Smooth ‘inverted-V-shaped’ & smooth ‘N-shaped’ pollution-income paths

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Abstract
We explore the idea of regime switching as a new methodological approach in the analysis of the emission-income relationship. A static smooth transition regression model is developed with fixed-effects. The basic idea is that when some threshold is passed, then the economy could move smoothly to another regime, with the emission-income relationship being different between the old and the new regime. We motive our methodology by proving that the quadratic or cubic polynomial model used in the literature derives from the smooth transition regression specification. The methodology is applied a panel dataset on US state-level sulfur dioxide and nitrogen oxide emissions covering 48 states over the period from 1929 to 1994. We find robust smooth N-shaped and smooth inverse-V-shaped pollution-income paths for the sulfur dioxide. For the nitrogen oxide emissions environmental pressure tends to rise with economic growth in the early stages of economic development then slows down but does not decline with further income growth.

Keywords: Environmental Kuznets Curve, smooth transition regression, regime switching, thresholds.

JEL Classification: C2, O1, Q2.
1. Introduction

In the analysis of emission-income relationship\(^1\), there exists a set of theoretical models, which derives inverted ‘V-shaped’ curves by having pollution increasing with income until some threshold point is passed, after which pollution is reduced. John and Pecchenino (1994) consider an overlapping generations model where economies with low income or high environmental quality are not engaged in environmental investment, that is, pollution abatement. When environmental quality deteriorates with growth, the economy moves to positive abatement, then environment improves with growth and the relationship is inverted V-shaped. Stokey (1998) generates an inverted V-shaped curve by considering a static optimization model where below a threshold income level only the dirtiest technologies are used. As economic activity and pollution increase, the threshold level is passed and cleaner activities are used. Jaeger (1998) derives the inverted V-shaped curve by considering a threshold in consumer preferences. Below the threshold the marginal benefits from improving environmental quality are small, whereas when pollution increases with growth and the threshold is passed, quality may be improved.

The main purpose of the present paper is to explore empirically the inverted V model by introducing the idea of regime-switching as a new methodological approach in the analysis of the emission-income relationship. More particular, econometric techniques appropriate for a static smooth transition regression (STR) with panel data are developed. In this model regression functions are not identical across all observations in a sample but fall into classes. The basic idea is that when some threshold is passed, then the economy could move smoothly to another regime, with the emission-income relationship being different between the old and the new regime. The low-income regime might correspond to an increasing emission-income relationship, while in the regime after the threshold the emission-income relationship might be decreasing. The abrupt regime switch is a special case of the smooth regime switch implying that the discrete inverse V-shaped emission-income paths are a special case of the more general smooth inverse V-shaped ones. Further, we motive our methodology by proving that the quadratic or cubic polynomial model used to examine the emission-income relationship derives from the STR model,

\(^1\) For a recent literature review, see, for example, Levinson (2002).
and therefore, the latter can be seen as the underlying structural specification in this literature. Finally, we also develop ‘N-shaped’ emission-income paths in a smooth transition regression framework where pollution can be increasing at low levels of income, decreasing at high levels, and then increasing again at even higher levels of national income.

The methodology is applied a panel dataset on US state-level sulfur dioxide ($SO_2$) and nitrogen oxide ($NO_x$) emissions covering 48 states over the period from 1929 to 1994. We find robust smooth N-shaped and smooth inverse-V-shaped pollution-income paths for $SO_2$. In particular, emissions of $SO_2$ are found to peak at a relatively early stage economic of development and then decrease at middle-to-high levels of income. What is more interesting, however, and perhaps policy relevant is that while in high-income states environmental quality continues to improve, in low income states pollution increases with the later increases in economic activity. As to $NO_x$ emissions, environmental pressure tends to rise with economic growth in the early stages of economic development then slows down but does not decline with further income growth.

