-2} - 0.1065 y_{t-3} + 0.785 y_{t-4} + \epsilon_t \]

\[ \epsilon_t \sim N.I.D.(0,0.25) \]

The impulse response function of the above process is given in figure 1. This impulse response has a duration of 141 periods and max=1.1 and min=-1.19.
In order to schematize the effects of temporal aggregation on the dynamic characteristics and the forecasting properties of the ARIMA(p,q) process (7)-(8) we applied 16 different level of temporal aggregation. For each of the 16 different level of temporal aggregation we estimate the process (7)-(8), selecting the appropriate lag order using the above analysed selection criteria and obtained the impulse responses and the analogous forecasts. The total number of the simulated observations was 1400 at the highest level of temporal disaggregation and the experiment was replicated 3000 times. The number of the forecasted periods at the highest level of temporal aggregation was 4.

Temporal aggregates are formed by averaging basic observations over nonoverlapping intervals. Let $y_T^A$ represent the temporally aggregated data:

$$y_T^A = Cy_t$$

where $y_T^A$ is the temporal aggregated data, $j = 1,2,3,...,16$ is the time aggregation level and $C$ is a time aggregation matrix of the form:

$$C = \begin{bmatrix}
11...1 & 00...0 & ... & 00...0 & 00...0 \\
00...0 & 11...1 & ... & 00...0 & 00...0 \\
00...0 & 00...0 & ... & 11...1 & 00...0 \\
. & . & . & . & . \\
. & . & . & . & . \\
00...0 & 00...0 & ... & 00...0 & 11...1
\end{bmatrix}$$

(10)

1For more about these Time - Aggregation relations using matrix approach, see: Gilbert C., 1977., pp. 223-225. A similar aggregation formulation is $y_T^A = \frac{1}{m} (\sum_{j=0}^{m-1} L^j) y_t$, where $L$ is the backshift operator on $t$. 

Figure 1. The Impulse Response of the stochastic process (7) & (8).
In the simulation experiment we performed 3,000 iterations at 16 different but nested time aggregation levels, in contrast with analogous research in which three or four (at the most) different time aggregation levels have been used.

The steps of realization of these Monte Carlo experiments are the following:

- On the basis of the relations (7)-(8) in each iteration, we obtained 1400 simulated observations of the dependent variable.
- For 16 different levels of time aggregation we estimated the ARIMA process using a Gauss-Newton nonlinear estimation technique.
- The estimated specification was then used to obtain ex-post forecasts for 4 observations at the highest level of time aggregation (j=16). These forecasts were compared with the corresponding simulated observations of the dependent variable at the corresponding level of time aggregation (j=2,3, …,16).

In addition, we conducted analogous forecasting comparisons of these forecasts at a specific time aggregation level (j=1,2, …,16), at the highest level of which j=16. To do this happen we aggregate the forecasts at the analogous temporal aggregation level, using the relation:

\[ y^F_T = Cy^F_i \]  
(11)

with C an aggregation matrix defined as in (10). The comparisons were made between the actual values at the highest temporal aggregation level and the aggregated forecasts.

In order to study the effects of the degree of time aggregation on the quantitative and qualitative characteristics of the ex-post forecasting ability of the model, we used some well known forecasting criteria.

Impulse responses comparisons were made after aggregating (for j=1,2,……,16) the simulated ‘actual’ responses at the highest level of temporal aggregation and the response on the estimated model and the \( j^{th} \) level of temporal aggregation. The criterion we used was the :

Mean Square Error :

\[ MSE_j = \frac{\sum_{j=1}^{Dur_j} (impul \_e^A_j - impul \_e^j)^2}{Dur_j} \]  
(12)

\[ j = 1,2,\ldots,16 \]

\[ impul \_e^A_j = C(impul \_e^j) \]  
(13)

Where:

\[ impul \_e^A_j = \text{Temporally Aggregated estimated impulse at the } j^{th} \text{ degree of temporal aggregation.} \]

\[ impul \_e^j = \text{estimated impulse at the } j^{th} \text{ degree of temporal aggregation.} \]

\[ Dur_j = \text{Estimated Duration of the impulse response at the } j^{th} \text{ level of temporal aggregation.} \]

In order to get an idea about this experiment in Figure 2 we present graphically the impulse responses between the estimated and the analogous time aggregated responses at the \( j=1,2,\ldots,16 \) different level of temporal aggregation.