2. Methodology

2.1 Model

By regime-switching behaviour we mean that the regression functions are not identical across all observations in the sample or they fall into discrete classes. One of the most prominent among the regime-switching models in the macroeconomics area has been the threshold class of models and its smooth transition generalization (STAR models) promoted by Teräsvirta and his co-authors (Teräsvirta, 1994, Teräsvirta and Anderson, 1992, Granger and Teräsvirta, 1993). Regime-switching models are flexible enough to allow several different types of effects that could be observed in the relationship between pollution and income. The structural equation of interest is the static one-threshold smooth transition regression (STR) model with state-specific fixed effects given by
\[ P_{it} = \mu_i + \beta_1 Y_{it} + (\beta_2 Y_{it}) F(\gamma, c; Y_{it}) + u_{it}, \quad i = 1, \ldots, N; \quad t = 1, \ldots, T \quad (1) \]

where \( P_{it} \) is a measure of per capita air pollution in state \( i \) in year \( t \), \( Y_{it} \) is per capita GDP in state \( i \) in year \( t \), \( \beta \equiv (\beta_1, \beta_2)' \) is the parameter vector, and \( u_{it} \) is an IID error term\(^2\). The function \( F(\gamma, c; Y_{it}) \) is the transition function continuous and bounded by zero and unity, \( \gamma \) and \( c \) are its parameters, whereas \( Y_{it} \) is assumed to act as the transition variable. By writing (1) as

\[ P_{it} = \mu_i + (\beta_1 + \beta_2 F) Y_{it} + u_{it} \]

it is seen that the model is locally linear in \( Y_{it} \) and that the combined parameter \( \beta_1 + \beta_2 F \) is a function of the transition variable \( Y_{it} \). If \( F \) is bounded between zero and one, the combined parameter fluctuate between \( \beta_1 \) and \( \beta_1 + \beta_2 \). Values of zero by the transition function identify the one regime and values of unity identify the alternative. In the analysis of emission-income relationship, this property makes it possible, for example, to derive smooth inverted V-shaped curves by having pollution increasing with income \( (\beta_1 > 0) \) until income passes some threshold, after which pollution is reduced \( (\beta_1 + \beta_2 < 0) \). It is also reasonable to assume that in utility terms the disutility\(^3\) of pollution is related to the flow of new pollutants and it is thus adequate to use a static model. Another alternative way of writing (1) is

\[ P_{it} = \begin{cases} 
\mu_i + \beta Y_{it} + u_{it} & F = 0 \\
\mu_i + (\beta_1 + \beta_2) Y_{it} + u_{it} & F = 1
\end{cases} \]

\(^2\) Note that we do not include year-specific effects to also let each year to have its own intercept for aggregate time effects such as technical progress since we argue that both the composition and technology effects imply increasing per capita income, so we focus solely on the relationship between pollution and income.

\(^3\) Since the flow of pollution affects utility.
Obviously, a weighted mixture of these two models applies if $0 < F < 1$.

The practical applicability of the above specification depends on how $F$ is defined. One form of transition function used in the literature is the logistic function

$$F(\gamma, c; Y_{it}) = \left(1 + \exp(-\gamma(Y_{it} - c))\right)^{-1}, \quad \gamma > 0 \quad (2)$$

where the parameter $c$ is the threshold between the two regimes or the location of the transition function, and the parameter $\gamma$ determines the smoothness of the change in the value of the logistic function and thus the speed of the transition from one regime to the other. When $\gamma \to \infty$, $F$ becomes a step function ($F = 0$ if $Y_{it} \leq c$ and $F = 1$ if $Y_{it} > c$), and the transition between the regimes is abrupt. In that case, the model approaches a threshold model (Hansen, 1999).

A smooth transition between the two extremes may be an attractive parameterization because from a theoretical point of view, the assumption of two regimes may sometimes be too restrictive compared to the STR alternative. For instance, instead of assuming that in the emission-income relationship there are two discrete regimes, degradation and improvement, say, it may be more convenient and realistic to assume a continuum of states between the two extremes. Another argument is that economic agents may not all act promptly and uniformly at the same time probably due to heterogeneous beliefs. Nevertheless, the two viewpoints are not competitors since the abrupt switch is a special case of an STR model and can therefore be treated in that framework.

Model (1) has a single threshold. An obvious extension could be to permit multiple thresholds. For example, the double threshold or three-regime STR model takes the form

$$P_{it} = \mu_i + \beta_3 Y_{it} + (\beta_2 Y_{it})F_1(\gamma_2, c_1; Y_{it}) + (\beta_3 Y_{it})F_2(\gamma_2, c_2; Y_{it}) + u_{it}$$

where $\beta \equiv (\beta_1, \beta_2, \beta_3)'$ is the parameter vector and $\gamma_1$, $c_1$ and $\gamma_2$, $c_2$ are the parameters of $F_1$ and $F_2$, respectively. It is assumed that income $Y_{it}$ determines both transitions, while the second transition function is defined analogously to (2).
assumed that $c_1 < c_2$, the parameters of this model change smoothly from $\beta_1$ via $\beta_2$ to $\beta_3$ for increasing values of $Y_{it}$.

2.2 Estimation

One traditional method to eliminate the individual effect $\mu_i$ is to remove individual-specific means. While straightforward in linear models, the non-linear specification (1) calls for a more careful treatment. Once we have removed individual-specific means to estimate the STR model it is computationally convenient to first concentrate on the transition function parameters. Note that giving fixed values to the parameters in the transition function makes the STR model linear in parameters. That is, conditional on the transition function, the parameters of the STR can be estimated by OLS. We first carried out a two-dimensional grid search procedure using 50 values of $\gamma$ (1 to 50) and at least 100 equally spaced values of $c$ within the observed range of the transition variable. Essentially, $Y_{it}$ is ordered by value, extremes are ignored by omitting the most extreme 15 values at each end and the 100 values are specified over the range of the remaining values. This procedure attempts guarantee to that the values of the transition function contains enough sample variation for each choice of $\gamma$ and $c$. The model with the minimum RSS value from the grid search is used to provide $\gamma$ and $c$. Following Teräsvirta (1994) the exponent of the transition function is standardised by the sample standard deviation of the transition variable. This makes $\gamma$ scale-free and helps in determining a useful set of grid values for this parameter. Specification of the double threshold model involves an analogous modeling procedure to the single transition case. Here, a four dimensional grid search is performed over $\gamma_{1}, \gamma_{2}, \gamma_1 = 1, \ldots, 50$ and 8 values of $c_1, c_2$ over the range of the transition variable

We have described an algorithm to estimate a STR static model with individual-specific fixed effects. As far as the consistency of the estimator vector $\beta$ is concerned we argue the following: In linear static models with individual-specific fixed effects this

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4 Essentially, the first threshold is considered over the left part of the observed range of income series whereas the second threshold over the right part.
estimator is consistent. If we assume that the dependence on $\gamma$ and $c$ is not of first-order asymptotic importance, then inference on $\beta$ can proceed as if the estimates $\hat{\gamma}$ and $\hat{c}$ were the true values. Hence, $\beta$ is asymptotically normal and conventional standard errors can be reported.

2.3 Inference

It is important to determine whether the threshold effect is statistically significant. The null hypothesis of no regime-switching effect in (1) is $H_0: \gamma = 0$ against $H_1: \gamma > 0$. It is seen also that the null hypothesis can be equally well expressed as $H_0^\beta: \beta = 0$. This is an indication of an identification problem in (1); the model is identified under the alternative but not under the null hypothesis, so classical tests have non-standard distribution. This is typically called the ‘Davies’ Problem (see Davies, 1987); for later contributions in the econometrics literature see Shively (1988), Granger and Teräsvirta (1993), King and Shively (1993), Andrews and Ploberger (1994) and Hansen (1996). The fixed-effects equation (1) fall in the class of models considered by Granger and Teräsvirta (1993), who find a way of solving the identification problem by circumventing.

To discuss this idea, take the logistic transition function in (2) and its first-order Taylor series approximation with the null hypothesis $\gamma = 0$ as the expansion point. It can be written as

$$T_t = \delta_0 + \delta_1 Y_{it} + R_1(\gamma, c; Y_{it})$$

where $R_1$ is the remainder and $\delta_0 = F_{\gamma=0}$, $\delta_1 = F_{\gamma=0}'$ are constants. Substituting $T_t$ for $F$ in (1) yields

$$P_{it} = \mu_i + \theta Y_{it} + \theta^2 Y_{it}^2 + u_{it}^*$$  (3)
where \( \theta_1 = (\beta_1 + \delta \beta_2) \), \( \theta_2 = \delta \beta_2 \) and \( \mu^* = u + (\beta_2 Y) R_1(\gamma, c; Y) \). Use of this approximation amounts to giving up information about the structure of alternative in order to circumvent the identification problem and obtain a simple test of the null hypothesis. Thus the null hypothesis in (1) \( H_0: \gamma = 0 \) implies \( H'_0: \theta = 0 \) and \( H'_1: \theta \neq 0 \) within (3). Standard asymptotic inference is used to test the null hypothesis since (3) is linear in the parameters and therefore a \( t \)-type test is performed.