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3 For more see the Appendix of this paper.
Impulse Responses at Different Levels of Temporal Aggregation

Simulation Results

Figure 2. Impulse Responses at Different Levels of Temporal Aggregation
THE SIMULATION RESULTS

According the simulations conducted in the previous section , we may conclude very briefly the following about the effects of temporal aggregation on the forecasting properties and the dynamic characteristics of the estimated ARIMA process.

**Effects on the forecasting ability of the ARIMA process.**

- As the level of temporal aggregation increases , the mean Root Mean Square Forecasts Error (RMSE⁴) which results from the forecast of an ARIMA (p,q) process, increases as well.(Table 1). These results confirms that the forecast can be more effective when a variable is used in its highest level of temporal aggregation. The smallest RMSE is given by the Hannan & Quinn selection criterion.

- The quality of the forecast made at the different level of temporal aggregation is analysed in Tables 1A and 1B. It is obvious that as the degree of temporal aggregation increase, the Mean Bias Proportion of the Mean Forecasts Error, increases rapidly, indicating that the temporal aggregations influence negatively the quality of the forecast made (Table 1A). The same time, according to our results (Table 1B), as the degree of temporal aggregation increases, the variance proportion of the MSE decomposition increases as well, but not the same degree for all the selection criteria.

- In Tables 2, 2A and 2B we present the same results with the previous tables, but with the only difference, that the comparisons are made on the higher degree of temporal aggregation. So, we have transformed(aggregate) our forecasts into forecasts to the highest level of temporal aggregation which is level 16. This means that each forecast has aggregated accordingly, so that to reach the level 16 of temporal aggregation and to allow for the analogous comparisons.

In Table 2, we present the mean Root Mean Square Forecasts Error between the actual and the forecasted time series, having aggregated the forecast to the highest level of temporal aggregation as follows:

\[ y^F_T = C y^F_t \]

with the matrix C as defined in (10).

Based on the results on Table 2,2A and 2B we may conclude that in most of the times when we want to do effective forecasts is preferable to use our data to the highest level of temporal disaggregation and after to aggregate them to the level we wanted them to forecast. Moreover, the results in Tables 2A and 2B, confirm that the use of data of high level of temporal aggregation decreases the effectiveness of our forecasts not only quantitative but also qualitative.

- Finally, in Table 3 we present the Mean RMSE between the estimated actual response at the \( f^{th} \) degree of temporal aggregation and the aggregated to the same degree ‘actual’ response. According to our results, it is obvious that the degree of temporal aggregation affects the impulse response functions of the specified ARIMA model. This means that when we are interested for effective dynamic analysis results it is important to use our data in the highest level of temporal aggregation.

**CONCLUSION**

In this short paper we analyse the effects of Temporal Aggregation on the dynamic characteristics and the forecasting properties of a stable ARMA(p,q) process. Using Monte Carlo techniques we may conclude that when we are interested for effective dynamic analysis and forecasts through an ARIMA process, it is important

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⁴ The Root Mean Square Forecasts Error is defined as:

\[ RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (y^A_t - y^F_t)^2} \]

where \( y^A_t = \text{Actual data} \), \( y^F_t = \text{Forecasted data} \) and \( T = \text{the number of forecasting periods} \).  

⁵ Simulated in Figure 1 impulse response.
As the degree of temporal aggregation increases, the Mean Root Mean Square Forecasts Error increases rapidly when we compare our forecasts in the highest and the lowest degree of temporal aggregation. The same time the impulse responses functions exhibit a high Mean Root Mean Square Error between the simulated (actual) response and the temporal aggregated response at the analogous degree of temporal aggregation. This means that when we are interested for effective dynamic analysis and effective forecasts it is important to use our data in the highest level of temporal aggregation.

The results of our experiments complements other analogous theoretical researches for the effects of temporal aggregation on estimating an ARIMA (p,q) process.

**TABLES**

**Table 1.** Mean Root Mean Square Forecasts Error (RMSE\(^6\)) of ARIMA (p,q) forecasts at different level of Temporal Aggregation and different selections criteria.

<table>
<thead>
<tr>
<th>Temporal Aggregation Level</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
<th>SC</th>
<th>HQ</th>
<th>SBC</th>
<th>Adj _ (R^2)</th>
<th>(R^2)</th>
<th>MSE</th>
</tr>
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<tr>
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<td>1,190377</td>
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<td>1,197098</td>
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<td>5,576265</td>
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</tbody>
</table>

Source: Our Estimates (Simulation Results).

AIC & AIC\(_c\)= Akaike’s information criterion and its corrected form, respectively; BIC = Bayesian information criterion; HQ = Hannan and Quinn information criterion; SBC=Schwarz criterion; \(Adj\ _ {R^2}\) = Adjusted \(R^2\); \(R^2\); MSE = Mean Square Error

**Table 1A.** Mean Bias Proportion\(^7\) of the ARIMA (p,q) forecast at different level of Temporal Aggregation and different selections criteria.

<table>
<thead>
<tr>
<th>Temporal Aggregation level</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
<th>SC</th>
<th>HQ</th>
<th>SBC</th>
<th>Adj _ (R^2)</th>
<th>(R^2)</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
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<td>0,002481</td>
<td>0,00305</td>
<td>0,002896</td>
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<td>0,200364</td>
<td>0,169778</td>
<td>0,169373</td>
<td>0,172413</td>
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</tbody>
</table>

Source: Our Estimates (Simulation Results).

AIC & AIC\(_c\)= Akaike’s information criterion and its corrected form, respectively; BIC = Bayesian information criterion; HQ = Hannan and Quinn information criterion; SBC=Schwarz criterion; \(Adj\ _ {R^2}\) = Adjusted \(R^2\); \(R^2\); MSE = Mean Square Error

**Table 1B.** Mean Variance Proportion\(^8\) of the MSE forecast decomposition at different level of Temporal Aggregation

\[ \text{RMSE} = \left( \frac{1}{T} \sum_{t=1}^{T} (y_t^A - y_t^F)^2 \right)^{1/2} \]

where \(y_t^A\) = Actual data, \(y_t^F\) = Forecasted data, \(\overline{y}_t^A, \overline{y}_t^F\) = mean values and \(T\)=the number of forecasting periods.

\[ U^M = \left( \frac{1}{T} \sum_{t=1}^{T} (y_t^F - \overline{y}_t^F)^2 \right) \]

where \(y_t^A\) = Actual data, \(y_t^F\) = Forecasted data and \(T\)=the number of forecasting periods

\[ U^S = \left( \frac{1}{T} \sum_{t=1}^{T} (S_t^F - S_t^A)^2 \right)^{1/2} \]

\[ \frac{1}{T} \sum_{t=1}^{T} (y_t^F - y_t^A)^2 \]

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\(^6\) The Root Mean Square Forecasts Error is defined as:

\[^7\] The Bias Proportion of the Mean Forecasts Error is defined as:

\[^8\] The Variance Proportion of the Mean Forecasts Error is defined as:
and different selections criteria.

<table>
<thead>
<tr>
<th>Temporal Aggregation Level</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
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<th>Adj $R^2$</th>
<th>$R^2$</th>
<th>MSE</th>
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Source: Our Estimates (Simulation Results).

AIC & AICc = Akaike’s information criterion and its corrected form, respectively; BIC = Bayesian information criterion; HQ = Hannan and Quinn information criterion; SBC = Schwarz criterion; $Adj \ R^2 = $ Adjusted $R^2$; $R^2$; MSE = Mean Square Error

Table 2: Mean Root Mean Square Forecasts Error (RMSE) between the actual and the forecasted time series at the highest (j=16) level of temporal aggregation at different selections criteria.

<table>
<thead>
<tr>
<th>Temporal Aggregation Level</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
<th>SC</th>
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<th>SBC</th>
<th>Adj $R^2$</th>
<th>$R^2$</th>
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</table>

Source: Our Estimates (Simulation Results).

AIC & AICc = Akaike’s information criterion and its corrected form, respectively; BIC = Bayesian information criterion; HQ = Hannan and Quinn information criterion; SBC = Schwarz criterion; $Adj \ R^2 = $ Adjusted $R^2$; $R^2$; MSE = Mean Square Error

Table 2A. Mean Bias Proportion between the actual and the forecasted time series at the highest (j=16) level of temporal aggregation at different selections criteria.