Next, consider a second-order Taylor series approximation to \( F \). This can be written as

\[
T = \delta_0 + \delta_1 Y + \delta_2 Y^2 + R_2(\gamma, c; Y)
\]

where \( R_2 \) is the remainder and \( \delta_0 = F(\gamma = 0) \), \( \delta_1 = F'(\gamma = 0) \), \( \delta_2 = 0.5F''(\gamma = 0) \) are constants.

Substituting \( T \) for \( F \) in (1) yields

\[
P = \mu + \theta Y + \theta Y^2 + \theta Y^3 + \mu^*
\]

where \( \theta = (\beta_1 + \delta \beta_2) \), \( \theta_2 = \delta \beta_2 \), \( \theta_3 = \delta \beta_2 \) and \( \mu^* = u + (\beta_2 Y) R_2(\gamma, c; Y) \). The null hypothesis of linearity is a straightforward \( F \)-test of \( H_0: \theta = \theta_3 = 0 \) within (4).

It turns out from equations (3) and (4) that these are the benchmark econometric specifications used in environmental Kuznets curve (EKC) studies. Therefore, we motive empirically the idea of regime-switching in the analysis of the emission-income relationship by proving that the above auxiliary regressions derive from a STR specification. In this light, the STR model can be seen as the underlying structural specification in this literature.

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5 In many studies the full specification also includes the average GDP per capita over the prior three years and other covariates, which can be recovered in our model if included in (1).
3. Empirical results

We use a long-term panel dataset on US state-level sulfur dioxide (SO\textsubscript{2}) and nitrogen oxide (NO\textsubscript{x}) emissions studied by List and Gallet (1999). The data source is the US Environmental Protection Agency (EPA) in their National Air Pollution Emission Trends. For both emissions, there are 3168 annual observations from 48\textsuperscript{6} states over the period from 1929 to 1994\textsuperscript{7}.

The estimated single-threshold log-STR\textsuperscript{8} model for SO\textsubscript{2} and NO\textsubscript{x} are presented in Table 1. To determine whether the threshold effect is statistically significant, we also report results for the test of no threshold effect by estimating the auxiliary regression (4). We find that the test is highly significant with a \( p \)-value of 0.000 in both emissions implying strong regime-switching behaviour. In the first panel, the model for SO\textsubscript{2} gives a threshold at per capita GDP of $3,200, which is a relatively small value in the empirical distribution of GDP transition variable. What is interesting, however, is that there are two classes of states those with ‘low-income’ (associated with \( F(GDP) = 0 \)) where pollution increases with economic growth, and those with ‘middle-to-high’ income (associated with \( F(GDP) = 1 \)) where pollution begins to decline. Also it seems that there is a continuum of states between the two extremes, where the transition from one class to the other is smooth. In the second panel, the model for NO\textsubscript{x} estimates a threshold at per capita GDP of $15,184 implying two classes of states, those with low-to-middle income and those with high-income. The effect of income on pollution is positive throughout the sample, though smaller in magnitude in high-income states. According to this specification the transition from one class to the other is abrupt (\( \hat{\gamma} = 50 \)), and therefore the model behaves similar to a threshold model (Hansen, 1999).

To ease the interpretation of the models Graph 1 shows the shape of the estimated transition functions. Every point indicates an observation so that one can readily see which values the transition function has obtained and how frequently. It can be seen that the location parameters (thresholds) are not distributed equally between the left-side and

\textsuperscript{6} Data are not available for Alaska, Hawai and Washington DC.
\textsuperscript{7} See List and Gallet (1999), for more details on the data.
\textsuperscript{8} We consider logarithmic transformations of Eq. (1).
right-side tails of the functions. In particular, in the $NO_x$ graph (second panel) the high-income states class applies to 8.3% of the sample.

On the other hand, the double-threshold models presented in Table 2 are more intuitive. In the first panel, the model for $SO_2$ which estimates thresholds at $3,368 and $17,292 implies roughly three classes of states. It is interesting to note that the coefficient estimates generate a smooth N-shaped pollution-income path, with trough and peak pollution levels somewhere in the range of low-income and high-income states, respectively. In other words, we observe a pollution-income path increasing at low-income states, decreasing at middle-to-high income states, and then increasing again at very-high income states. However, the first panel of Graph 2 shows that the very-high income class contains sparse data (associated with $F_1(GDP) = 1$ and $F_2(GDP) ‘close’ to 1). Grossman and Krueger (1995) dismiss the upper tail of this pattern as an artificial construct of the fact that they use a cubic functional form. Millimet and Stengos (1999), find a similar pattern to ours with a semi-parametric specification. As before, $NO_x$ emissions seem to increase at decreasing rates in relation to income.

A consistent result obtained from the above models is that the thresholds occur at reasonable values and consequently split the states into groups. To make our findings more robust and to provide a more elaborate presentation of these groups we employ a dummy variable, which indicates whether a state’s per capita $GDP$ is above or below the average per capita $GDP$ of all states in every year from 1929 to 1994. This way we try to develop regime-switching specifications for low-income and high-income states separately, and consequently address possible output composition and technology effects. The final double-threshold specifications are illustrated in Table 3, while the transition functions in Graph 3. Our major finding concerns the model for $SO_2$ which implies that low income states follow a smooth N-shaped pollution-income path, whereas high-income states seem to follow a smooth inverse-V-shaped path. One possible explanation of this finding could be that low-income states, which try to catch up with high-income states, may have received less attention from policymakers later in the

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9 For a detailed explanation of these effects, see Grossman and Krueger (1995).
10 We also considered single-threshold models before settling on the proposed double-threshold specifications. The results are similar to the first and second regime obtained from the double-threshold specifications and are available from the authors upon request.
pollution process perhaps because of an increase in the marginal costs of abating emissions. Thus, the scale of economic activities appears to deteriorate environmental quality. On the other hand, in high-income states structural economic changes and abatement activities seem to offset this effect even during the later stage of economic growth and thus improve environmental quality. Once, again the model for \( NO_x \) shows that in both low and high-income states environmental pressure tends to rise with economic growth in the early stages of economic development then slows down but does not decline with further income growth.

These results can be compared and contrasted with the original papers in the literature. Selden and Song (1994) confirm the EKC hypothesis for suspended particulate matter (SPM), \( SO_2 \) and \( NO_x \) but not for \( CO \) emissions. Our finding concerning \( NO_x \) is not line with the above work, however, Selden and Song employ a panel of 30 countries so they use different data. Grossman and Krueger (1995) find a robust inverse-U-shaped pollution-income path for \( SO_2 \), smoke and for most of the 11 indicators of water quality. Shafik and Bandyopadhyay (1992) find only two types of environmental pressure, namely SPM and \( SO_2 \), out of ten follow an EKC according to their estimates. These studies have allowed for intercept heterogeneity, but have ignored the possibility of slope heterogeneity running a higher risk of producing inconsistent and biased parameter estimates. Only, List and Gallet (1999) address this issue by allowing US states to have heterogeneous slopes and provide evidence of the quadratic and cubic polynomial model for most of the states for both \( SO_2 \) and \( NO_x \). However, we challenge their approach by allowing for two and three slopes (regimes) and a further continuum of slopes between the extremes. US states may vary in terms of resource endowments, infrastructure, technological developments, public pressures, etc., but it may be too strict, perhaps, to assume that every state has undergone a distinct pollution-income path. List and Gallet actually employ an \( F \) statistic where the null hypothesis tests for identical slopes across all states. Our argument is that the null is constructed too strictly since it is not surprising that at least one state has different slope. In this light, the smooth transition regression model can be seen a more general and flexible specification than the quadratic or cubic

\[ ^{11} \text{However, this effect gets weaker: compare the coefficients of -1.008 and -0.471.} \]
polynomial model with different slope parameters. Therefore, although our finding that $NO_x$ does not follow an EKC seems a first in contrast with List and Gallet, is most probably due to different model specifications.

4. Concluding remarks
The main contribution of this study is that it re-addresses the pollution-income path from a different angle. First, we motivate the idea of regime-switching and develop the smooth transition model as a more general specification than the quadratic or cubic polynomial model used in the literature. Second, in the empirical part we find robust smooth N-shaped and smooth inverse-V-shaped pollution-income paths for $SO_2$. The thresholds, which can be viewed as turning points, occur at reasonable values. Emissions of $SO_2$ are found to peak at a relatively early stage economic of development (before a state reaches a per capita income of $3,500), and then decrease at middle to high levels of income. What is more interesting, however, and perhaps policy relevant is that while in high-income states environmental quality continues to improve, in low-income states pollution increases with the later increases in economic activity. As to $NO_x$ emissions, environmental pressure tends to rise with economic growth in the early stages of economic development then slows down but does not decline with further income growth.