<table>
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<tr>
<th>Temporal Aggregation Level</th>
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<th>AICc</th>
<th>BIC</th>
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<td>0.568298</td>
<td>0.57756</td>
<td>0.60592</td>
<td>0.615559</td>
<td>0.558725</td>
<td>0.561178</td>
<td>0.565312</td>
</tr>
</tbody>
</table>

Source: Our Estimates (Simulation Results).

AIC & AICc = Akaike’s information criterion and its corrected form, respectively; BIC = Bayesian information criterion; HQ = Hannan and Quinn information criterion; SBC = Schwarz criterion; $Adj \ R^2 = $ Adjusted $R^2$; $R^2$; MSE = Mean Square Error

where $y^A_t =$ Actual data , $y^F_t =$ Forecasted data , $S^A_y , S^F_y =$ Standard Deviation of Forecasted and Actual values and T=the number of forecasting periods

$^9$ The Root Mean Square Forecasts Error is defined as:

$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (y^A_t - y^F_t)^2}$ where $y^A_t =$ Actual data , $y^F_t =$ Forecasted data and T=the number of forecasting periods.
Table 2B. Mean Variance Proportion between the actual and the forecasted time series at the highest (j=16) level of temporal aggregation at different selections criteria.

<table>
<thead>
<tr>
<th>Temporal Aggregation Level</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
<th>SC</th>
<th>HQ</th>
<th>SBC</th>
<th>Adj _$\bar{R}^2$</th>
<th>$R^2$</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.467827</td>
<td>0.480843</td>
<td>0.49922</td>
<td>0.471341</td>
<td>0.489719</td>
<td>0.474023</td>
<td>0.471256</td>
<td>0.472771</td>
<td>0.231649</td>
</tr>
<tr>
<td>5</td>
<td>0.314252</td>
<td>0.402894</td>
<td>0.476395</td>
<td>0.319246</td>
<td>0.425831</td>
<td>0.340518</td>
<td>0.316414</td>
<td>0.319033</td>
<td>0.052632</td>
</tr>
<tr>
<td>10</td>
<td>0.229254</td>
<td>0.290149</td>
<td>0.345188</td>
<td>0.234713</td>
<td>0.304719</td>
<td>0.227774</td>
<td>0.245326</td>
<td>0.241740</td>
<td>0.083425</td>
</tr>
<tr>
<td>15</td>
<td>0.244395</td>
<td>0.281403</td>
<td>0.338897</td>
<td>0.256921</td>
<td>0.281542</td>
<td>0.217250</td>
<td>0.261934</td>
<td>0.262348</td>
<td>0.127556</td>
</tr>
<tr>
<td>16</td>
<td>0.23368</td>
<td>0.294474</td>
<td>0.329176</td>
<td>0.244087</td>
<td>0.259037</td>
<td>0.259445</td>
<td>0.259597</td>
<td>0.166335</td>
<td></td>
</tr>
</tbody>
</table>

Source: Our Estimates (Simulation Results).

AIC & AICc= Akaike’s information criterion and its corrected form, respectively; BIC = Bayesian information criterion; HQ = Hannan and Quinn information criterion; SBC=Schwarz criterion; Adj _$\bar{R}^2$ = Adjusted $R^2$; $R^2$ ; MSE = Mean Square Error

Table 3. Mean MSE between the estimated actual response at the jth degree of temporal aggregation and the aggregated to the same temporal aggregation degree of the actual response.

<table>
<thead>
<tr>
<th>Temporal Aggregation Level</th>
<th>AIC</th>
<th>AICc</th>
<th>BIC</th>
<th>SC</th>
<th>HQ</th>
<th>SBC</th>
<th>Adj _$\bar{R}^2$</th>
<th>$R^2$</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.760533</td>
<td>1.72424</td>
<td>1.718443</td>
<td>1.693042</td>
<td>1.803462</td>
<td>1.692019</td>
<td>1.692709</td>
<td>1.692019</td>
<td>1.760388</td>
</tr>
</tbody>
</table>

Source: Our Estimates (Simulation Results).

REFERENCES


Marcellino, M. (1999), Some Consequences of Temporal Aggregation in Empirical Analysis, Journal of