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12 Many studies in the literature find very high turning points that are not achievable for the majority of the world population.
Bibliography


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<table>
<thead>
<tr>
<th>Table 1: Single-threshold STR models</th>
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</thead>
<tbody>
<tr>
<td><strong>Fixed-state effects STR model for SO(_2):</strong></td>
</tr>
<tr>
<td>( SO_2 = 0.156 \times GDP + (-0.648 \times GDP) \times F(GDP) )</td>
</tr>
<tr>
<td>(3.982) \hspace{1cm} (-13.39)</td>
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<tr>
<td>Classification of regimes</td>
</tr>
<tr>
<td>( SO_2 = 0.156 \times GDP, ) when ( F(GDP) = 0 )</td>
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<tr>
<td>( SO_2 = -0.492 \times GDP, ) when ( F(GDP) = 1 )</td>
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<td>( \hat{c} = $3,200 (\text{antilog of 8.071}), \hat{\gamma} = 6 )</td>
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<td>( R\text{-sq} = 0.1247 )</td>
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<td>Test for threshold effect (p-value) \hspace{1cm} 0.000</td>
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| **Fixed-state effects STR model for NO\(_x\):** |
| \( NO_x = 0.579 \times GDP + (-0.163 \times GDP) \times F(GDP) \) |
| (50.61) \hspace{1cm} (-6.065) |
| Classification of regimes |
| \( NO_x = 0.579 \times GDP, \) when \( F(GDP) = 0 \) |
| \( NO_x = 0.416 \times GDP, \) when \( F(GDP) = 1 \) |
| \( \hat{c} = \$15,184 (\text{antilog of 9.628}), \hat{\gamma} = 50 \) |
| \( R\text{-sq} = 0.5004 \) |
| Threshold effect (p-value) \hspace{1cm} 0.000 |

*Notes:* Models are estimated in logarithmic levels; threshold effect tests for the null of no regime-switching behaviour; values in parentheses are t-ratios.
Table 2: Double-threshold STR models

Fixed-state effects STR model for $SO_2$

\[
SO_2 = 0.281 \times GDP + (-1.107 \times GDP) \times F_1(GDP) + (1.510 \times GDP) \times F_2(GDP)
\]
\[
(6.945) \quad (-18.45) \quad (13.84)
\]

Classification of regimes

- $SO_2 = 0.281 \times GDP$, when $F_1(GDP) = 0 \& F_2(GDP) = 0$
- $SO_2 = -0.826 \times GDP$, when $F_1(GDP) = 1 \& F_2(GDP) = 0$
- $SO_2 = 0.684 \times GDP$, when $F_1(GDP) = 1 \& F_2(GDP) \approx 1$

\[
\hat{c}_1 = \$3,368\text{ (antilog of 8.122)}, \quad \hat{\gamma}_1 = 3
\]
\[
\hat{c}_2 = \$17,292\text{ (antilog of 9.758)}, \quad \hat{\gamma}_2 = 5
\]

$R$-sq = 0.1706

Fixed-state effects STR model for $NO_x$

\[
NO_x = 0.579 \times GDP + (1.251 \times GDP) \times F_1(GDP) + (-1.496 \times GDP) \times F_2(GDP)
\]
\[
(42.81) \quad (8.721) \quad (-8.863)
\]

Classification of regimes

- $NO_x = 0.579 \times GDP$, when $F_1(GDP) = 0 \& F_2(GDP) = 0$
- $NO_x = 1.830 \times GDP$, when $F_1(GDP) = 1 \& F_2(GDP) = 0$
- $NO_x = 0.334 \times GDP$, when $F_1(GDP) = 1 \& F_2(GDP) = 1$

\[
\hat{c}_1 = \$6,608\text{ (antilog of 8.796)}, \quad \hat{\gamma}_1 = 5
\]
\[
\hat{c}_2 = \$9,284\text{ (antilog of 9.136)}, \quad \hat{\gamma}_2 = 2
\]

$R$-sq = 0.5126

Notes: Models are estimated in logarithmic levels; values in parentheses are t-ratios.
Table 3: Single-threshold STR models with dummy for high-income states

### Fixed-state effects STR model for SO₂

\[
SO_2 = 0.216 \times GDP + 0.261 \times GDP \times Dh + (-0.815 \times GDP - 0.670 \times GDP \times Dh) \times F_1(GDP) + (0.885 \times GDP - 0.348 \times GDP \times Dh) \times F_2(GDP)
\]

<table>
<thead>
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<th>Coefficient</th>
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<th>p-value</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
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<td>F_2(GDP)</td>
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<td>0.016</td>
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</tbody>
</table>

Classification of regimes:

- \( SO_2 = 0.216 \times GDP \), when \( F_1(GDP) = 0 \) & \( F_2(GDP) = 0 \) low income states
- \( SO_2 = 0.477 \times GDP \), when \( F_1(GDP) = 0 \) & \( F_2(GDP) = 0 \) high income states
- \( SO_2 = -0.599 \times GDP \), when \( F_1(GDP) = 1 \) & \( F_2(GDP) = 0 \) low income states
- \( SO_2 = -1.008 \times GDP \), when \( F_1(GDP) = 1 \) & \( F_2(GDP) = 0 \) high income states
- \( SO_2 = 0.286 \times GDP \), when \( F_1(GDP) = 1 \) & \( F_2(GDP) = 1 \) low income states
- \( SO_2 = -0.471 \times GDP \), when \( F_1(GDP) = 1 \) & \( F_2(GDP) = 1 \) high income states

\( \hat{\gamma}_1 = $3,368 \) (antilog of 8.122), \( \hat{\gamma}_2 = 3 \)

\( \hat{\gamma}_2 = $15,063 \) (antilog of 9.620), \( \hat{\gamma}_2 = 11 \)

R-sq = 0.1988

### Fixed-state effects STR model for NOₓ

\[
NO_x = 0.535 \times GDP - 0.183 \times GDP \times Dh + (0.731 \times GDP - 0.515 \times GDP \times Dh) \times F_1(GDP) + (-0.869 \times GDP + 0.430 \times GDP \times Dh) \times F_2(GDP)
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>30.51</td>
<td>0.000</td>
<td>GDP*Dh</td>
<td>-5.282</td>
<td>0.000</td>
</tr>
<tr>
<td>F_1(GDP)</td>
<td>-11.81</td>
<td>0.000</td>
<td>F_2(GDP)</td>
<td>3.668</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Classification of regimes:

- \( NO_x = 0.535 \times GDP \), when \( F_1(GDP) = 0 \) & \( F_2(GDP) = 0 \) low income states
- \( NO_x = 0.352 \times GDP \), when \( F_1(GDP) = 0 \) & \( F_2(GDP) = 0 \) high income states
- \( NO_x = 1.266 \times GDP \), when \( F_1(GDP) = 1 \) & \( F_2(GDP) = 0 \) low income states
- \( NO_x = 0.568 \times GDP \), when \( F_1(GDP) = 1 \) & \( F_2(GDP) = 0 \) high income states
- \( NO_x = 0.397 \times GDP \), when \( F_1(GDP) = 1 \) & \( F_2(GDP) = 1 \) low income states
- \( NO_x = 0.129 \times GDP \), when \( F_1(GDP) = 1 \) & \( F_2(GDP) = 1 \) high income states

\( \hat{\gamma}_1 = $5,464 \) (antilog of 8.606), \( \hat{\gamma}_1 = 2 \)

\( \hat{\gamma}_2 = $13,508 \) (antilog of 9.511), \( \hat{\gamma}_2 = 4 \)

R-sq = 0.5558

**Notes:** Models are estimated in logarithmic levels; the dummy \( Dh \) indicates whether a state’s per capita GDP is above or below the average per capita GDP of all states in each year from 1929 to 1994; the values in parentheses are t-ratios.
Graph 1: Transition function $F$ of single-threshold $SO_2$ (upper panel) and $NO_x$ (lower panel) models versus $GDP$ (in logarithms).

Graph 2: Transition functions $F_1$ and $F_2$ of double-threshold $SO_2$ (upper panel) and $NO_x$ (lower panel) models versus $GDP$ (in logarithms).
Graph 3: Transition functions $F_1$ and $F_2$ of double-threshold (with high-income states dummy) $SO_2$ (upper panel) and $NO_x$ (lower panel) models versus GDP (in logarithms).